Example nº 7
Prestressed Beam

CivilFEM Manual of Advanced Examples
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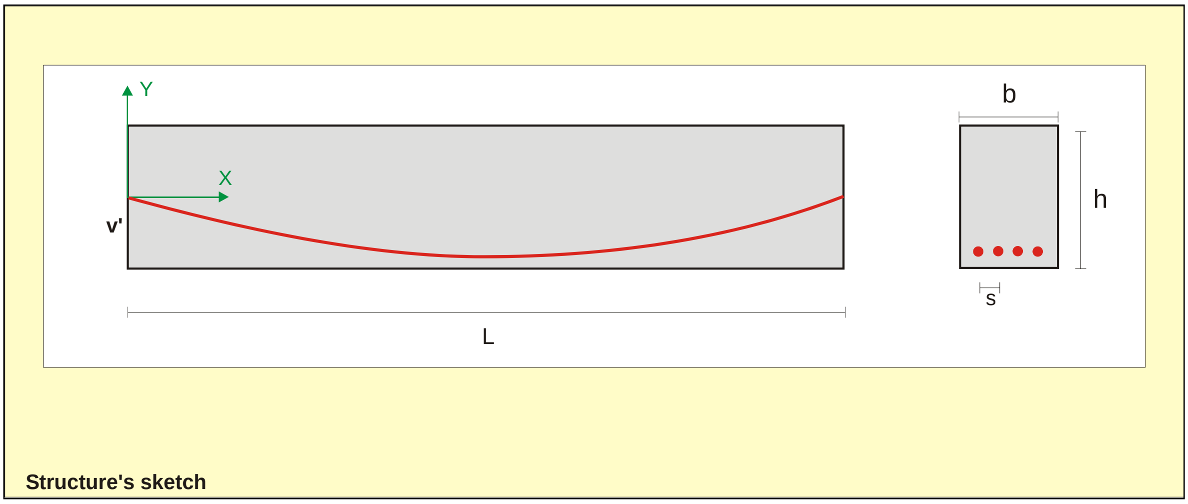
7 EXAMPLE Nº 7: PRESTRESSED BEAM

7.1 AIM

This aim of this example, first of a series of four dedicated to prestressed concrete, is to serve as an introduction to this complex subject, solving a simple case which consists of an isostatic beam that is reinforced by means of a system of active reinforcement, whose losses are to obtain, according to the EHE code.

7.2 DESCRIPTION OF THE EXAMPLE

The beam of the attached figure, that has a length of L = 25.0 m



is prestressed with 4 post-tensioned tendons T 5 ϕ 0,6" (A_p = 28 cm^2) whose layout, referred to the axes of the figure, is the parabolic equation

$$y = 0.0032\,x^2 - 0.08\,x$$

The section dimensions are b = 0.60 m, h = 1.25 ms, s = 0.13 m.

The properties of the used materials are:

Concrete

Type	HA-45 (EHE code)	
Elasticity modulus (E_c)	40000	MPa
Shrinkage final strain (ε_{cs})	-0.0003	
Creep coefficient (φ)	2	
Aging coefficient (χ)	0.8	
Creep stress	5	MPa

Active reinforcement

Prestressing initial tension before anchoring	2200	kN
Anchorage slip	2	mm
Type of tensioning	Two by two, from both ends	
Maximum strength (f_{pmax})	1860	MPa
Characteristic strength, elastic limit (f_{pyk})	1700	MPa
Elasticity modulus (E_p)	200000	MPa
Friction coefficient (μ)	0.2	
Equivalent friction (k/μ)	0.015	rad/m

The size of the tendons casing is ignored.

7.3 RESULTS TO BE OBTAINED

Calculate with ANSYS + CivilFEM the short term and long term prestressin losses, at the central sections and supports, according to the Spanish concrete code EHE. Compare the results with the analytical solution shownin the following table

	Support		Center of span	
	kN	%	kN	%
Initial prestressing force	2200.00	100.0	2200.00	100.0
Friction ΔP_1	0.00	0.0	114.61	5.2
Anchorage slip ΔP_2	202.98	9.2	0.00	0.0
Elastic deformation ΔP_3	5.27	0.2	16.28	0.7
Concrete shrinkage ΔP_4	163.25	7.4	154.83	7.0
Creep ΔP_5	136.04	6.2	129.03	5.9
Steel relaxation ΔP_6	0.00	0.0	0.00	0.0
Short term total losses	208.25	9.5	130.89	5.9
Long term total losses	299.24	13.6	283.86	12.9
Total losses	507.54	23.1	414.75	18.9
Final prestressing force	1692.46	76.9	1785.25	81.1

7.4 ANALYTICAL RESOLUTION

7.4.1 Friction losses

They are obtained from the following expression:

$$\Delta P_1 = P_0 \left[1 - e^{-\mu \cdot \alpha - K \cdot x} \right]$$

At the center of the span x = 12.5m and α = y'(12.5)-y'(0) = 0.08. Entering these values in the previous equation ΔP_1 = 114,61 kN is obtained.

It is obvius that in the section at the support α and x are null, and therefore $\Delta P_{1(L/2)}$ = 0 kN.

7.4.2 Anchorage slip losses

Being *a* the anchorage slip:

$$a = \int_0^l \frac{\Delta P(x)}{E_P \cdot A_P} dx$$

Assuming a linear distribution for the tendon stresses, and an affection length of the anchorage slip of l_p, the previous expression becomes:

$$a \approx \frac{\Delta P_2 \cdot l_P}{E_P \cdot A_P}$$

Where ΔP_2 is the anchorage slip loss.

$$\Delta P_2 = 2(P_0 - \Delta P_1)\left[1 - e^{-\mu \cdot \alpha - K \cdot l_P}\right]$$

Isolating l_p of the previous expressions: l_p = 5.52 m and ΔP_2 = 202.98 kN.

As the affection length of the anchorage slip exceed the center of the span (l_p < L/2), it is not necessary to calculate the anchorage slip losses at the center of the span, because they are null.

7.4.3 Losses by elastic shortening of concrete

Assuming a uniform shortening in the tendons, the losses by elastic shortening of concrete are obtained by the expression:

$$\Delta P_3 = \sigma_{cp} \frac{(n-1) \cdot A_p \cdot E_p}{2 \cdot n \cdot E_c}$$

where n is the number of prestressing sets (2 in this case).

In the support section the only load is the one due to prestressing, which acts at the center of gravity of the section, reason why

$$\sigma_{cp} = -\frac{P_0 - \Delta P_1 - \Delta P_2}{A_c} = -5.53 \text{ MPa}$$

And therefore ΔP_3 = -19.355 kN

In the section located at the center of span, the prestressing load is not applied at the center of gravity. The existing eccentricity (e) will produce a bending moment that must be considered for the calculation of σ_{cp}. Self weight also introduces a bending moment (M_w) in the section:

$$\sigma_{cp} = -\frac{P_0 - \Delta P_1 - \Delta P_2}{A_c} - \frac{(P_0 - \Delta P_1 - \Delta P_2)\cdot e^2}{I_c} - \frac{M_w \cdot e}{I_c} = -8.55\ \text{MPa}$$

And therefore $\Delta P_{3(L/2)}$ = -29.925 kN

7.4.4 Long term losses

The long term losses are calculate using the formulation of the EHE code.

$$\Delta P_{4,5,6} = \frac{\frac{E_p}{E_c}\cdot \varphi \cdot \sigma_{cpg} + E_p \cdot \varepsilon_{cs} + 0.8 \cdot \Delta\sigma_{pr}}{1 + \frac{E_p}{E_c}\cdot\frac{A_p}{A_c}\cdot\left(1 + \frac{A_c \cdot y_p^2}{I_c}\right)\cdot(1 + \chi \cdot \varphi)} \cdot A_p$$

being able to detach the previous expression in each one of the three long term losses:

$$\Delta P_5 = \frac{\frac{E_p}{E_c}\cdot \varphi \cdot \sigma_{cpg}}{1 + \frac{E_p}{E_c}\cdot\frac{A_p}{A_c}\cdot\left(1 + \frac{A_c \cdot y_p^2}{I_c}\right)\cdot(1 + \chi \cdot \varphi)} \cdot A_p$$

$$\Delta P_4 = \frac{E_p \cdot \varepsilon_{cs}}{1 + \frac{E_p}{E_c}\cdot\frac{A_p}{A_c}\cdot\left(1 + \frac{A_c \cdot y_p^2}{I_c}\right)\cdot(1 + \chi \cdot \varphi)} \cdot A_p$$

$$\Delta P_6 = \frac{0.8 \cdot \Delta\sigma_{pr}}{1 + \frac{E_p}{E_c}\cdot\frac{A_p}{A_c}\cdot\left(1 + \frac{A_c \cdot y_p^2}{I_c}\right)\cdot(1 + \chi \cdot \varphi)} \cdot A_p$$

The losses due to steel relaxation are not considered, therefore ΔP_6 = 0 kN.

The section at the supports will have ΔP_4 = 163.25 kN and ΔP_5 = 136.04 kN, whereas for the section at the center of span $\Delta P_{4(L/2)}$ = 129.03 kN and $\Delta P_{5(L/2)}$ = 129.03 kN.

7.5 CALCULATION LOG

7.5.1 Introduction

All the preprocess of the example has been arranged in a single LOG file. The materials are defined as *library materials*, and are later transformed into *user materials*, thus to be able to change code properties that otherwise would be blocked.

Since it is a 2D model it has been chosen to use a BEAM3 type of beam.

The geometry of the tendon is described by an analytical formula, but since it is a vertical axis parabola, it is possible to exactly represent it by means of Bèzier curves, defining the height and slope of the tendon at the ending points. These values are:

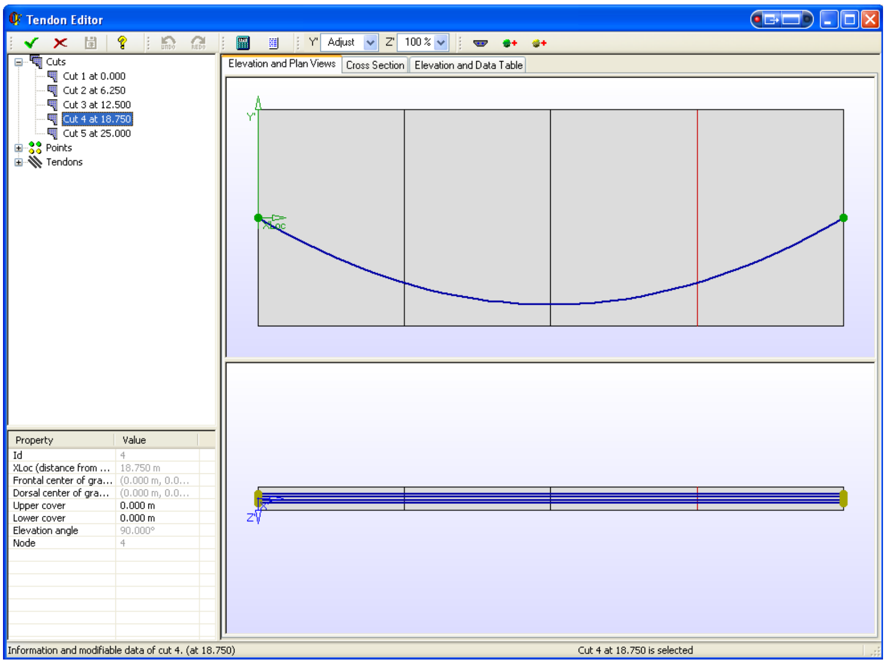
	Height (meters)	Slope
Starting point (x = 0.00)	0.00	-0.08 (downwards)
Ending point (x = 25.00)	0.00	0.08 (upwards)

For the resolution of this example it is not necessary to solve the model. Nevertheless boundary conditions of have been arranged in case the reader wishes to continue with the analysis of the prestressed beam.

7.5.2 Log

```
FINISH
~CFCLEAR,,1
~CODESEL,,EHE,EHE
~UNITS,SI

/PREP7
! Parameters
b  = 0.60    ! Section width (m)
h  = 1.25    ! Section height (m)
s  = 0.13    ! Distance between tendons (m)
L  = 25.00   ! Beam length (m)
p0 = 2200e3  ! Total prestressing force (N)
Ec = 4e10    ! Concrete Elasticity modulus (Pa)
Cs = 5e6     ! Creep stress (Pa)
Ep = 2e11    ! Prestr. steel Elasticity modulus (Pa)
Ecs = 3e-4   ! Final creep strain
Anch = 2e-3  ! Anchorage slip (m)
Ap = 28e-4   ! Total prestressing steel area (m)
Mu = 0.2     ! Friction oefficient
K  = 0.015   ! Equivalent friction (rad/m)
CC = 2       ! Creep Coefficient
Chi = 0.8    ! Ageing Coefficient.
Fpmax = 1860e6 ! Character. tensile strength (Pa)
Fpyk  = 1700e6 ! Character. yield stress (Pa)

! Material definition
~CFMP,1,LIB,CONCRETE,EHE,HA-45
~CFMP,2,LIB,PREST,EHE,Y1860S7
~CFMP,1,USER
~CFMP,2,USER
~CFMP,1,Concr,ExLn   ,,Ec
~CFMP,1,EHE_C,E0j    ,,Ec,3
~CFMP,1,EHE_C,Ej     ,,Ec,3
~CFMP,2,USER
~CFMP,2,DatGen,Ex    ,,Ep
~CFMP,2,Prest,A      ,,Anch
~CFMP,2,Prest,EPSsr  ,,Ecs
~CFMP,2,Prest,Mu     ,,Mu
~CFMP,2,Prest,K      ,,K*Mu
~CFMP,2,Prest,PHI    ,,CC
~CFMP,2,EHE_Pres,fpk,,Fpyk
~CFMP,2,EHE_Pres,fmax,,Fpmax

! Beam properties definition
~CSECDMS,1,REC,b,h,1
~BMSHPRO,1,BEAM,1,1,,,3,1,1,Beam 1

! Nodes
N,1 $ N,2,L/4 $ N,3,L/2 $ N,4,3*L/4 $ N,5,L
```

```
! Elements
MAT,1 $ ET,1,BEAM3 $ TYPE,1 $ REAL,1
E,1,2 $ E,2,3 $ E,3,4 $ E,4,5

! Boundary conditions
D,1,UX $ D,1,UY
D,5,UY

! Support Beam
~SBBMDEF

! Tendon Geometry
~PCEPDEF,1,1,0,-0.08
~PCEPDEF,2,5,0,0.08
~PCPPDEF,1,1,-s/2,0
~PCPPDEF,2,5,-s/2,0
~PCPPDEF,3,1,-s-s/2,0
~PCPPDEF,4,5,-s-s/2,0
~PCPPDEF,5,1,s/2,0
~PCPPDEF,6,5,s/2,0
~PCPPDEF,7,1,s+s/2,0
~PCPPDEF,8,5,s+s/2,0
~PCTNDEF,1,2,Ap/4,0.001,p0/4,p0/4,Cs
~PCTNDEF,2,2,Ap/4,0.001,p0/4,p0/4,Cs
~PCTNDEF,3,2,Ap/4,0.001,p0/4,p0/4,Cs
~PCTNDEF,4,2,Ap/4,0.001,p0/4,p0/4,Cs
~PCTNMDF,1,ORDER,2
~PCTNMDF,3,ORDER,2
~PCTNMDF,1,EADD,1
~PCTNMDF,1,EADD,2
~PCTNMDF,2,EADD,1
~PCTNMDF,2,EADD,2
~PCTNMDF,3,EADD,1
~PCTNMDF,3,EADD,2
~PCTNMDF,4,EADD,1
~PCTNMDF,4,EADD,2
~PCTNMDF,1,PADD,1
~PCTNMDF,1,PADD,2
~PCTNMDF,2,PADD,3
~PCTNMDF,2,PADD,4
~PCTNMDF,3,PADD,5
~PCTNMDF,3,PADD,6
~PCTNMDF,4,PADD,7
~PCTNMDF,4,PADD,8

! Losses Calculation
~PCLOSS,1000000,1,EHE,Chi

! List losses
~PCTNLST,0,0,1,LOSSES,ALL
```



7.6 RESULTS

After performing the calculation, the following values are obtained:

	Apoyo		Centro de vano	
	kN	%	kN	%
Initial prestressing force	2200.00	100.0	2200.00	100.0
Friction ΔP_1	0.00	0.0	109.07	5.0
Anchorage slip ΔP_2	200.67	9.1	0.00	0.0
Elastic deformation ΔP_3	5.60	0.3	17.10	0.8
Concrete shrinkage ΔP_4	163.25	7.4	154.83	7.0
Creep ΔP_5	136.04	6.2	129.03	5.9
Steel relaxation ΔP_6	0.00	0.0	0.00	0.0
Short term total losses	206.27	9.4	126.16	5.7
Long term total losses	299.28	13.6	283.86	12.9
Total losses	505.56	23.0	410.03	18.6
Final prestressing force	1694.44	77.0	1789.98	81.4

As it can be seen there are some differences between the analytical losses calculation and the one given by CivilFEM. The friction losses is slightly different because of the internal discetization that CivilFEM does of each tendon. Although the overall value is correct, the values at certain locations may not be the same because there may be no control point at the location of the tendon where the results are being read. The differences in the calculation of anchorage slip losses is due to the fact that CivilFEM makes a more precise calculation, since it does not consider a linear distribution of stresses in each tendon, as it has been done in the analytical calculation. Regarding the concrete elastic shortening losses, CivilFEM assumes that the tendons are tensioned without any load on the structure, reason why a small difference in this result is appraised. In addition, in the manual analysis, it is assumed that all the tendons have the same uniform loss, whereas CivilFEM does the calculation real assigning to each tendon the loss that.

7.7 SUMMARY

With this example, the reader has learned to define a simple tendon with the tendon editor, or by means of *APDL* commands and to calculate and to interpret the originated tension losses.