Theory Manual

Table of Contents

Chapter 1
General Aspects

1.1. Introduction

Welcome to the Theory Manual for CivilFEM Powered by Marc. This manual presents the theoretical descriptions of every calculation procedure used by the program and describes the relationship between the input data and the results given by CivilFEM. This manual is essential for understanding how the program calculates results as well as how to interpret the results correctly.

The Theory Manual provides the theoretical basis of the algorithms included in the program. With knowledge of the underlying theory, the user can perform analyses efficiently and confidently on CivilFEM by using the capabilities to their full potential while being aware of the limitations.

Reading the whole manual will not be necessary; it is recommended to read only the paragraphs containing the specific algorithms being utilized.

Take into account that there is a online CivilFEM help manual.

1.1.1. References

- MSC. Software Help Documentation

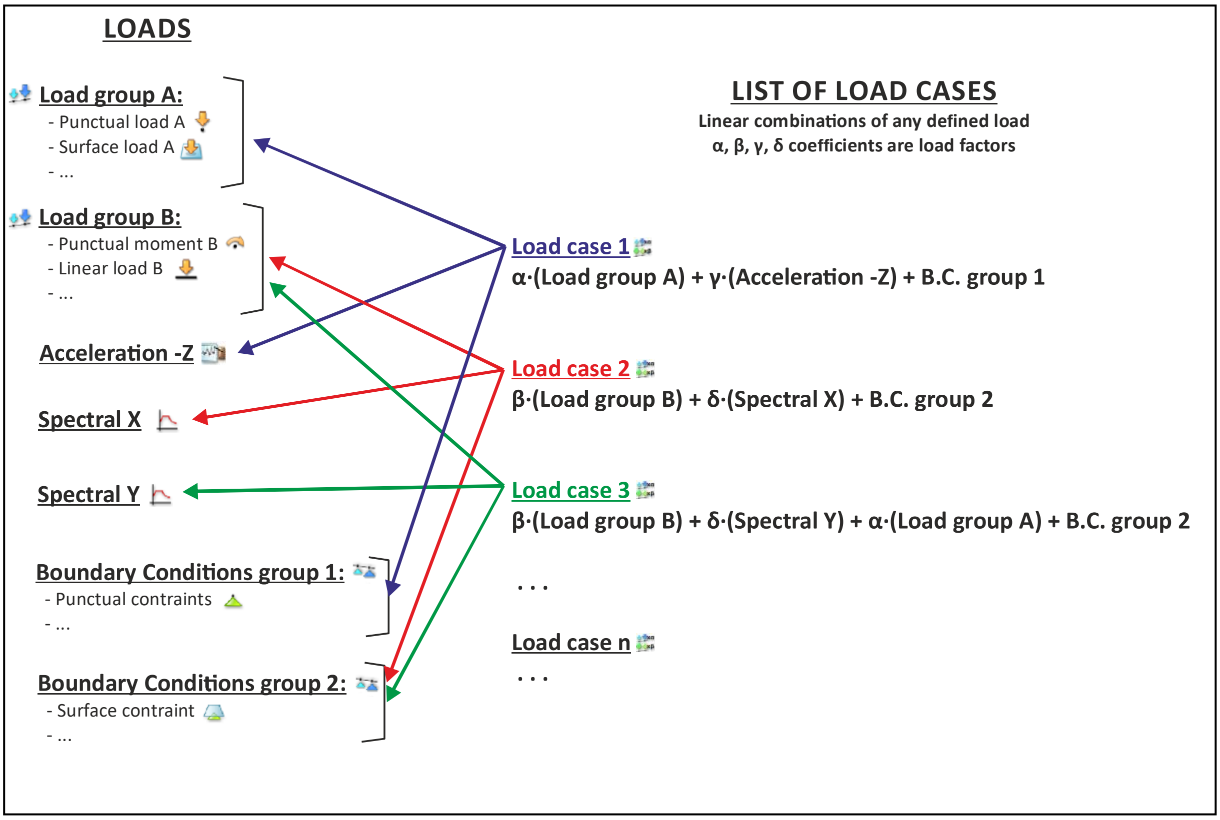
Chapter 2
Loading

2.1. Introduction

Linear finite element analysis is characterized by a force-displacement relationship that only contains linear terms. Linear system of equations always produces a unique solution while in nonlinear analysis does not guarantee a unique solution. In fact, there may be multiple solutions or no solution at all. The task of providing analysis directives (i.e. controls by which the program will come to a solution) is far from simple. Solving nonlinear equations is an incremental and iterative process.

A linear static structural analysis with a known external load can be performed in one step. If nonlinearities are expected, it may be necessary to apply the load in increments and let each load increment iterate to the equilibrium state, within a specified tolerance, using a particular iteration scheme such as Newton-Raphson.

In this CivilFEM Theory Manual version only **linear static loads** are considered. In the figure below there is an example of loading workflow covering all that civil engineering needs.



The word *loads* in CivilFEM terminology includes load groups, accelerations, spectra and boundary conditions.

Chapter 3
Solution

3.1. Linear analysis

Linear analysis is the type of stress analysis performed on linear elastic structures. Because linear analysis is simple and inexpensive to perform and generally gives satisfactory results, it is the most commonly used structural analysis.

Nonlinearities due to material, geometry, or boundary conditions are not included in this type of analysis. The behavior of an isotropic, linear, elastic material can be defined by two material constants: Young's modulus, and Poisson's ratio.

CivilFEM allows user to perform linear elastic analysis using any element type in the program. Various kinematic constraints and loadings can be prescribed to the structure being analyzed; the problem can include both isotropic and anisotropic elastic materials.

The principle of superposition holds under conditions of linearity. Therefore, several individual solutions can be superimposed (summed) to obtain a total solution to a problem.

Linear analysis does not require storing as many quantities as does nonlinear analysis; therefore, it uses the core memory more sparingly.

3.2. Finite Element Technology

This section describes the basic concepts of finite element technology. CivilFEM solver was developed on the basis of the displacement method. The stiffness methodology used addresses force-displacement relations through the stiffness of the system.

The force displacement relation for a linear static problem can be expressed as:

$$Ku = f$$

Where is the system stiffness matrix, is the nodal displacement, and is the force vector.

Assuming that the structure has prescribed boundary conditions both in displacements and forces, the governing equation can be written as:

$$\begin{bmatrix} K_{11} K_{12} \\ K_{21} k_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}$$

u_1 is the unknown displacement vector, f_1 is the prescribed force vector,u_2 is the prescribed displacement vector, and f_2 is the reaction force. After solving for the displacement vector, the strains in each element can be calculated from the strain-displacement relation in terms of element nodal displacement as:

$$\varepsilon_{el} = \beta u_{el}$$

The stresses in the element are obtained from the stress-strain relations as:

$$\sigma_{el} = L\varepsilon_{el}$$

Where σ_{el} and ε_{el} are stresses and strains in the elements, and u_{el} is the displacement vector associated with the element nodal points; β and L are strain-displacement and stress-strain relations, respectively.

In a dynamic problem, the effects of mass and damping must be included in the system. The equation governing a linear dynamic system is:

$$M\ddot{u} + D\dot{u} + Ku = f$$

Where M is the system mass matrix, D is the damping matrix, following equation is the acceleration vector, and is the velocity vector. The equation governing an undamped dynamic system is:

$$M\ddot{u} + Ku = f$$

The equation governing undamped free vibration is:

$M\ddot{u} + Ku = 0$

Natural frequencies and modal shapes of the structural system are calculated using this equation.

$$K\emptyset - \omega^2 M\emptyset = 0$$

3.3. Dynamic analysis

CivilFEM's dynamic analysis capability allows the user to perform the following calculations:

1. Modal analysis.
2. Harmonic analysis.
3. Spectrum analysis.
4. Transient analysis.

Damping and nonlinear effects, including material nonlinearity, and boundary nonlinearity, can be incorporated. All nonlinear problems should be analyzed using direct integration methods.

3.3.1. Modal Analysis

CivilFEM uses the Lanczos method to extract eigenvalues (natural frequencies) and eigenvectors (mode shapes), optimal for several modes. After the modes are extracted, they can be used in a transient analysis or spectrum response calculation.

In dynamic eigenvalue analysis, we find the solution to an undamped linear dynamics problem:

$$(\mathrm{K}-\omega^2\mathrm{M})\emptyset = 0$$

Where K is the stiffness matrix, M is the mass matrix, ω are the eigenvalues (frecuencies) and Ø are the eigenvectors. In CivilFEM, if the extraction is performed after increment zero, K is the tangent stiffness matrix, which can include material and geometrically nonlinear contributions. The mass matrix is formed from both distributed mass and point masses.

The Lanczos algorithm converts the original eigenvalue problem into the determination of the eigenvalues of a tri-diagonal matrix. The method can be used either for the determination of all modes or for the calculation of a small number of modes. For the latter case, the Lanczos method is the most efficient eigenvalue extraction algorithm. A simple description of the algorithm is as follows. Consider the eigenvalue problem:

$$-\omega^2\mathrm{M}\,\mathrm{u} + \mathrm{K}\,\mathrm{u} = 0$$

Previous equation can be rewritten as:

$$\frac{1}{\omega^2}\mathrm{M}\,\mathrm{u} = \ \mathrm{M}\,\mathrm{K}^{-1}\mathrm{M}\,\mathrm{u}$$

Consider the transformation:

$$u = Q\,\eta$$

Substituting last equation into previous one and premultiplying by the matrix Q^T on both sides of the equation, we have:

$$\frac{1}{\omega^2} Q^T M\, Q\, \eta = Q^T M\, K^{-1} M\, Q\, \eta$$

The Lanczos algorithm results in a transformation matrix Q such that:

$$Q^T\, M\, Q = I$$

$$Q^T\, M\, K^{-1} M Q = T$$

where the matrix T is a symmetrical tri-diagonal matrix of the form:

$$\underline{T} = \begin{bmatrix} \alpha_1 \beta_2 & & 0 \\ \beta_2 \alpha_2 \beta_3 & & \\ & & \beta m \\ 0 & \beta_m \alpha_m & \end{bmatrix}$$

Consequently, the original eigenvalue problem is reduced to the following new eigenvalue problem:

$$\frac{1}{\omega^2} \eta = T\, \eta$$

The eigenvalues can be calculated by the standard QL-method.

Within CivilFEM it can be selected either the number of modes to be extracted, or a range of modes to be extracted. The Sturm sequence check can be used to verify that all of the required eigenvalues have been found.

In addition, user can select the lowest frequency to be extracted to be greater than zero.

Eigenvalue extraction is controlled by the maximum number of iterations for all modes in the Lanczos iteration method in convergence controls.

3.3.1.1. Modal stresses and reactions

After the modal shapes (and frequencies) are extracted, it is allowed to recover stresses and reactions for a specified number of modes during a modal or a buckling analysis.

The stresses are computed from the modal displacement vector ϕ ; the nodal reactions are calculated from:

$$F = K\phi - \omega^2 M\phi$$

The nodal vector of modal mass is calculated as m = Mϕ.

3.3.1.2. Participation factors and effective modal masses

The participation factor for a given mode is defined as

$$c_{nj} = \left(\{\Phi_{n,i}\}^T [M]\{T_{i,j}\}\right) / \left(\left(\{\Phi_{n,i}\}^T [M]\right)\{\Phi_{n,i}\}\right)$$

Where:

c_{nj}	is the participation factor for mode *n* in the *j*th direction.
$\{\Phi_{n,i}\}$	is the eigenvector value for mode n and degree of freedom *i.*
$[M]$	is the mass matrix.
$\{T_{i,j}\}$	Defines the magnitude of the rigid body response of degree of freedom *i* to impose rigid body motion in the jth direction and takes the following form:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & (Z-Z_0) & -(Y-Y_0) \\ 0 & 1 & 0 & -(Z-Z_0) & 0 & (X-X_0) \\ 0 & 0 & 1 & (Y-Y_0) & -(X-X_0) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \{e_j\}$$

Where:

X, Y, and Z	are the coordinates of the respective node.
X_0, Y_0, Z_0	are the coordinates of center of rotation.
e_j	is the unit vector (carrying 1 for row j and the rest being zeros)

The effective modal masses are calculated as squares of the participation factors.

$$m_{n,j}^{eff} = [c_{n,j}]^2$$

Where $m_{n,j}^{eff}$ is the effective modal mass for mode *n* in the j^{th} direction.

While the nodal vector of modal masses gives the significance of mass participation of the node for the given mode in the given direction, the effective modal mass gives an idea about the mass contribution of the whole structure (or model) for the mode in the given direction.

3.3.2. Harmonic Analysis

Any sustained cyclic load will produce a sustained cyclic response in a structural system. Harmonic response analysis gives the ability to predict the sustained dynamic behavior of structures, thus enabling to verify whether or not designs will successfully overcome resonance, fatigue, and other harmful effects of forced vibrations.

Harmonic response analysis allows to analyze structures vibrating around an equilibrium state. This equilibrium state can be unstressed or statically prestressed. Statically prestressed equilibrium states can include material and/or geometric nonlinearities. User can compute the damped response for prestressed structures at various states.

In many practical applications, components are dynamically excited. These dynamic excitations are often harmonic and usually cause only small amplitude vibrations. CivilFEM linearizes the problem around the equilibrium state. If the equilibrium state is a nonlinear, statically prestressed situation, CivilFEM considers all effects of the nonlinear deformation on the dynamic solution. These effects include the following:

- Initial stress.
- Change of geometry.
- Influence on constitutive law.

The vibration problem can be solved as a linear problem using complex arithmetic.

The analytical procedure consists of the following steps:

1. CivilFEM calculates the response of the structure to a static preload (which can be nonlinear) based on the constitutive equation for the material response. In this portion of the analysis, the program ignores inertial effects.

2. CivilFEM calculates the complex-valued amplitudes of the superimposed response for each given frequency, and amplitude of the boundary tractions and/or displacements. In this portion of the analysis, the program considers both material behavior and inertial effects.

3. You can apply different loads with different frequencies or change the static preload at your discretion. All data relevant to the static response is stored during calculation of the complex response.

3.3.2.1. Small amplitude vibration problem

The small amplitude vibration problem can be written with complex arithmetic as follows:

$$[K + i\omega D - \omega^2 M]\bar{u} = \bar{P}$$

Where:

$\bar{u} = u_{re} + iu_{im}$ is the complex response vector,

$\overline{P} = P_{re} + iP_{im}$ is the complex load vector,

$i = \sqrt{-1}$,

ω is the excitation frequency.

$$K = \sum K_{el} + \sum K_{sp}$$

Where:

K_{el} are element stiffness matrices,

K_{sp} are the spring stiffness matrices.

$$M = \sum M_{el} + \sum M_{mp}$$

Where:

M_{el} are element mass matrices,

M_{mp} are mass point contributions

$$D = \sum D_{el} + \sum D_d + \sum \left(\alpha M + \left(\beta + \frac{2\gamma}{\omega}\right) K\right)$$

Where:

D_{el} are element damping matrices,

D_d are damper contributions,

α is the mass damping coefficient,

β is the stiffness damping coefficient,

γ is the numerical damping coefficient.

If all external loads and forced displacements are in phase and the system is undamped, this equation reduces to:

$$(K - \omega^2 M) u_{re} = P_{re}$$

The element damping matrix (D_{el}) can be obtained for any material with the use of a material damping matrix which allows the user to input a real (elastic) and imaginary (damping) stress-strain relation. The material response is specified with the constitutive equation.

$$\sigma = B\varepsilon + C\dot{\varepsilon}$$

Where B and C can be functions of deformation and/or frequency.

The global damping matrix is formed by the integrated triple product. The following equation is used:

$$D = \sum_{el} \int \beta^T \, C dV_{el}$$

Where β is the strain-displacement relation.

Similarly, the stiffness matrix K is based on the elastic material matrix B.

The output of CivilFEM consists of stresses, strains, displacements and reaction forces, all of which may be complex quantities. The strains are given by

$$\bar{\varepsilon} = \beta \, \bar{u}$$

and the stresses by

$$\sigma = B\bar{\varepsilon} + C\dot{\bar{\varepsilon}}$$

The reaction forces are calculated with

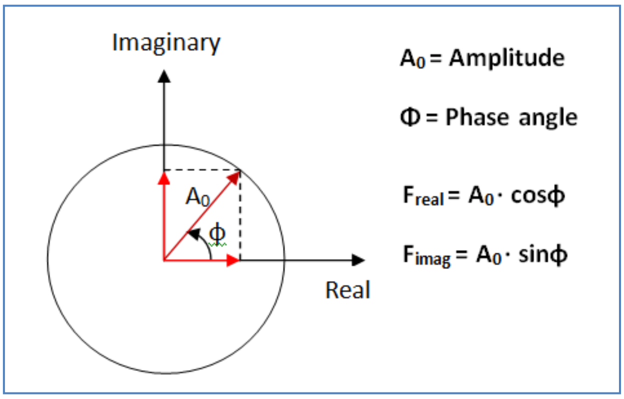
$$\bar{R} = K\bar{u} - \omega^2 M\bar{u} + i\omega \sum D_{el}\bar{u} + i\omega \sum D_d \bar{u}$$

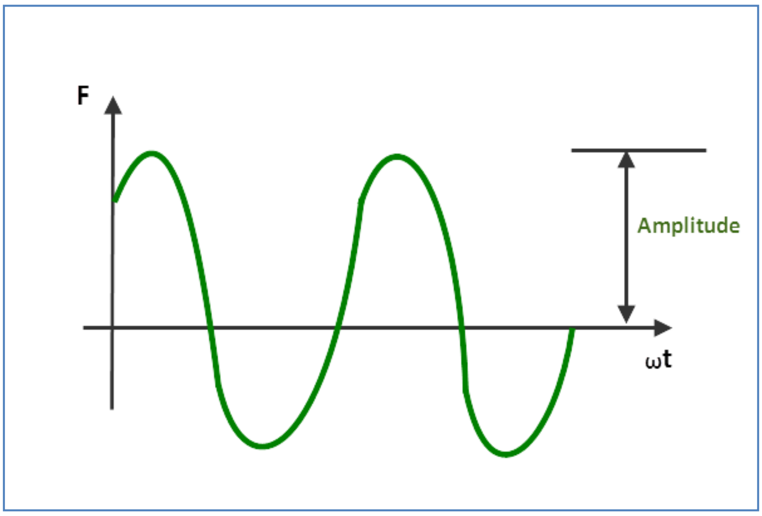
The printout of the nodal values consists of the real and imaginary parts of the complex values, but you can request that the amplitude and phase angle be printed.

3.3.2.2. Performing a Harmonic Response Analysis

Harmonic response analysis is a technique used to determine the steady-state response of a linear structure to loads that vary sinusoidally (harmonically) with time. The idea is to calculate the structure's response at several frequencies and obtain a graph of some response quantity (usually displacements) versus frequency.

A harmonic analysis, by definition, assumes that any applied load varies harmonically (sinusoidally) with time. To completely specify a harmonic load, three pieces of information are usually required: the amplitude, the phase angle, and the forcing frequency range.





$$A_0 = \sqrt{F_{real}^2 + F_{imag}^2}$$

$$\Phi = tan^{-1}\left({F_{imag}}/{F_{real}} \right)$$

The amplitude is the maximum value of the load.

The phase angle is a measure of the time by which the load lags (or leads) a frame of reference. On the complex, it is the angle measured from the real axis. The phase angle is required only if you have multiple loads that are out of phase with each other.

Increments in frequency can be linear or logarithmic:

- If linear:

$$n = (\omega_{HIGH} - \omega_{LOW})/(\Delta\omega)$$

$$\Delta\omega_\mathrm{i} = \text{entered value of} \frac{(\omega_{HIGH} - \omega_{LOW})}{n} - 1$$

- If logarithmic increments in frequency:

$$fac = \left(\frac{\omega_{HIGH}}{\omega_{LOW}}\right)^{\left(\frac{1}{n}-1\right)}$$

$$\Delta\omega_{\mathrm{i}} = \mathrm{fac} ** (i-1)$$

3.3.3. Spectrum Analysis

The spectrum response capability allows obtaining maximum response of a structure subjected to know spectral base excitation response. This is of particular importance in earthquake analysis and random vibration studies. You can use the spectrum response option at any point in a nonlinear analysis and, therefore, ascertain the influence of material nonlinearity or initial stress.

The spectrum response capability technique operates on the eigenmodes previously extracted to obtain the maximum nodal displacements, velocities, accelerations, and reaction forces. You can choose a subset of the total modes extracted by either specifying the lowest n modes or by selecting a range of frequencies.

Enter the displacement response spectrum $\mathrm{S_D}(\omega)$ for a particular digitized value of damping through the RESPONSE SPECTRUM model definition. CivilFEM performs the spectrum analysis based on the latest set of modes extracted. The program lumps the mass matrix to produce $\overline{\mathrm{M}}$. It hen obtains the projection of the inertia forces onto the mode $\emptyset_{\mathrm{j}}$

$$\mathrm{P_j} = \overline{\mathrm{M}}\emptyset_{\mathrm{j}}$$

The spectral displacement response for the $\mathrm{j^{th}}$ mode is

$$\alpha_{\mathrm{j}} = \mathrm{S_D}(\omega_{\mathrm{j}})\mathrm{P_j}$$

CivilFEM then calculates the square roots of the sum of the squares as

$$\mathrm{u} = \left[\textstyle\sum_{\mathrm{j}}\left(\alpha_{\mathrm{j}}\emptyset_{\mathrm{j}}\right)^2\right]^{1/2}$$ DISPLACEMENT

$$\mathrm{v} = \left[\textstyle\sum_{\mathrm{j}}\left(\alpha_{\mathrm{j}}\omega_{\mathrm{j}}\emptyset_{\mathrm{j}}\right)^2\right]^{1/2}$$ VELOCITY

$$\mathrm{a} = \left[\textstyle\sum_{j}\left(\alpha_j\omega_j^2\emptyset_j\right)^2\right]^{1/2}$$ ACCELERATION

$$\mathrm{f} = \left[\textstyle\sum_{\mathrm{j}}\left(\alpha_{\mathrm{j}}\omega_{\mathrm{j}}^2\overline{\mathrm{M}}\emptyset_{\mathrm{j}}\right)^2\right]^{1/2}$$ FORCE

The internal forces given by Force equation are identified as reaction forces on the post file. The force transmitted by the structure to the supporting medium (also referred to as base shear) is only reported in the out file and is given by

$tf = \sum_j S_D\left(\omega_j\right)\omega_j^2 P_j^2$ TRANSMITTED FORCE

3.3.4. Transient Analysis

Transient dynamic analysis deals with an initial-boundary value problem. In order to solve the equations of motion of a structural system, it is important to specify proper initial and boundary conditions. You obtain the solution to the equations of motion by using either modal superposition (for linear systems) or direct integration (for linear or nonlinear systems). In direct integration, selecting a proper time step is very important. For both methods, you can include damping in the system.

The following sections discuss the seven aspects of transient analysis listed below.

1. Direct Integration
2. Time Step Definition
3. Initial Conditions
4. Time-Dependent Boundary Conditions
5. Mass Matrix
6. Damping

3.3.4.1. Direct Integration

Direct integration is numerical method for solving the equations of motion of a dynamic system. It is used for both linear and nonlinear problems. In nonlinear problems, the nonlinear effects can include geometric, material, and boundary nonlinearities. For transient analysis, CivilFEM offers two direct integration operators listed below.

a) Newmark-beta Operator
b) Generalized-Alpha Operator

Consider the equations of motion of a structural system:

$$Ma + Cv + Ku = F$$

where M, C, and K are mass, damping, and stiffness matrices, respectively, and a, v, u, and F are acceleration, velocity, displacement, and force vectors. Various direct integration operators can be used to integrate the equations of motion to obtain the dynamic response of the structural system. The technical background of the two direct integration operators available in CivilFEM is described below.

a) Newmark-Beta Operator

This operator is probably the most popular direct integration method used in finite element analysis. For linear problems, it is unconditionally stable and exhibits no numerical damping. The Newmark-beta operator can effectively obtain solutions for linear and nonlinear problems for a wide range of loadings. The procedure allows for change of time step, so it can be used in problems where sudden impact makes a reduction of time step desirable. This operator can be used with adaptive time step control. Although this method is stable for linear problems, instability can develop if nonlinearities occur. By reducing the time step and/or adding (stiffness) damping, you can overcome these problems.

The generalized form of the Newmark-beta operator is

$$u^{n+1} = u^n + \Delta t v^n + (1/2 - \beta)\Delta t^2 a^n + \beta\Delta t^2 a^{n+1}$$

$$v^{n+1} = v^n + (1-\gamma)\Delta t a^n + \gamma\Delta t a^{n+1}$$

where *superscript*^{n} denotes a value at the nth time step and u, v, and a take on their usual meanings.

The particular form of the dynamic equations corresponding to the trapezoidal rule

$$\Upsilon = 1/2, \quad \beta = 1/4$$

results in

$$\left(\frac{4}{\Delta t^2}M + \frac{2}{\Delta t}C + K\right)\Delta u = F^{n+1} - R^n + M\left(a^n + \frac{4}{\Delta t}v^n\right) + Cv^n$$

where the internal force R is

$$R = \int_v \beta^T \sigma dv$$

Dynamic equation allows implicit solution of the system

$$u^{n+1} = u^n + \Delta u$$

Notice that the operator matrix includes K, the tangent stiffness matrix, Hence, any nonlinearity results in a reformulation of the operator matrix. Additionally, if the time step changes, this matrix must be recalculated because the operator matrix also depends on the time step. It is possible to change the values of γ and β through the global solution controls.

b) Generalized Alpha Operator

One of the drawbacks of the existing implicit operators is the inability to easily control the numerical dissipation. While the Newmark-Beta method has no dissipation and works well for regular vibration problems. A single scheme that easily allows zero/small dissipation for regular structural dynamic problems and high-frequency numerical dissipation for dynamic contact problems is desirable. In (Chung, J. and Hulbert, G.M., "*A time integration algorithm for structural dynamics with improved numerical dissipation: The Generalized-α Method*", Journal of Applied Mechanics, Vol. 60, pp. 371 - 375, June 1993) a Generalized-alpha method has been presented as an unconditionally stable, second-order algorithm that allows user-controllable numerical dissipation. The dissipation is controlled by choosing either the spectral radius S of the operator or alternatively, two parameters α_f and α_m. The choice of the parameters provides a family of time integration algorithms that encompasses the Newmark-Beta and the Hilber-Hughes-Taylor time integration methods as special cases.

The equilibrium equations for the generalized alpha method can be expressed in the form

$$M_a^{n+1+\alpha_m} + C_v^{n+1+\alpha_f} + K_u^{n+1+\alpha_f} = F^{n+1\alpha_f}$$

where

$$u^{n+1+\alpha_f} = (1+\alpha_f)u^{n+1} - \alpha_f u^n$$

$$v^{n+1+\alpha_f} = (1+\alpha_f)v^{n+1} - \alpha_f v^n$$

$$a^{n+1+\alpha_m} = (1+\alpha_m)a^{n+1} - \alpha_m a^n$$

The displacement and velocity updates are identical to those of the Newmark algorithm

$$u^{n+1} = u^n + \Delta t v^n + (1/2 - \beta)\Delta t^2 a^n + \beta \Delta t^2 a^{n+1}$$

$$v^{n+1} = v^n + (1-\gamma)\Delta t a^n + \gamma \Delta t a^{n+1}$$

where optimal values of the parameters β and γ are related to α_f and α_m by

$$\beta = \frac{1}{4}(1 + \alpha_m - \alpha_f)^2$$

$$\gamma = \frac{1}{2} + \alpha_m - \alpha_f$$

It is seen that the α_f and α_m parameters can be used to control the numerical dissipation of the operator. A simpler measure is the spectral radius ρ. This is also a measure of the numerical dissipation; a smaller spectral radius value corresponds to greater numerical dissipation. The spectral radius of the generalized alpha operator can be related to the α_f and α_m parameters as follows

$$\alpha_f = -\frac{\rho}{1+\rho}$$

$$\alpha_m = -\frac{1+2\rho}{1+\rho}$$

ρ varies between 0 and 1. Accordingly, the ranges for the α_f and α_m parameters are given by

$-0.5 \leq \alpha_f \leq 0.0$ and $-0.5 \leq \alpha_m \leq 1$. $\alpha_f = -0.5, \alpha_m = 1$ corresponds to a spectral radius of 0.0.

It can also be noted that the case of ρ = 1 has no dissipation and corresponds to a mid-increment Newmark-beta operator.

3.3.4.2. Time Step Definition

In a transient dynamic analysis, time step parameters are required for integration in time.

Enter parameters to specify the time step size and period of time for this set of boundary conditions.

When using the Newmark-beta operator, decide which frequencies are important to the response. The time step in this method should not exceed 10 percent of the period of the highest relevant frequency in the structure. Otherwise, large phase errors will occur. The phenomenon usually associated with too large a time step is strong oscillatory accelerations. With even larger time steps, the velocities start oscillating. With still larger steps, the displacement eventually oscillates. In nonlinear problems, instability usually follows oscillation. When using adaptive dynamics, you should prescribe a maximum time step.

As in the Newmark-beta operator, the time step in Houbolt integration should not exceed 10 percent of the period of the highest frequency of interest. However, the Houbolt method not only causes phase errors, it also causes strong artificial damping. Therefore, high frequencies are damped out quickly and no obvious oscillations occur. It is, therefore, completely up to the engineer to determine whether the time step was adequate.

For the Generalized-alpha operator, depending on the chosen parameters, the integration scheme can vary between the Newmark-beta operator and the Single-step houbolt operator. For spectral radii < 1, there is artificial damping in the system. Depending on the type of problem, the Generalized-alpha parameters and the associated time step should be carefully chosen to reduce phase errors and effects of artificial damping.

In nonlinear problems, the mode shapes and frequencies are strong functions of time because of plasticity and large displacement effects, so that the above guidelines can be only a coarse approximation. To obtain a more accurate estimate, repeat the analysis with a significantly different time step (1/5 to 1/10 of the original) and compare responses.

3.4. Nonlinear analysis

Nonlinear analysis is usually more complex and expensive than linear analysis. Also, a nonlinear problem can never be formulated as a set of linear equations. In general, the solutions of nonlinear problems always require incremental solution schemes and sometimes require iterations within each load/time increment to ensure that equilibrium is satisfied at the end of each step. Superposition cannot be applied in nonlinear problems.

Newton-Raphson is the iterative procedures supported in CivilFEM.

A nonlinear problem does not always have a unique solution. Sometimes a nonlinear problem does not have any solution, although the problem can seem to be defined correctly.

Nonlinear analysis requires good judgment and uses considerable computing time. Several runs are often required. The first run should extract the maximum information with the minimum amount of computing time. Some design considerations for a preliminary analysis are:

- Minimize degrees of freedom whenever possible.
- Halve the number of load increments by doubling the size of each load increment.
- Impose a coarse tolerance on convergence to reduce the number of iterations. A coarse run determines the area of most rapid change where additional load increments might be required. Plan the increment size in the final run by the following rule of thumb: there should be as many load increments as required to fit the nonlinear results by the same number of straight lines.

CivilFEM solves nonlinear static problems according to one of the following two methods: tangent modulus or initial strain. Examples of the tangent modulus method are elastic-plastic analysis, nonlinear springs, nonlinear foundations, large displacement analysis and gaps. This method requires at least the following three controls:

- A tolerance on convergence.
- A limit to the maximum allowable number of iterations.
- Specification of a minimum number of iterations.

3.4.1. Load Incrementation

In many nonlinear analyses, it is useful to have CivilFEM automatically determine the appropriate load step size. For an adaptive scheme, the load step size changes from one increment to the other and also within an increment depending on convergence criteria and/or user-defined physical criteria.

Selecting a proper load step increment is an important aspect of a nonlinear solution scheme. Large steps often lead to many iterations per increment and, if the step is too large, it can lead to inaccuracies and nonconvergence. On the other hand, using too small steps is inefficient.

When a fixed step fraction scheme is used, it is important to select an appropriate step fraction size that captures the loading history and allows for convergence within a reasonable number of recycles. For complex load histories, it is necessary to prescribe the loading through time tables while setting up the run.

For fixed stepping, there is an option to have the load step automatically cut back in case of failure to obtain convergence. When an increment diverges, the intermediate deformations after each iteration can show large fluctuations and the final cause of program exit can be any of the following: maximum number of iterations reached, elements going inside out or, in a contact analysis and nodes sliding off a rigid contact body. If the cutback feature is activated and one of these problems occur, the state of the analysis at the end of the previous increment is restored and the increment is subdivided into a number of subincrements. The step size is halved until convergence is obtained or the user-specified number of cutbacks has been performed. Once a subincrement is converged, the analysis continues to complete the remainder of the original increment. No results are written to the post file during subincrementation. When the original increment is finished, the calculation continues to the next increment with the original increment count and time step maintained.

3.4.2. Newton-Raphson Method

The Newton-Raphson method can be used to solve the nonlinear equilibrium equations in structural analysis by considering the following set of equations:

$$\mathrm{K(u)\delta u = F - R(u)}$$

Where u is the nodal-displacement vector, F is the external nodal-load vector, R is the internal nodal-load vector (following from the internal stresses), and K is the tangent-stiffness matrix. The internal nodal-load vector is obtained from the internal stresses as

$$R = \sum_{\mathrm{Elem}} \int_{\mathrm{v}} \beta^{\mathrm{T}} \sigma \mathrm{dv}$$

In this set of equations, both R and K are functions of u. In many cases, F is also a function of u (for example, if F follows from pressure loads, the nodal load vector is a function of the orientation of the structure). The equations suggest that use of the Newton-Raphson method is appropriate. Suppose that the last obtained approximate solution is termed δu^i, where ^{i} indicates the iteration number. First equation can then be written as

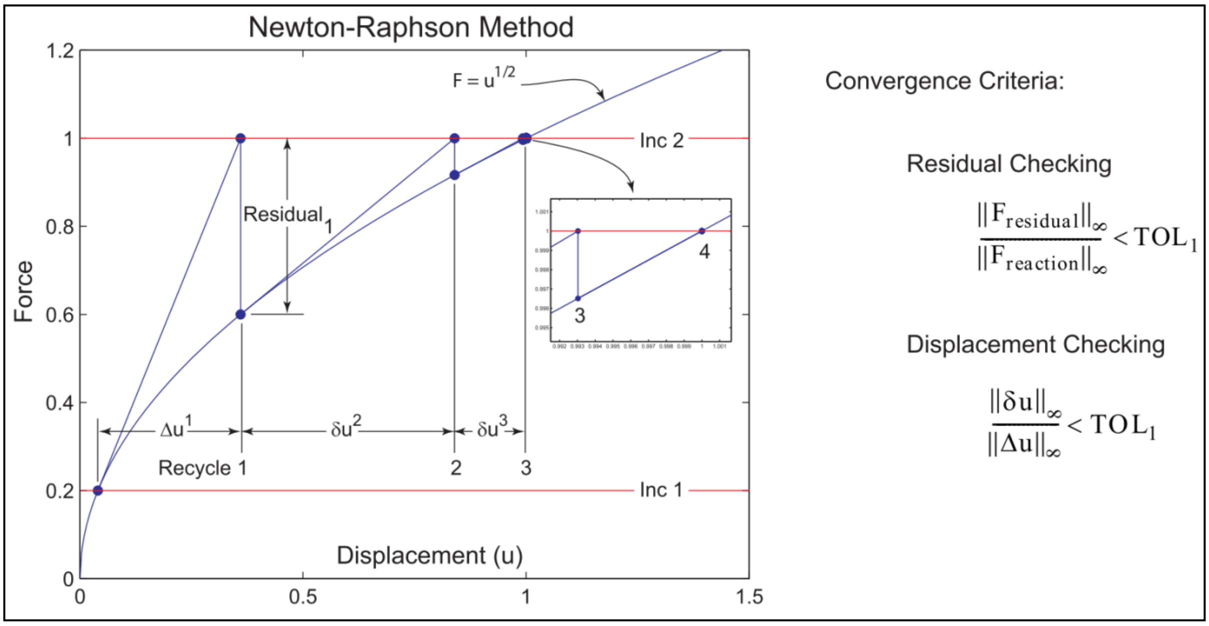
$$K\left(u_{n+1}^{i-1}\right)\delta u = F - R\left(u_{n+1}^{i-1}\right)$$

This equation is solved for δu^i and the next appropriate solution is obtained by

$$\Delta u^i = \Delta u^{i+1} + \delta u^i \text{and } u_{n+1}^i = u_n + \Delta u^i$$

Solution of this equation completes one iteration, and the process can be repeated. The subscript $_n$ denotes the increment number representing the state t = n. Unless stated otherwise, the subscript *n+1* is dropped with al quantities referring to the current state.

The Newton-Raphson method is the default in CivilFEM (see figure below).



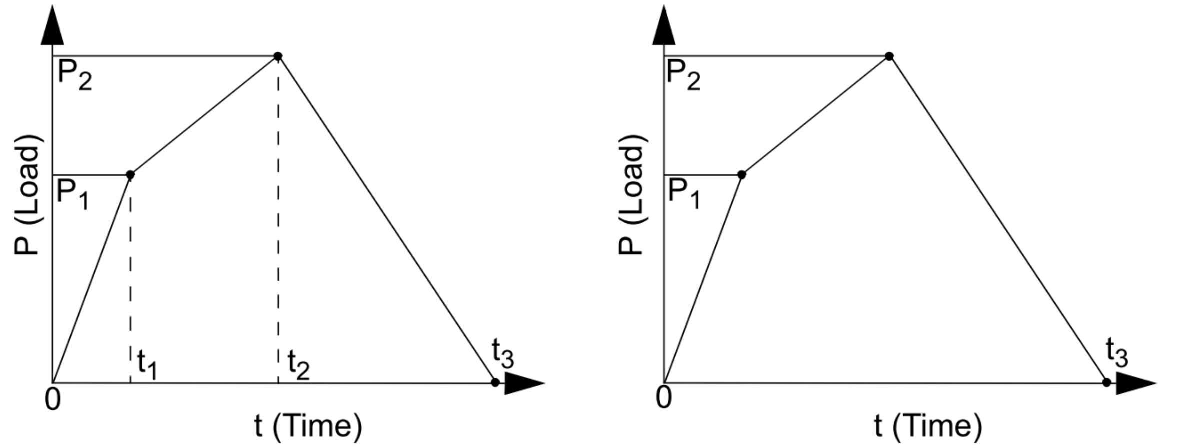
The Newton-Raphson method provides good results for most nonlinear problems, but is expensive for large, three-dimensional problems, when the direct solver is used. The computational problem is less significant when the iterative solvers are used. Figure above illustrates the graphical interpretation of the Newton-Raphson iteration technique in one dimension to find the roots of the function $F(u) - 1 = \sqrt{u} - 1 = 0$ starting from increment 1 where $F(u_0) = 0.2$ to increment 2 where $F(u_{last}) = 1.0$.

The iteration process stops when the convergence criteria are satisfied.

3.4.3. Arc-Length

The arc-length procedures assume that the control of the nonlinear behavior and possible instabilities is due to mechanical loads, and that the objective is to obtain an equilibrium position at the end of the loadcase. Hence, while the program may increase or decrease the load, the load can always be considered to be $F = F_b + \lambda\,(F_e - F_b)$, where F_b and F_e are the loads at the beginning and end of the loadcase. The scale factor does not necessarily vary linearly from 0 to 1 over the increments, and may, in fact, become negative.

Mechanical loads, as shown above, are applied in a proportional manner and thermal loads are applied instantaneously.

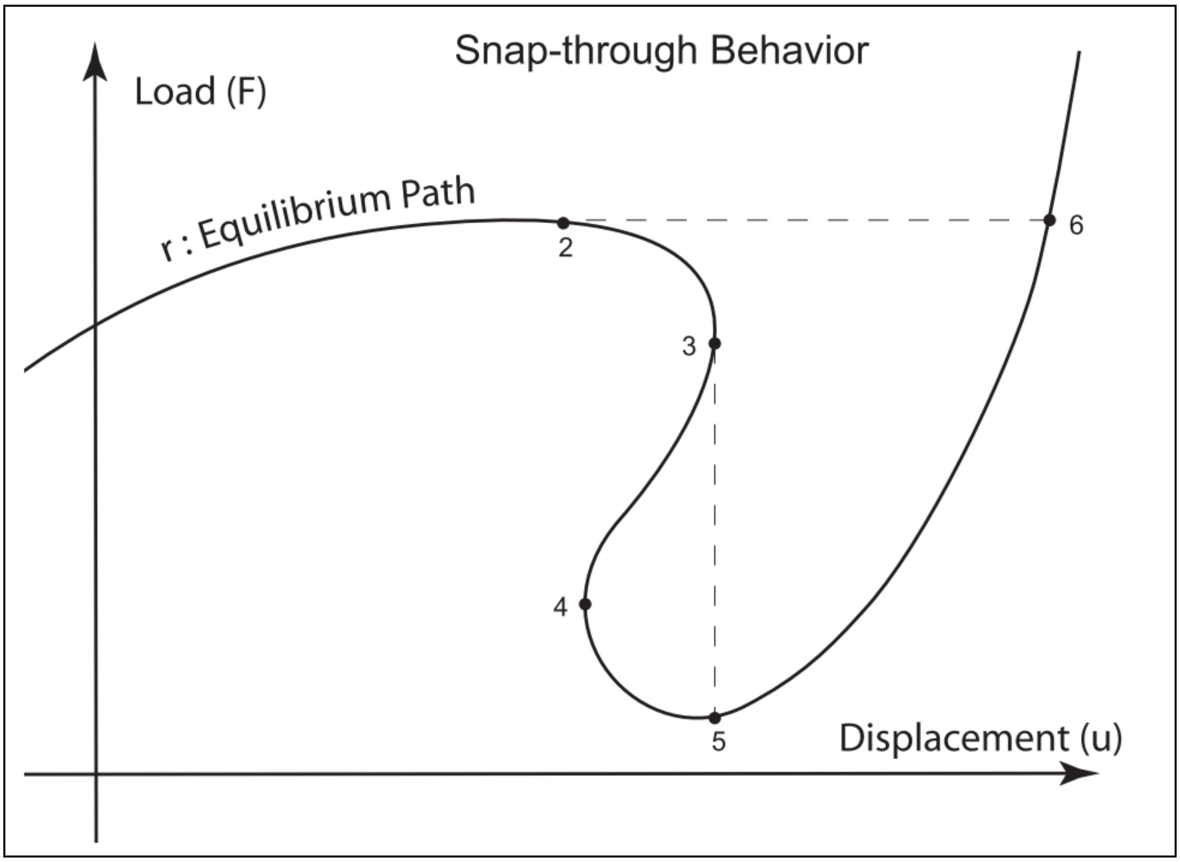


a. Three Defined Loadcases b. One Defined Loadcase

This means that any automatic load incrementation method is limited to mechanical input histories that only have linear variations in load or displacement and thermal input histories that have immediate change in temperature. For example, one may not use a rigid body with a linearly changing velocity, since the resulting displacement of the rigid body would give parabolically changing displacements. In this case, one would need to use a constant velocity for the arc length method to work properly.

For the arc length method, care must be taken to appropriately define the loading history in each loadcase. The load case should be defined between appropriate break points in the load history curve. For example, in figure above, correct results would be obtained upon defining three distinct loadcases between times 0-t_1, t_1-t_2, and t_2-t_3 during the model preparation. However, if only one load case is defined for the entire load history between 0-t_3, the total applied load for the loadcase is zero.

The solution methods described above involve an iterative process to achieve equilibrium for a fixed increment of load. Besides, none of them have the ability to deal with problems involving snap-through and snap-back behavior. An equilibrium path as shown in figure below displays the features possibly involved.



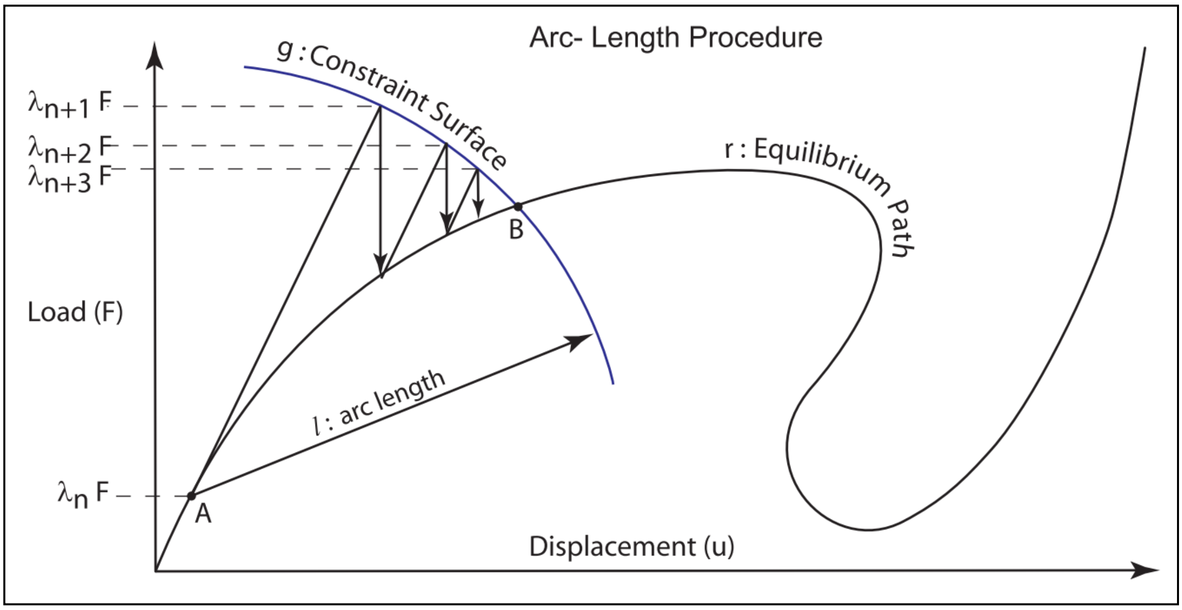
The issue at hand is the existence of multiple displacement vectors, u , for a given applied force vector, F . The arc-length methods provide the means to ensure that the correct displacement vector is found by CivilFEM. If you have a load controlled problem, the solution tends to jump from point 2 to 6 whenever the load increment after 2 is applied. If you have a displacement controlled problem, the solution tends to jump from 3 to 5 whenever the displacement increment after 3 is applied. Note that these problems appear essentially in quasi-static analyses. In dynamic analyses, the inertia forces help determine equilibrium in a snap-through problem.

Thus, in a quasi-static analysis sometimes it is impossible to find a converged solution for a particular load (or displacement increment):

$$\lambda_{n+1}F - \lambda_n F = \Delta\lambda F$$

This is illustrated in previous figure where both the phenomenon of snap-through (going from point 2 to 3) and snap-back (going from point 3 to 4) require a solution procedure which can handle these problems without going back along the same equilibrium curve.

As shown in figure below, assume that the solution is known at point A for load level $\lambda_n F$. For arriving at point B on the equilibrium curve, you either reduce the step size or adapt the load level in the iteration process.



To achieve this end, the equilibrium equations are augmented with a constraint equation expressed typically as the norm of incremental displacements. Hence, this allows the load level to change from iteration to iteration until equilibrium is found.

3.4.4. Convergence Controls

The default procedure for convergence criterion in CivilFEM is based on the magnitude of the maximum residual load compared to the maximum reaction force. This method is appropriate since the residuals measure the out-of-equilibrium force, which should be minimized. This technique is also appropriate for Newton methods, where zero-load iterations reduce the residual load. The method has the additional benefit that convergence can be satisfied without iteration.

The basic procedures are outlined below.

3.4.4.1. Residual Checking

$$\frac{\|F_{residual}\|_\infty}{\|F_{reaction}\|_\infty} < TOL_1$$

$$\frac{||F_{residual}||\infty}{||F_{reaction}||\infty} < TOL_1 \text{and} \frac{||M_{residual}||\infty}{||M_{reaction}||\infty} < TOL_2$$

$$||F_{residual}||_\infty < TOL_1$$

$$||F_{residual}||_\infty < TOL_1 \text{ and} ||M_{residual}||\infty < TOL_2$$

Where F is the force vector, and M is the moment vector, TOL_1 and TOL_2 are control tolerances. $||F||_\infty$ indicates the component of F with the highest absolute value.

3.4.4.2. Displacement Checking

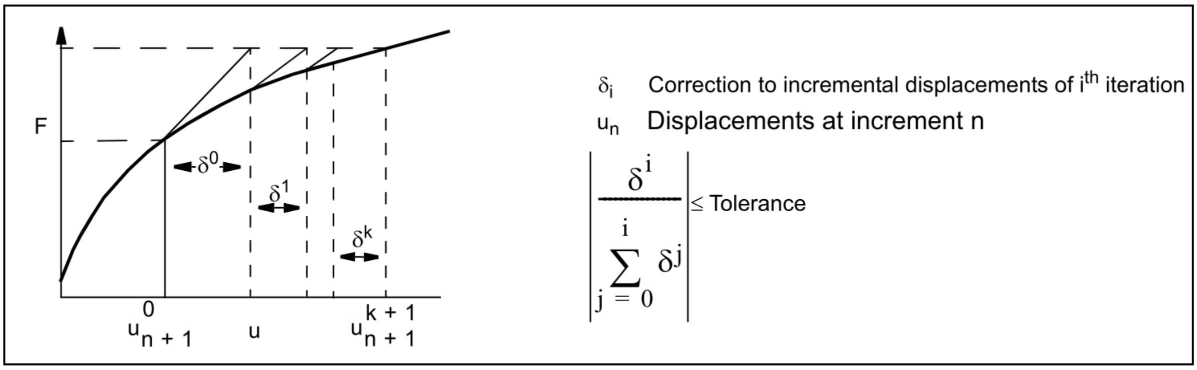
$$\frac{||\delta u||_\infty}{||\Delta u||_\infty} < TOL_1$$

$$\frac{||\delta u||_\infty}{||\Delta u||_\infty} < TOL_1 \text{ and } \frac{||\delta \emptyset||_\infty}{||\Delta \emptyset||_\infty} < TOL_2$$

$$||\delta u||_\infty < TOL_1$$

$$||\delta u||_\infty < TOL_{1 \text{ and }} ||\delta \emptyset||_\infty < TOL_2$$

Where Δu is the displacement increment vector, δu is the correction to incremental displacement vector, $\Delta \emptyset$ is the correction to incremental rotation vector, and $\delta \emptyset$ is the rotation iteration vector. With this method, convergence is satisfied if the maximum displacement of the last iteration is small compared the actual displacement change of the increment. A disadvantage of this approach is that it results in at least one iteration, regardless of the accuracy of the solution.



3.4.5. Construction process analysis

Staged construction of many structures as tunnels, excavations and bridges involve that certain elements in your model may become come into existence or cease to exist.

Using the activation/deactivation time capability allows the manual deactivation of elements during the course of an analysis, which can be useful to model ablation, excavation and other problems. By default, after the elements are deactivated, they demonstrate zero stresses and strains on the post file. However, internally, they retain the stress state in effect at the time of deactivation and this state can be postprocessed or printed at any time. At the later stage in the analysis, the elements can again be activated.

By default the activated elements will appear in their original position (will be reactivated in their originally specified geometric configuration) unless the behaviour of the contruction process is changed and then free motion of deactivated elements will be allowed. To achieve this effect, the program does not actually remove deactivated elements. Instead, it deactivates them by multiplying their stiffness by a severe reduction factor. This factor is set to 1.0E-9 by default, but can be given other values.

Element loads associated with deactivated elements are zeroed out of the load vector, but only if the construction process behaviour option is not checked. In this case, loading must be set accordingly in the corresponding structural elements and timing. The mass and energy of deactivated elements are not included in the summations over the model. An element's strain is also set to zero as soon as that element is deactivated.

In like manner, when elements are activated they are not actually added to the model; they are simply reactivated. User must create all elements, including those to be activated in later stages of your analysis.

When an element is reactivated, its stiffness, mass, element loads, etc. return to their full original values. Elements are reactivated with no record of strain history (or heat storage, etc.); that is, a reactivated element is generally strain-free. Initial strain defined as a real constant, however, is not be affected by birth and death operations.

Large-deflections effects should be included to obtain meaningful results.

3.5. Solvers

The finite element formulation leads to a set of linear equations. The solution is obtained through numerically inverting the system. Because of the wide range of problems encountered with CivilFEM, there are several solution procedures available.

Most analyses result in a system which is real, symmetric, and positive definite. While this is true for linear structural problems, assuming adequate boundary conditions, it is not true for all analyses.

Each iteration of the Newton-Raphson Method requires solving the system of equations. This can be done with a Direct Solver or with an Iterative Solver.

With recent advances in solver technology, the time spent in assembly and recovery now exceeds the time spent in the solver.

Which solution method to use depends very much on the problem. In some cases, one method can be advantageous over another; in other cases, the converse might be true.

Whether a solution is obtainable or not with a given method, usually depends on the character of the system of equations being solved, especially on the kind on nonlinearities that are involved.

As an example in problems which are linear until buckling occurs, due to a sudden development of nonlinearity, it is necessary to guide the arc-length algorithm by making sure that the arc length remains sufficiently small prior to the occurrence of buckling.

Even if a solution is obtainable, there is always the issue of efficiency. The pros and cons of each solution procedure, in terms of matrix operations and storage requirements have been discussed in the previous sections. A very important variable regarding overall efficiency is the size of the problem. The time required to assemble a stiffness matrix, as well as the time required to recover stresses after a solution, vary roughly linearly with the number of degrees of freedom of the problem. On the other hand, the time required to go through the direct solver varies roughly quadratically with the bandwidth, as well as linearly with the number of degrees of freedom.

In small problems, where the time spent in the solver is negligible, user can easily wipe out any solver gains, or even of assembly gains, with solution procedures such as a line search which requires a double stress recovery. Also, for problems with strong material or contact nonlinearities, gains obtained in assembly in modified Newton-Raphson can be nullified by increased number of iterations or nonconvergence.

3.5.1. Basic Theory

A linear finite element system is expressed as:

Ku = F

And a nonlinear system is expressed as:

$$K^{T}\Delta u = F - R = r$$

Where K is the elastic stiffness matrix, K^{T} is the tangent stiffness matrix in a nonlinear system, Δu is the displacement vector, F is the applied load vector, and r is the residual.

The linearized system is converted to a minimization problem expressed as:

$$\Psi(u) = 1 / 2u^{T}Ku - u^{T}F$$

For linear structural problems, this process can be considered as the minimization of the potential energy. The minimum is achieved when

$$u = K^{-1}F$$

The function ψ decreases most rapidly in the direction of the negative gradient,

$$\nabla\psi(u) = F - Ku = r$$

The objective of the iterative techniques is to minimize function, ψ, without inverting the stiffness matrix. In the simplest methods,

$$u_{k+1} = u_{k+}\alpha_k r_k$$

Where

$$\alpha_k = r_k^T r_k / r_K^T K r_k$$

The problem is that the gradient directions are too close, which results in poor convergence.

An improved method led to the conjugate gradient method, in which

$$u_{K+1} = u_k + \alpha_k P_k$$

$$\alpha_k = P_k^T r_{k-1} / P_k^T K P_k$$

The trick is to choose P_K to be K conjugate to P_1,P_2, …,P_{k-1}.

Hence, the name “conjugate gradient methods”. Note the elegance of these methods is that the solution may be obtained through a series of matrix multiplications and the stiffness matrix never needs to be inverted.

Certain problems which are ill-conditioned can lead to poor convergence. The introduction of a preconditioner has been shown to improve convergence. The next key step is to choose an appropriate preconditioner which is both effective as well as computationally efficient. The easiest is to use the diagonal of the stiffness matrix. The incomplete Cholesky method has been shown to be very effective in reducing the number of required iterations.

3.5.2. Pardiso Direct Sparse method

Traditionally, the solution of a system of linear equations was accomplished using direct solution procedures, such as Cholesky decomposition and the Crout reduction method. These methods are usually reliable, in that they give accurate results for virtually all problems at a predictable cost. For positive definite systems, there are no computational difficulties. For poorly conditioned systems, however, the results can degenerate but the cost remains the same. The problem with these direct methods is that a large amount of memory (or disk space) is required, and the computational costs become very large.

The solution of the linear equations may be solved using multi-processors using the hardware provided solver, the multifrontal solver, the Pardiso solver. If a multiprocessor machine is available, then Pardiso solver is recommended.

3.5.3. Parallel processing

CivilFEM can make use of multiple processors when performing an analysis in parallel mode. The type of parallelism used is based upon domain decomposition. A commonly used name for this is the Domain Decomposition Method (DDM). The model is decomposed into domains of elements, where each element is part of one and only one domain. The nodes which are located on domain boundaries are duplicated in all domains at the boundary. These nodes are referred to as inter-domain nodes below. The total number of elements is thus the same as in a serial (nonparallel) run but the total number of nodes can be larger. The computations in each domain are done by separate processes on the machine used. At various stages of the analysis, the processes need to communicate data between each other. This is handled by means of a communication protocol called MPI (Message Passing Interface). MPI is a standard for how this communication is to be done and CivilFEM makes use of different implementations of MPI on different platforms. CivilFEM uses MPI regardless of the type of machine used.

The types of machines supported are shared memory machines, which are single machines with multiple processors and a memory which is shared between the processors and cluster of separate workstations connected with some network. Each machine (node) of a cluster can also be a multiprocessor machine.

Only Pardiso solver supports shared memory machines and out-of-core solution in parallel on a cluster of workstations. The main reason for running an analysis in parallel on a shared memory machine is speed. Since all processes run on the same machine sharing the same memory, the processes all compete for the same memory. There is an overhead in memory usage so some parts of the analysis need more memory for a parallel run than a serial analysis. The matrix solver, on the other hand, needs less memory in a parallel analysis. Less memory is usually needed to store and solve several smaller systems than a single large one.

In the case of a cluster, the picture is somewhat different. Suppose a number of workstations are used in a run and one process is running on each workstation. The process then has full access to the memory of the workstation. If a analysis does not fit into the memory of one workstation, the analysis could be run on, say, two workstations and the combined memory of the machines may be sufficient.

The amount of speed-up that can be achieved depends on a number of factors including the type of analysis, the type of machine used, the size of the problem, and the performance of communications. For instance, a shared memory machine usually has faster communication than a cluster (for example, communicating over a standard Ethernet). On the other hand, a shared memory machine may run slower if it is used near its memory capacity due to memory access conflicts and cache misses etc.

The conjugate gradient iterative solver operates simultaneously on the whole model. It works to a large extent like in a serial run. For each iteration cycle, there is a need to synchronize the residuals from the different domains.

3.6. Obtaining solution

Load cases must be generated when all load groups are defined and prior the solving process in order to obtain results. Only with load groups definition is not enough to solve and an error message will appear if at least one load case is not created.

Then user is ready to solve the analysis, a prompt message is displayed in order to save a backup copy of the model (a file name and directory path must be specified).

Load cases are solved independently following the sequence of specified *Calculation Time* variables. Each load case one generates its corresponding results file (.*RCF* with the same name as the load case).

Increments are points within a load case at which solutions are calculated. They are used for different reasons:

- In a nonlinear static or steady-state analysis, increments are used to apply the loads gradually so that an accurate solution can be obtained.
- In a linear or nonlinear transient analysis, increments are used to satisfy transient time integration rules (which usually dictate a minimum integration time step for an accurate solution).

In a linear static analysis increments have no meaning and a single increment is solved for each load case.

Iterations are additional solutions calculated at a given increment for equilibrium convergence purposes. They are iterative corrections used only in nonlinear analyses (static or transient), where convergence plays an important role.

3.6.1. Program messages

The messages provided by CivilFEM at various points in the output show the current status of the problem solution. Several of these messages are listed below.

- *Initializing solver engine*.
  Start the solution process.
- *Checking the model*.
  Checks the consistency of the model.
- *Creating input for Marc.*
  Links with the external Marc solver.

- *Solving load case n.*
  Indicates solver is about to enter the stiffness matrix assembly.
- *Solving increment x.*
  Indicates the start of the solution of the linear system.
- *Increment x has been solved.*
  Indicates the end of matrix decomposition and completion of increment number x.
- *Marc run completed successfully.*
  All load cases are solved without singularities.
- *Finished solving.*
  Indicates results file has been written to disk.

In addition to these messages, exit messages indicate normal and abnormal exists from solver. Following table shows the most common exit messages:

MARC EXIT 13	Input data errors were detected by the program.
MARC EXIT 2004	Operator matrix (for example, stiffness matrix in stress analysis) has become non-positive definite and the analysis terminated.
MARC EXIT 3002	Convergence has not occurred within the allowable number of iterations.
EXIT 3015	If the minimum time step is reached and the analysis still fails to converge.

Failure to satisfy user-defined physical criteria can occur due to two reasons: the maximum number of cutbacks allowed by the user can be exceeded, or the minimum time step can be reached. In this case, the analysis terminates with exit 3002 and exit 3015, respectively. These premature terminations can be avoided by using the option to continue the analysis even if physical criteria are not satisfied.

3.6.2. Stop solution

Solution process can be terminated anytime and writing data of results file will be skipped.

3.6.3. Output results

User can control the solution data written on the results file when solving (*.RCF*). It writes outthe specified solution results item for every load case. By default all solution results will be written and available to list and plot. The list of results is the following:

NODAL RESULTS	
UT	Displacements
UR	Rotations
RF	Reaction forces
RM	Reaction moments
CPRESS	Contact normal stress
CSHEAR	Contact shear stress
CNORMF	Contact normal force
CSHEARF	Contact shear force
CSTATUS	Contact status

TRUSS/BEAM/SHELL/SOLID RESULTS	
S	Stresses
E	Total strain
EE	Elastic strain
PE	Plastic strain
PEEQ	Equivalent plastic strain
MISES	Von Mises equivalent stress
PRESS	Equivalent pressure stress
SF	Forces (O.BS.)
SM	Moments (O.BS.)

SE	Generalized strains (O.BS.)
SK	Curvatures (O.BS.)
CE	Cracking strain (N.B.)
SP	Principal stresses (O.S.)

(O.BS.) Only available in beam and shell elements.

(N.B.) Not available in beam elements.

(O.S.) Only available in solid elements.

3.7. Initial State

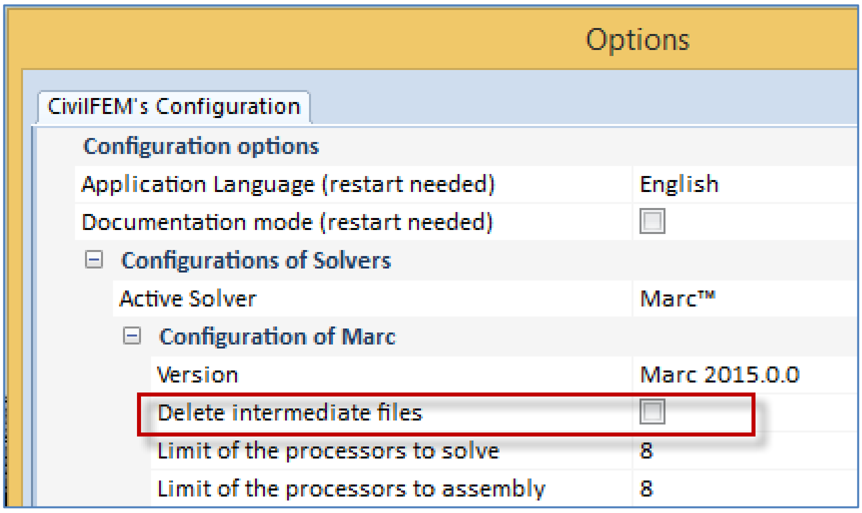
The term initial state refers to the state of a structure at the start of an analysis. Typically, the assumption is that the initial state of a structure is undeformed and unstressed. In many cases, it is necessary to analyze a nonlinear process in several stages. Each stage may involve different structural elements and boundary conditions, but history data such as displacements, stresses and strains have to be carried over for entities to be passed from one stage to another.

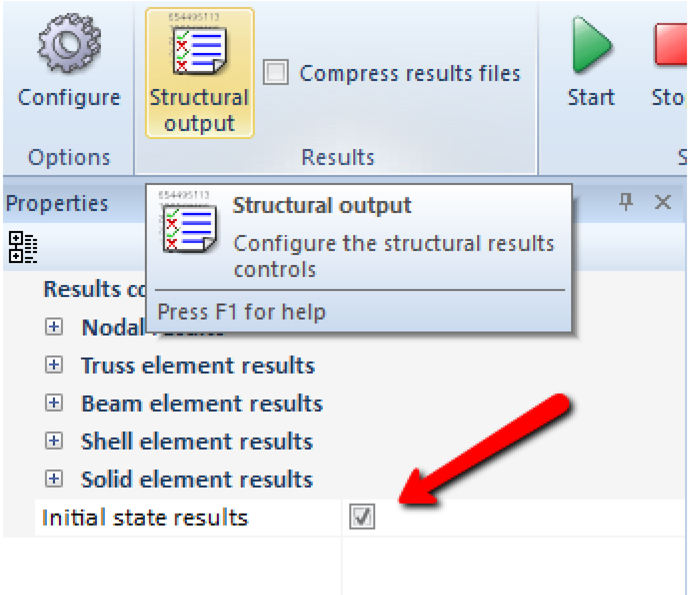
The Initial State option is designed to read data from a Marc results file solver (t16) and to use the data as initial conditions in the new analysis.

Typical analyses thay may need Initial State are construction or evolutive processes (tunnel, retaining walls, etc.)

This option takes several steps:

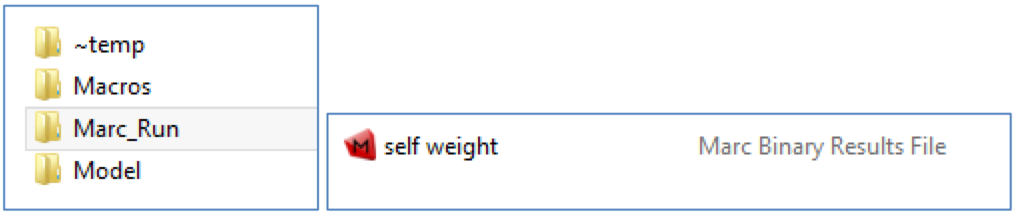
a) Run **first stage analysis** of the model to generate result files (for example solving a single lload case with just gravity in a). It is important to use a different model name to solve this stage. Make sure that Intermediate files are not deleted (uncheck corresponding box in *Configuration options*) and output results for initial state is activated:



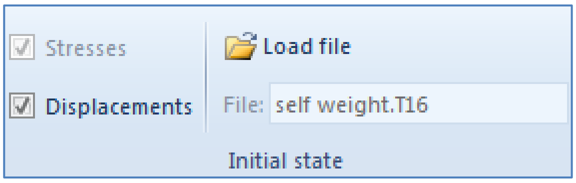


b) Start a new model (with a new model name) by adding new structural elements, contact pairs and other boundary conditions.

c) Load the *.t16* Marc results file. This file can be located inside *Marc_Run* folder:



d) Choose stresses and/or displacements to be included as Initial State.



e) Run a new analysis.

There are some conditions to be taken into account to carry out an analysis with Initial State:

- For 2D analyses, only plane strain behavior is supported.
- Linear structural elements as beam or cables are not supported.
- Node numbering must be correlative. If merge tool has been performed and nodes were fused then nodes must be re-numbered.
- If model includes different structural elements participating in a construction or evolutive analysis (using activation/deactivation material time) then individual mesh must be performed first on structural elements where Initial State will be studied. For example, if a tunnel construction analysis is carried out then original unexcavated soil must be meshed first.

3.8. Water conditions

Groundwater and pore pressures are very import to modelize the correct behavior of soils. CivilFEM can takes into account the underground water conditions in soils. There are three ways to indicate the water table in an initial water table condition :

- By using the result file generated by a seepage analysis.
- By defining the water table orthogonal to an axis.
- By defining the water with a geometry (surfaces for 3d and curves for 2d).

In all cases CivilFEM will compute the water pore pressure at the barycenter of each element. Then the internal water pressure calculated is applied to the soil skeleton by a distributed normal pressure on element edges (2D) or element faces(3D) in order to work with the effective stresses.

If water conditions generates external water pressure (i.e. water load on soil boundaries) user must introduce the hydrostatic pressure in a load group.

By default, suction (pore water pressure above phreatic level) is ignored. Pore water pressures are applied only to solid structural elements.

Different initial water conditions can be specified to each structural load case by assigning the different initial water table condition to each load case. This can be usefull to modelize the variation of phreatic level. For example, during a construction procces, users can first resolve the seepage problem for the different stages and then use this results files to define the pore water pressure in the structural analysis.

Chapter 4
Results

4.1. Introduction

Once solution process is completed successfully it is time to analyze the results and verify the criteria for acceptance. For each load case, the requested results are stored in a binary file. The following three basic steps are needed to gain access to the results.

- Step 1: Open the results file.
- Step 2: Select the desired information.
- Step 3: Select an appropriate display technique and display the results.

4.1.1. Read results

The first step is to read data from the results file into the model. The model should contain the same entities for which the solution was calculated, including the structural elements, nodes, elements, cross sections, material properties and coordinate systems.

Each load case is saved in an independent file. After choosing the desired load case it must be loaded (.RCF file) replacing any results previously displayed.

4.1.2. Results extrapolation

The solution of the finite element analysis involves a geometrical discretization of the object, and if applicable, also a temporal discretization. The geometrical discretization is obtained by creating the finite element mesh that consists primarily of nodes and elements. The results (depending on their nature) are supplied at either the nodes or the integration points of the elements. We make the distinction by referring to one as data at nodes, and the other as data from elements at integration points.

Data at nodes is a vector where the number of degrees of freedom of the quantity indicates the number of components in the vector. Data from elements at integration points is either scalar, vector, or tensor data.

The data from elements at integration points are not in a form that can be used directly in a graphics program.

A node may be shared by several elements. Each element contributes a potentially different value to that shared node. The values are summed and averaged by the number of contributing elements.

If a node is shared by elements of different materials, the averaging process may not be appropriate. To prevent the program from averaging values, do not use the AVERAGE option.

4.2. Element information

4.2.1. Result Types

In CivilFEM there are three Result Types:

- Node Results.
- End Results.
- Element Results.

Nodal results are displayed or listed according to the global coordinate system. The following quantities at each nodal point are available:

- Displacements and rotations.

- Reaction forces and moments at fixed boundary conditions.

End results are derived data as generalized stresses and strains:
- Forces and moments: axial, bending, shear, twist.

- Curvatures.

The system provides the element data for each node end (I,J for beams, I,J,K,L for shells). The orientation of these physical components depends on the structural element coordinate system.

Element results are derived data as stresses and strains.

The system provides the element data at each integration point. All quantities are total values at the current state (at the end of the current load case), and the physical components are printed for each tensor quantity (stress, strain). The orientation of these physical components depends on the structural element coordinate system.

In addition to the physical components, certain invariants are given, as follows:

von Mises intensity – calculated for strain type quantities as

$$\bar{\varepsilon} = \sqrt{\frac{2}{3}\varepsilon_{ij}\varepsilon_{ij}}$$

CivilFEM uses these measures in the plasticity and creep constitutive theories. For example, incompressible metal creep and plasticity are based on the equivalent von Mises stress. For beam, truss, and plane stress elements, an incompressibility assumption is made regarding the non calculated strain components.

For plane strain elements:

$$\varepsilon_{33} = -(\varepsilon_{11} + \varepsilon_{22})$$

Pressure – calculated as:

$$-(\sigma_{11} + \sigma_{22} + \sigma_{33})/3$$

Previous equation represents the negative hydrostatic pressure for stress quantities. For strain quantities, the equation gives the dilatational magnitude. This measurement is important in hydrostatically dependent theories (Mohr-Coulomb or extended von Mises materials), and for materials susceptible to void growth.

The principal values are calculated from the physical components. The eigenvalue problem is solved for the principal values using the Jacobi transformation method. Note that this is an iterative procedure and may give slightly different results from those obtained by solving the cubic equation exactly.

4.2.2. Shell elements

Conventional finite element implementation of Mindlin shell theory results in the transverse shear distribution being constant through the thickness of the element.

CivilFEM prints generalized stresses and generalized total strains for each integration point. The generalized stresses printed out for shell elements are:

– In-plane and transverse shell forces (per unit length)

$$\int_{-t/2}^{+t/2} \sigma_{ij}\, dz$$

– Shell moments (per unit length)

$$\int_{-t/2}^{+t/2} z\sigma_{ij}\, dz$$

The generalized strains printed are:

$E_{\alpha\beta;}$ $\alpha, \beta = 1{,}2$

(Stretch)

$K_{\alpha\beta;}$ $\alpha, \beta = 1{,}2$

(Curvature)

Physical stress values are output only for the extreme layers. In addition, thermal, plastic, creep, and cracking strains are printed for values at the layers, if applicable.
Although the total strains are not output for the layers, they can be calculated using the following equations:

$$\varepsilon_{11} = E_{11} + hk_{11}$$

$$\varepsilon_{22} = E_{22} + hk_{22}$$

$$\gamma_{12} = E_{12} + hk_{12}$$

Where h is the directed distance from the midsurface to the layer; are the stretches; and are the curvatures as printed.

More information about shell results in chapter Forces and Moments Sign Criteria.

4.2.3. Beam elements

The printout for beam elements is similar to shell elements, except that the section values are force, bending and torsion moment, and bimoment for open section beams. These values are given relative to the section axes (X, Y, Z).
Before a beam member can be designed, it is necessary to understand the section forces distribution along the axial direction of the beam. For example, if variations of shear force and moment along axial direction are plotted, the graphs are termed shear diagram and moment diagram, respectively.

4.2.4. Tensors and associated invariants

i ≤ j ≤ 3

TENSORS AND ASSOCIATED INVARIANTS
ij-component of stress
Von Mises equivalent stress

Pressure
ij-component of elastic strain
ij-component of plastic strain
Equivalent plastic strain

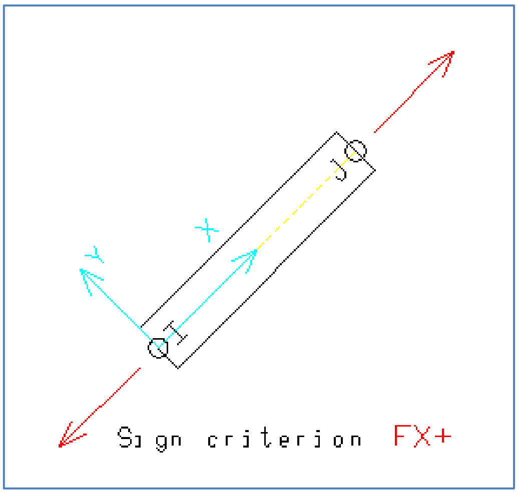
4.2.5. Forces and moments in beams

Forces and moments are calculated with respect to the coordinate system of the elements.

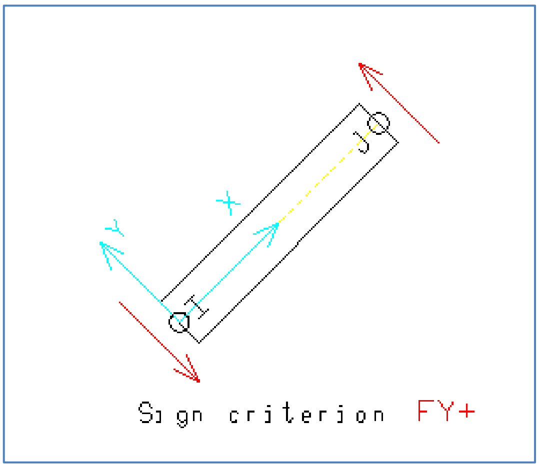
BEAM FORCES AND MOMENTS
Axial force
Transverse shear force in the local 2-direction
Transverse shear force in the local 3-direction
Bending moment about the local i-axis (i = 1:2)
Twisting moment about the beam axis.
Bimoment

Sign criteria of Force and Moment are explained below using a single element (I, J ends):

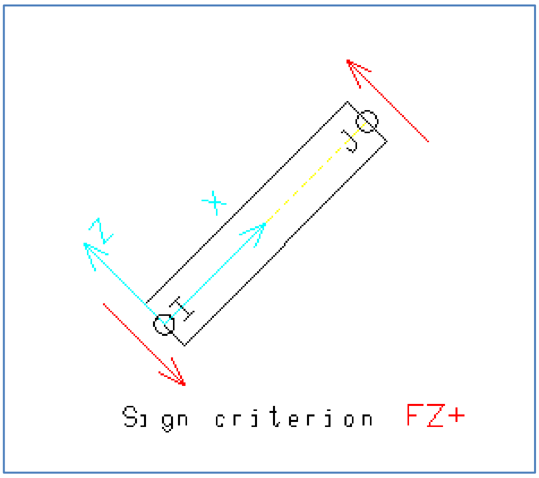
- Axial Force FX:



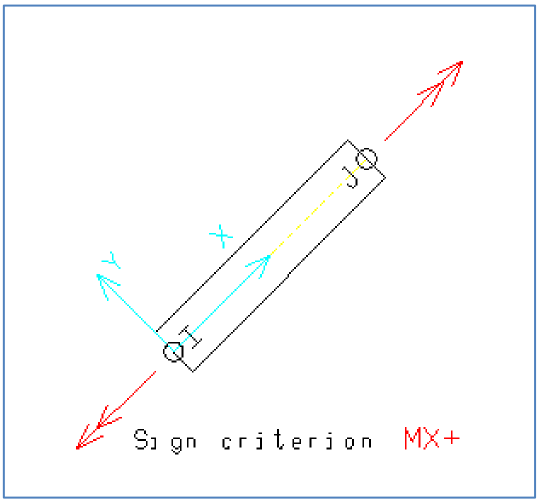
- Shear Force FY:



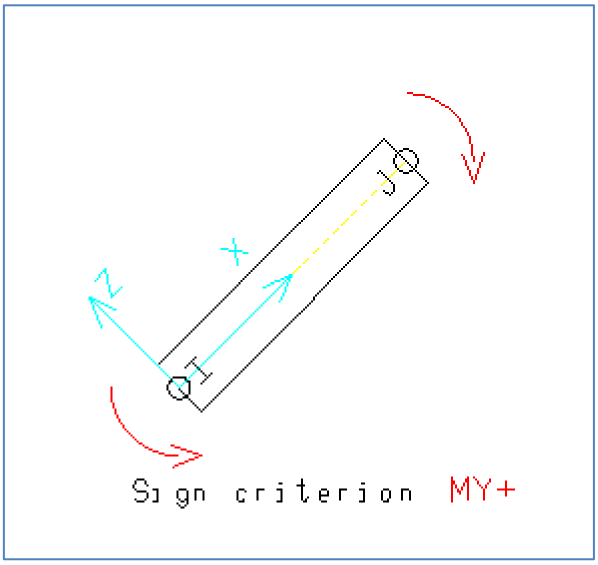
- Shear Force FZ:



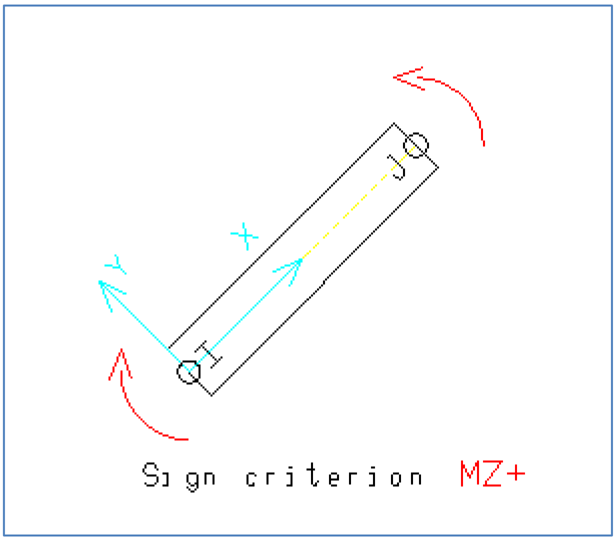
- Twisting moment MX:



- Bending moment MY:



- Bending moment MZ:



4.2.6. Generalized strains in beams and trusses

BEAM GENERALIZED STRAINS
Axial strain
Transverse shear strain in the local 2-direction
Transverse shear strain in the local 3-direction
Curvature about the local 1-axis
Curvature about the local 2-axis
Twist about the local 3-axis
Bicurvature

4.2.7. Forces and Moments in Shells

SHELL FORCES AND MOMENTS
Direct membrane force per unit width in local 1-direction
Direct membrane force per unit width in local 2-direction
Shear membrane force per unit width in local 1-2 plane
Transverse shear force per unit width in local 1-direction
Transverse shear force per unit width in local 2-direction
Bending moment force per unit width about local 2-axis
Bending moment force per unit width about local 1-axis
Twisting moment force per unit width in local 1-2 plane

4.2.8. Generalized Strains in Shells

SHELL GENERALIZED STRAINS
Direct membrane strain in local 1-direction
Direct membrane strain in local 2-direction
Shear membrane strain in local 1–2 plane
Transverse shear strain in the local 2-3 plane
Transverse shear strain in the local 1-3 plane
Curvature about local 2-axis
Curvature about local 1-axis
Surface twist in local 1–2 plane
Current section thickness

4.3. Nodal results

CivilFEM also prints out the following quantities at each nodal point (*i* = 1-3).

DISPLACEMENTS, ROTATIONS AND REACTION FORCES
i-component of displacement
i-component of rotation
i-component reaction force
i-component reaction moment component

Contact results.

Contact Status
Normal stress
Shear stress
Normal force
Shear force

Contact status: useful to detect when two surfaces have contacted. This result applies to nodes on contacting surfaces.

a) A value of 0 means that a node is not in contact.
b) A value of 1 means that a node is in contact.

Contact normal stress: component along the normal of the contact surface of the traction vector.

Contact shear stress: component along the tangent plane of the contact surface of the traction vector.

Contact normal force: component along the normal of the contact surface of the equivalent nodal force of traction vectors.

Contact shear force: component along the tangent plane of the contact surface of the equivalent nodal force of traction vectors.

These results are described in Friction Modeling chapter.

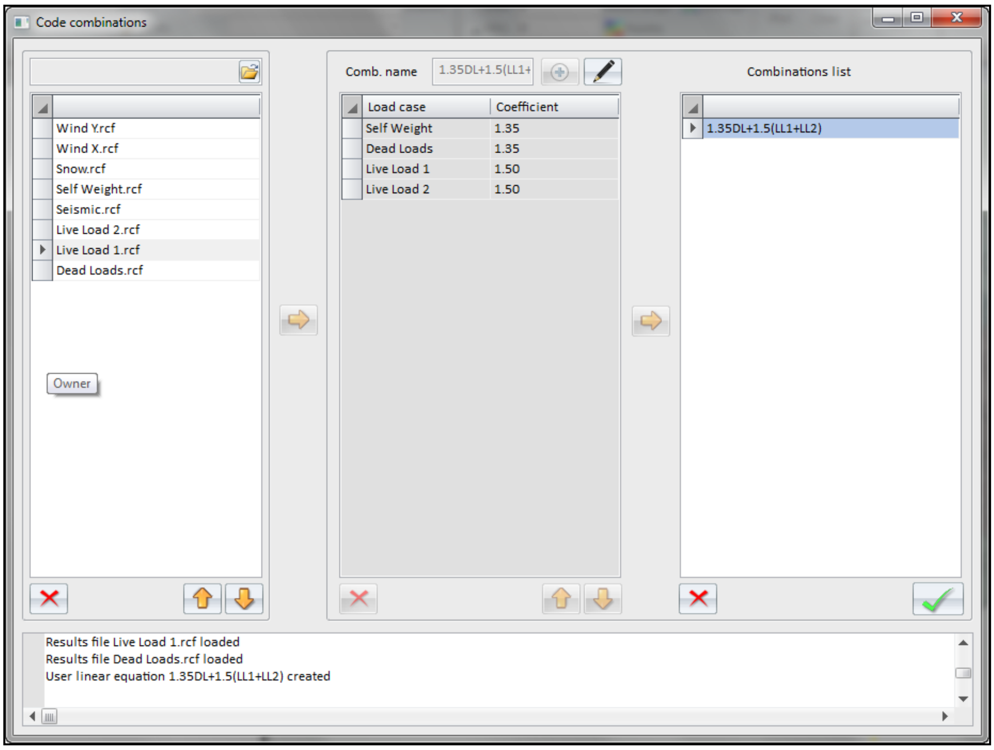
4.4. Combining Result Files

In a typical postprocessing process, user reads one results file (load case1 data, for instance) into the database and process it. Each time data is stored a new set of results (another .rcf file), program clears the results portion of the database and then brings in the new results data. If operations are different between sets of results data, (such as comparing and storing the maximum of two load cases), the load combinations must be performed.

A load combination is a linear postprocess operation between load cases already solved. The outcome of the operation creates a new results file, which permits user to display and list the load case combination as with any other standard results file.

The resulting load cases are obtained by combining linearly the initial load cases, as defined in the combination rules, with the desired coefficients.

The combination rules are defined in a new window (***Automatic User Combination Tool***) and single load cases must exist beforehand.



Summable data are those that can "participate" in the database operations. All primary data (DOF solutions) are considered summable. Among the derived data, component stresses, elastic strains, thermal gradients and fluxes, magnetic flux density, etc. are considered summable.

Sometimes, combining "summable" data may result in meaningless results, such as nonlinear data (plastic strains, hydrostatic pressures), thermal strains, etc. Therefore, exercise your engineering judgement when reviewing combined load cases.

4.5. Envelopes

The data stored in the CivilFEM results file are stored in two different types of data blocks: blocks of nodal results (displacements, reactions, etc.), element results (stresses, strains, etc.) and/or extreme results (forces and moments) in load cases (.rcf files) and blocks of code check/design results (explained in following chapters) as total criterion, allowable stresses, design resistance, etc. (.crcf files).

The utility ENVELOPE has been developed to create of other result files as envelope of others previously obtained. Envelopes have to be homogeneous; specifically, they must be obtained by the application of the same code and process to the same model. The new results file will be homogeneous with the previous ones, with a similar identification and the same utilities for reading, plotting and listing.

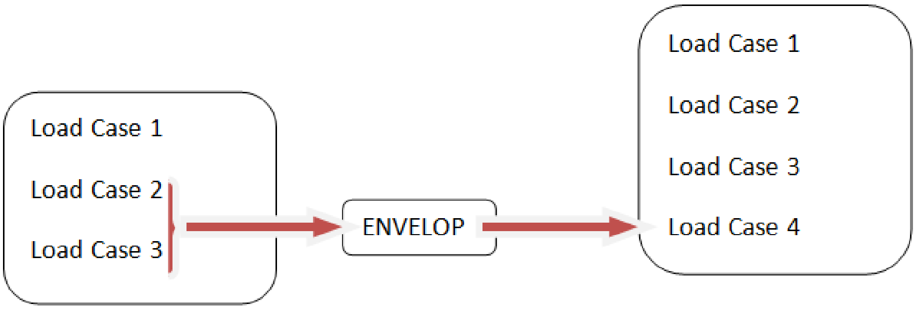
There are 3 types of envelopes:

- Maximum values envelope.
- Minimum values envelope.
- Maximum absolute value envelope.

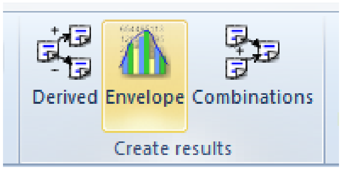
And the available result types:

- Displacements.
- Stresses and strains.
- Axial force.
- Bending moment.
- Torsional moment.
- Shear force.
- Reactions.
- Rest of results (code check/design).

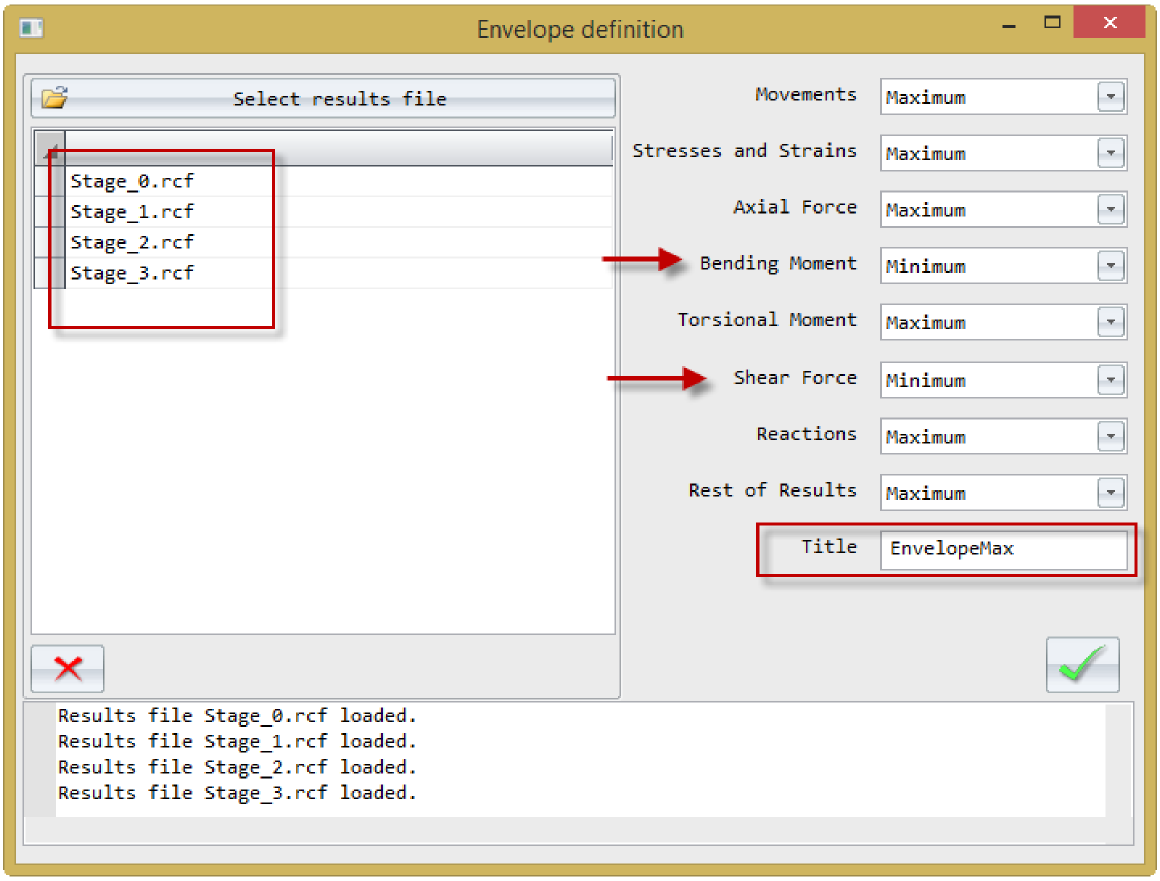
For example, a new envelope file from different homogeneous results files can store the maximum displacements in absolute value and minimum values for rest of results types.



The *Envelope* window is located in Results tab:



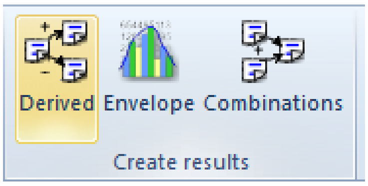
This results window is very user friendly and very easy to manage:



4.6. Derived results

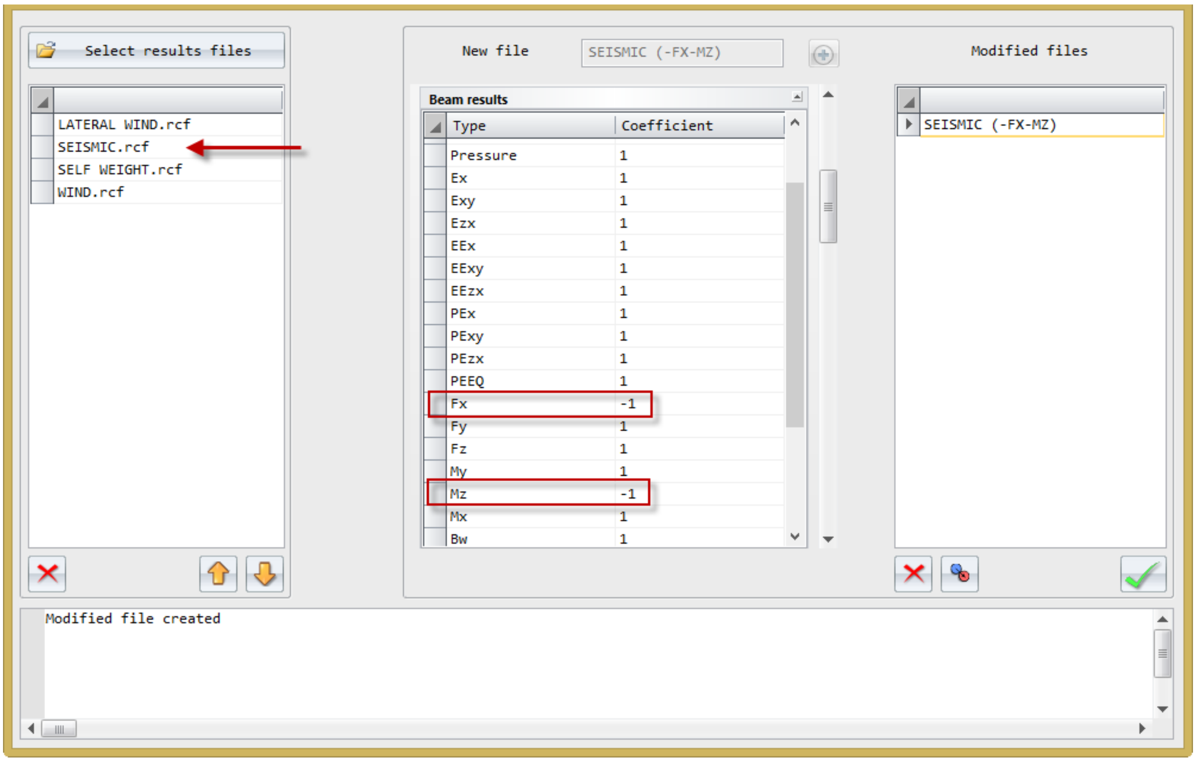
Once CivilFEM results data are stored, user can perform arithmetic operations among results data such as addition and subtraction of nodal and element results.

This tool is accessable through within Results tab:



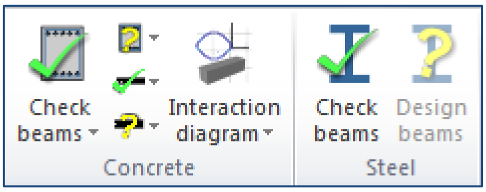
This results window is very user friendly and very easy to manage, user just need to select the appropriate results file and multiply any available result by a coefficient.

Then a new derived result file will be created.



4.7. Checking and Design results

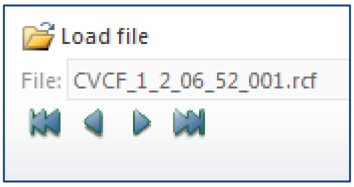
After loading a .rcf result file, the user can perform a code check or design using the results from the loaded result file. By clicking on the corresponding icon, the user will be able to select the desired option:



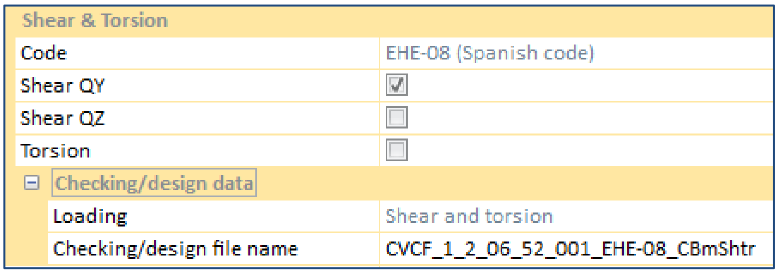
After choosing the kind of checking or design option (Axial, bending, torsion, shear, etc), CivilFEM will generate a new kind of file with the extension .crcf that can be loaded in the Results tab. Opening this file will generate a whole new result list that will contain the obtained checking/design results.

Here is an example of a shear/torsion check using a concrete beam.

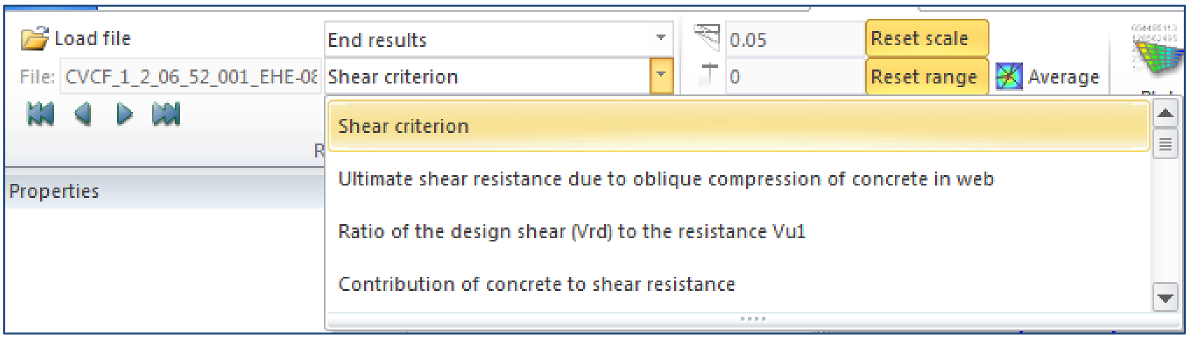
After solving the model we load the result file which is a .rcf file:



Then we select Concrete, Check Beams, Shear/Torsion and click Shear Qy and Torsion. The user can choose the file name in the window:



After clicking OK, the Check .crcf is generated and it can be loaded in the Result tab. Note that the extension needs to be changed to .crcf in the file window.



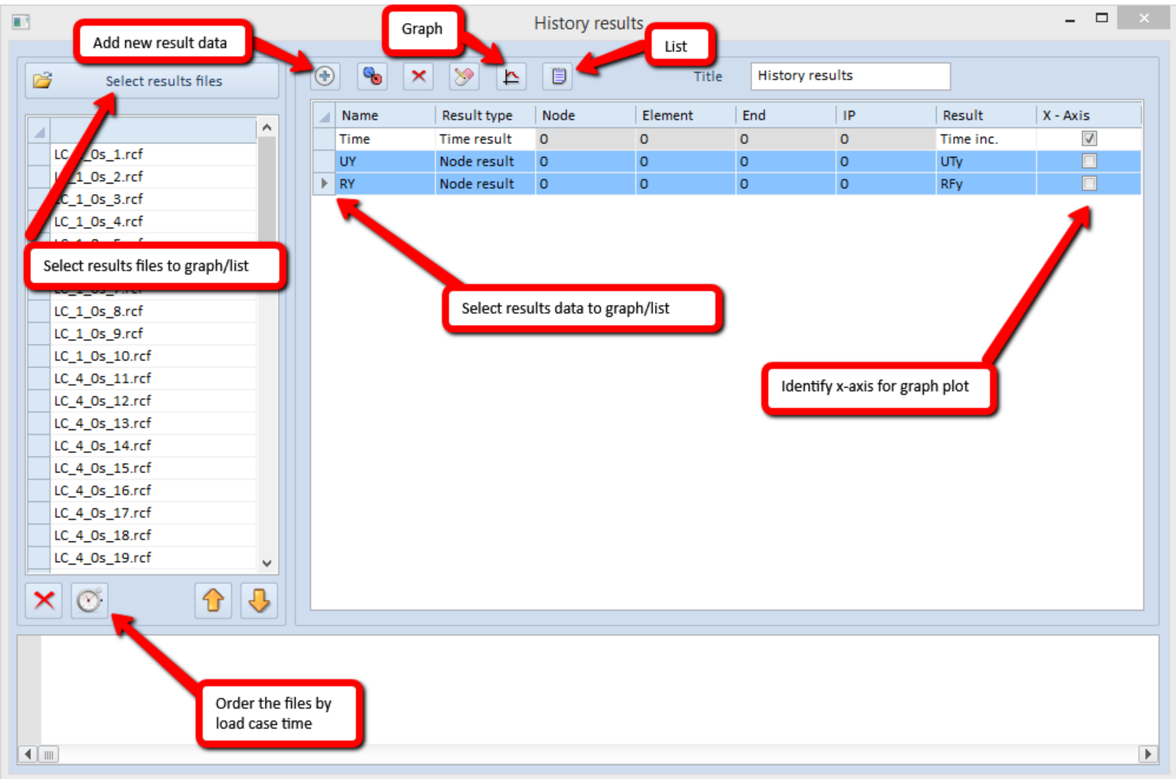
Now the different checking options can be selected and plotted or listed.

4.8. History results

History results is a tool created to review analysis results at specific locations as a function of time, frecuency or any other available result. For example, user can graph results versus time in transient analysis or graph force versus displacement results in a nonlinear analysis.

First, introduce the results files desired to be plotted. This results files can be sorted by load case calculation time automatically.

Add results data to define results and locations to graph/list. One of this result data must be identified as x-axis. Results data selected will be graphed versus the x-axis data.

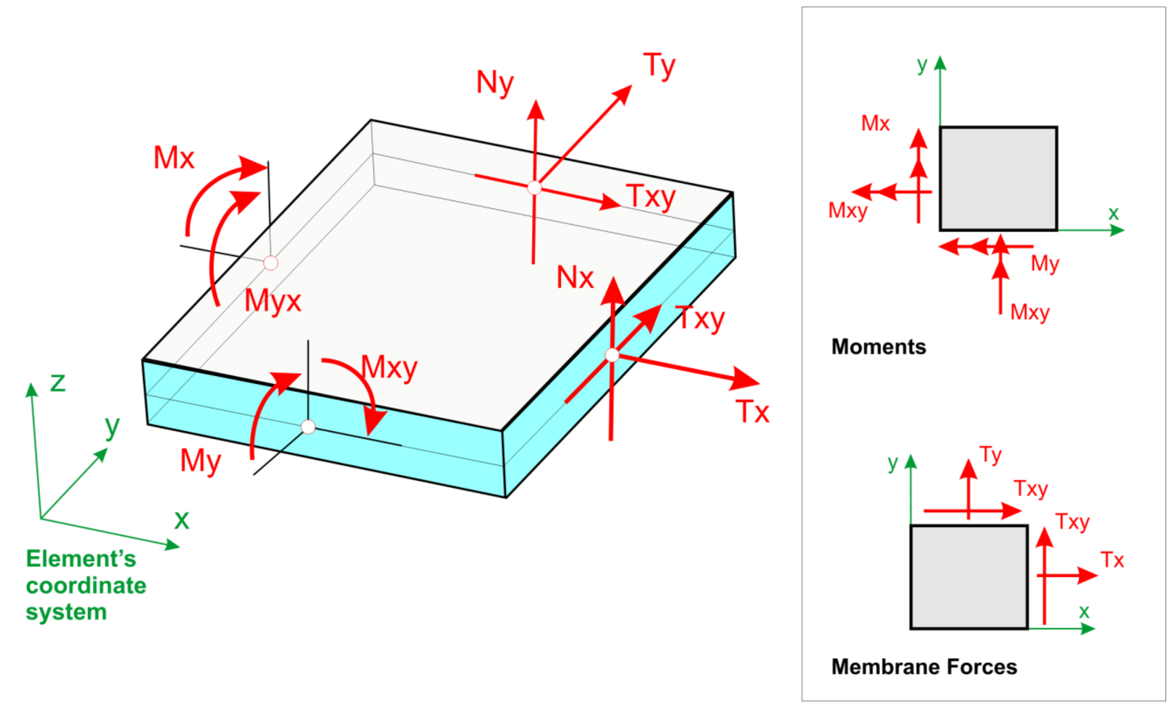


Chapter 5
Concrete Shells

5.1. General Concepts

5.1.1. Forces and Moments Sign Criteria

The following figure illustrates the sign criteria for forces and moments. The direction shown in the figure represents the positive direction of the force/moment.



Forces and moments sign criterion

T_x	Axial force in X direction
T_y	Axial force in Y direction
T_{xy}	Shear force in XY plane
M_x	Bending moment about Y axis (XZ plane)
M_y	Bending moment about X axis (YZ plane)
M_{xy}	Torsional moment XY
N_x	Shear force in X
N_y	Shear force in Y

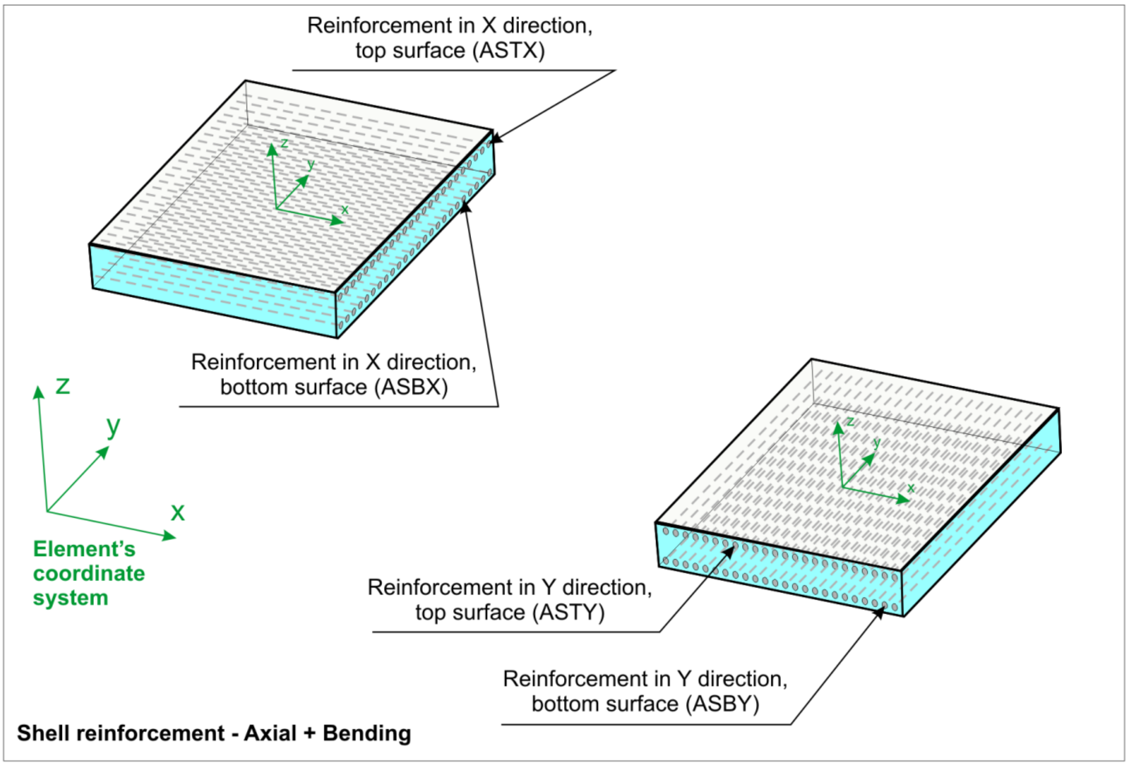
5.1.2. Reinforcement Directions

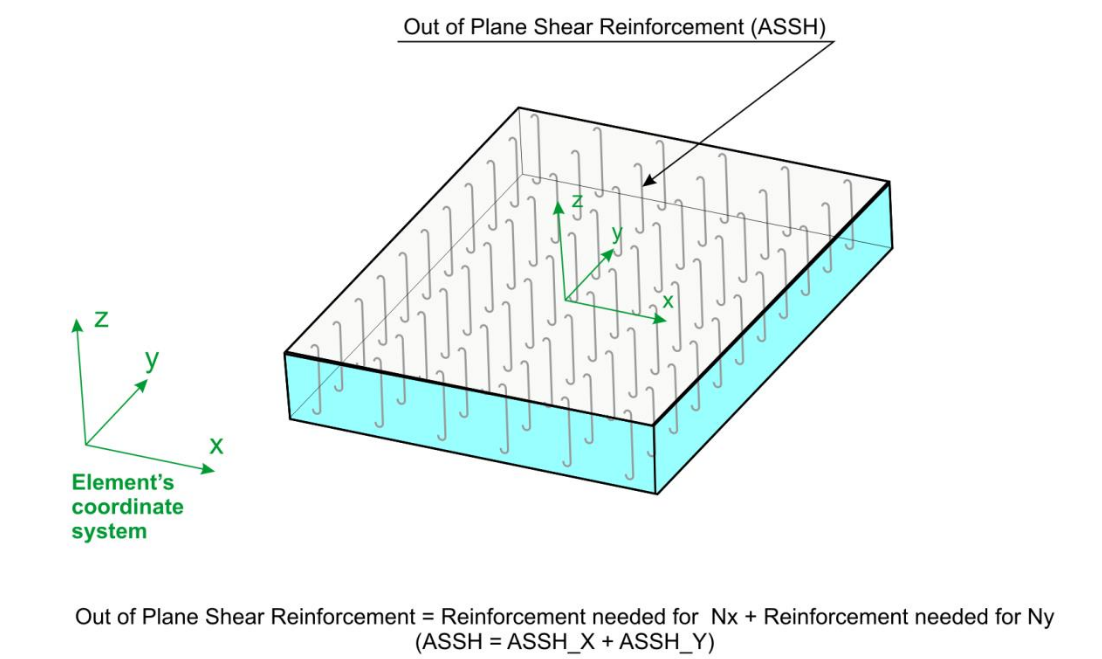
Three type of reinforcements are considered for concrete shells:

- Axial+Bending reinforcement.
- Out of plane shear reinforcement.
- In-plane shear reinforcement.

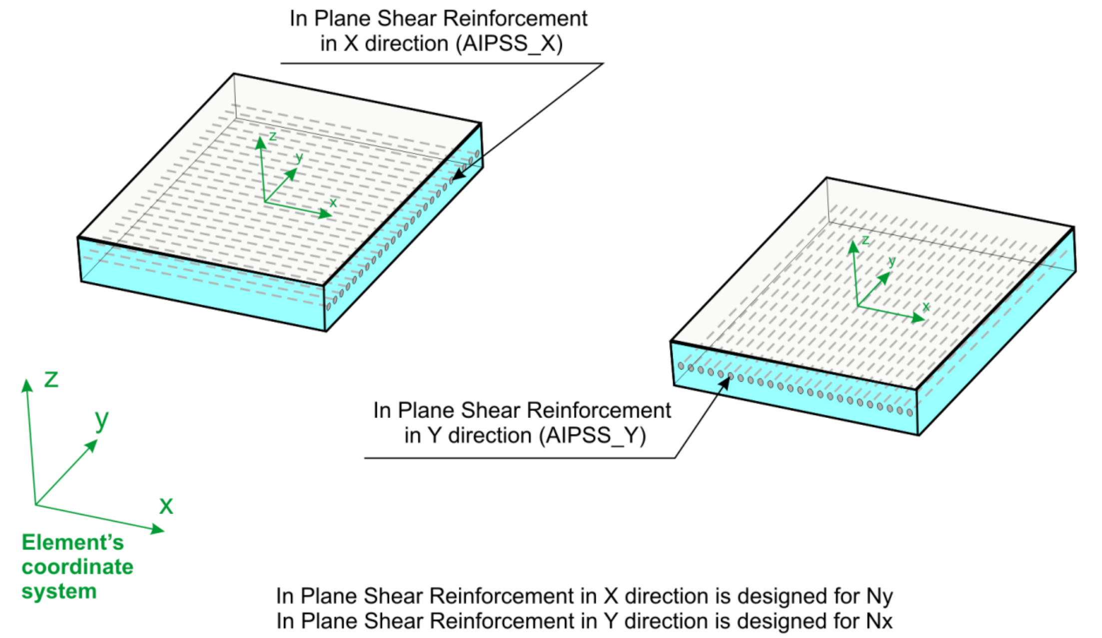
Note: Some design methods or codes consider in-plane shear together with axial+bending. In these cases, a single group of reinforcement is provided that covers these actions.

The following diagrams show the different reinforcements along with the axis on which they are defined.





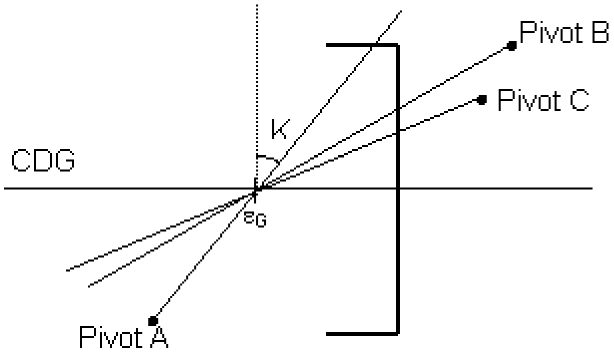
Shell reinforcement - Out of Plane Shear



Shell reinforcement - In Plane Shear

5.1.3. Interaction Diagram

The interaction diagram is a curve in space that contains the forces and moments (axial load, bending moment) corresponding to the shell ultimate strength states. In CivilFEM the ultimate strength states are determined through the pivots diagram.



A pivot is a strain limit associated with a material and its position in the shell vertex. If the strain in a section's pivot exceeds the limit for that pivot, the shell vertex is considered cracked. Thus, pivots establish the positions of the strain plane. So, in an ultimate strength state, the strain plane supports at least one pivot of the shell vertex.

In CivilFEM pivots are defined as material properties and these properties (pivots) are extrapolated to all the points through the thickness of the shell vertex, accounting for the particular material of each point (concrete or reinforcement). Therefore, for the section's strain plane determination, the following pivots and their corresponding material properties will be considered:

A Pivot	EPSmax. Maximum allowable strain in tension at any point of the shell vertex (the largest value of the maximum strains allowable for each point of the section in case there are different materials in the section).
B Pivot	EPSmin. Maximum allowable strain in compression at any point of the section (the largest value of the maximum strains allowable for each point of the section).
C Pivot	EPSint. Maximum allowable strain in compression at the interior points of the section.

Navier's hypothesis is assumed for the determination of the strains plane. The strains plane is defined according to the following equation:

$$\varepsilon(z) = \varepsilon_g + K \cdot z \qquad \text{EQN.1}$$

where:

$\varepsilon(z)$	Strain of a point of the shell vertex. Depends on its z location.
ε_g	Strain in the center of the section (center of gravity).
K	Curvature.

Diagram Construction Process

CivilFEM uses the elements (ε_g,K) to determine the strains plane (ultimate strength plane) of the shell vertex. The process is composed of the following steps:

1. Values of ε_g are chosen arbitrarily within the valid range:
   EPSmin (B pivot) < ε_g < EPSmax (A pivot)

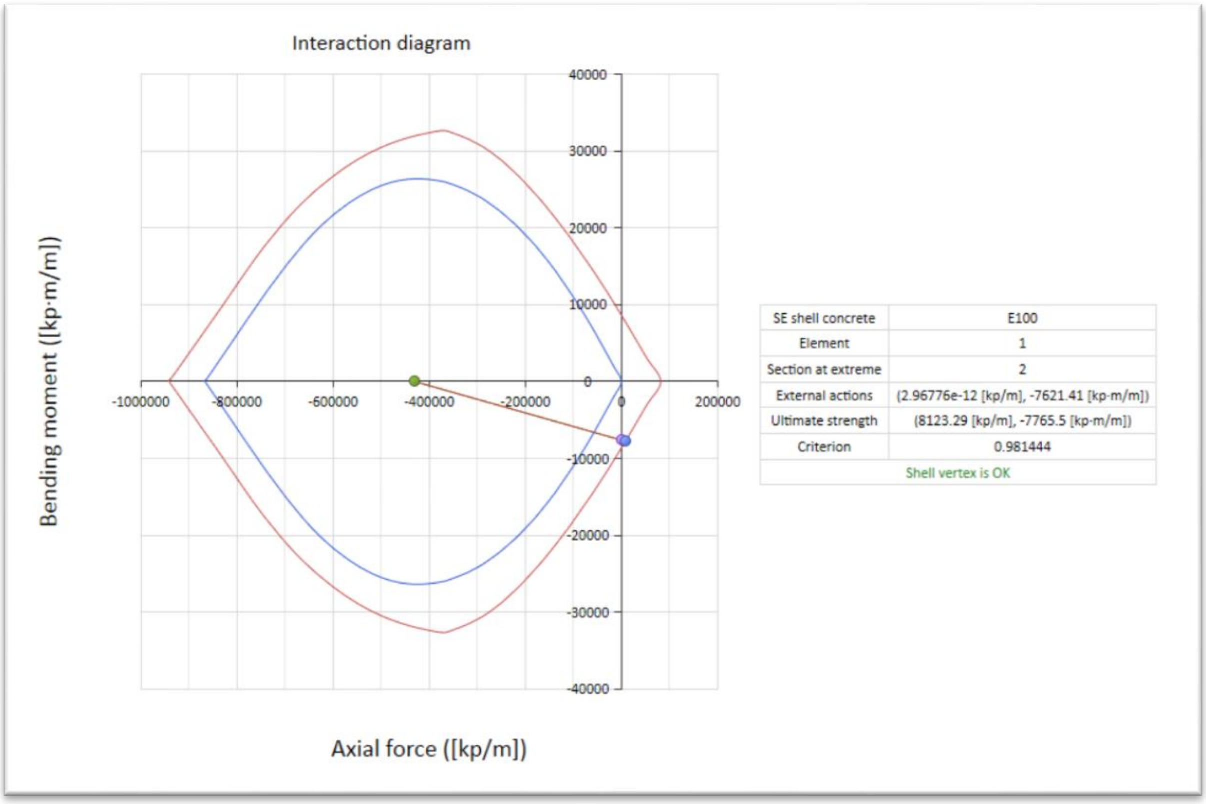
   If there is no A pivot, (no reinforcement steel or if the ACI, AS3600 or BS8110 codes are used) there is no tension limit, and this is considered as infinite.

2. Two extreme admissible strains (EPSmin and EPSmax) are defined (different strains for different materials)
3. For each point of the shell vertex, the minimum ultimate strength curvature (K) is calculated.
4. The K curvature adopted will be the minimum of all the curvatures of the shell vertex points, according to the condition $K \geq 0$.
5. From the obtained K curvature and ε_g (strain imposed at the center of gravity) the deformation corresponding to each of the shell vertex points $\varepsilon(z)$, is determined using EQN.1.
6. From the $\varepsilon(z)$ strain, the stress corresponding to each point of the shell vertex (σ_p) is calculated. With this method, the stress distribution inside the shell vertex will be determined.
7. The ultimate axial force and bending moment is obtained by integrating the resulting stresses.

Note: For the design process, two components of forces and moments will be calculated: the component relative to the fixed points (corresponding to the concrete) and the component relative to the scalable points (corresponding to the bending reinforcement). The final forces and moments will be equal to the sum of the forces and moments of both components. The forces and moments due to the component for scalable points will be multiplied by the reinforcement factor (ω).

$$(\mathbf{F}, \mathbf{M})_{real} = (\mathbf{F}, \mathbf{M})_{fixed} + \omega \cdot (\mathbf{F}, \mathbf{M})_{scalable}$$

Steps 1 to 7 are repeated, adjusting the ε_g value and calculating the corresponding ultimate axial force and bending moment. Therefore, each value of ε_g represents a point in the interaction diagram of the shell vertex.



5.1.4. Axial + Bending Check and Design

5.1.4.2 Calculation Hypothesis

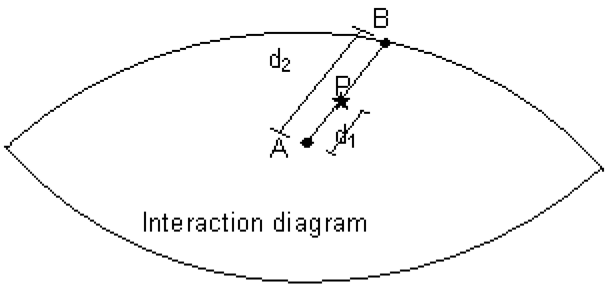
The checking procedure only verifies the shell vertex strength requirements; thus, requirements relating to the serviceability conditions, minimum reinforcement amounts or reinforcement distribution for each code and structural type will not be considered.

It is assumed that plane sections will remain plane. The longitudinal strain of concrete and steel will be proportional to the distance from the neutral axis.

5.1.4.2 Criterion Definition

Checking of elements with regards to axial force and bending moment is performed as follows:

1. Acting forces and moments on the shell vertex (F, M) are obtained from the CivilFEM results file (file .RCF).
2. To construct the interaction diagram of the shell vertex, the ultimate strain state is determined such that the ultimate forces and moments are homothetic to the acting forces and moments with respect to the diagram center.



3. The strength criterion of the shell vertex is defined as the ratio between two distances. As shown above, the distance to the “center” of the diagram (point A of the figure) from the point representing the acting forces and moments (point P of the figure) is labeled as d_1 and the distance to the center from the point representing the homothetic ultimate forces and moments (point B) is d_2.

$$Criterion = \frac{d_1}{d_2}$$

If the criterion is less than 1.00, the forces and moments acting on the shell vertex will be inferior to its ultimate strength, and the shell vertex will be safe. On the contrary, for criterion higher than 1.00, the shell vertex will be considered as not valid.

9.1.4.3 Reinforcement Design

The reinforcement designs produced by the various design methods designed in this chapter will be valid for a criterion value of 1.00 within a tolerance of 1%.

5.2. Wood-Armer Design Method

5.2.1. Hypothesis of the Calculation

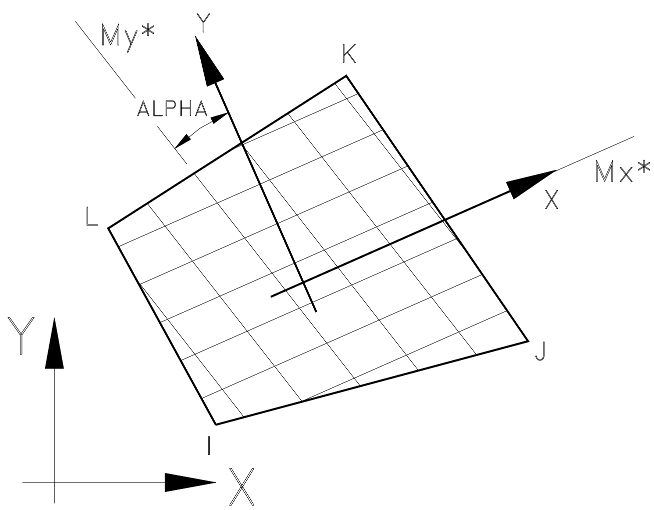
The reinforcement design of shells under bending moments is accomplished by the method developed by R.H. Wood and G.S.T. Armer.

Once the reinforcement design moments have been calculated, a design for flexure is performed for each shell vertex.

5.2.2. Calculation Process of the Reinforcement Design Moments

Bending moments *Mx* and *My* and torsional moments Mxy are calculated from the shell calculation and obtained from the CivilFEM results file. Once these moments are obtained, the program searches for the pair of design moments *Mx** and *My**. This pair of moments is necessary for the reinforcement design and must include all the possible moments generated by *Mx*, *My* and *Mxy* in every direction.

CivilFEM provides the possibility of placing the reinforcement in two oblique directions: in the X direction of the element or in a direction at an angle α with the element Y direction.



Design moments for the bottom reinforcement:

$$Mx^* = Mx - 2Mxy \cdot \tan\alpha + My \cdot \tan^2\alpha + \left|\frac{Mxy - My \cdot \tan\alpha}{\cos\alpha}\right|$$

$$My^* = \frac{My}{\cos^2\alpha} + \left|\frac{Mxy - My \cdot \tan\alpha}{\cos\alpha}\right|$$

If either moment is negative, they will be defined as:

1. If Mx* < 0

$$Mx^* = 0$$

$$My^* = \frac{1}{\cos^2\alpha}\left(My + \left|\frac{(Mxy - My \cdot \tan\alpha)^2}{Mx - 2Mxy \cdot \tan\alpha + My \cdot \tan^2\alpha}\right|\right)$$

2. If My* < 0

$$Mx^* = Mx - 2Mxy \cdot \tan\alpha + My \cdot \tan^2\alpha + \left|\frac{(Mxy - My \cdot \tan\alpha)^2}{My}\right|$$

$$My^* = 0$$

Design moments for the top reinforcement:

$$Mx^* = Mx - 2Mxy \cdot \tan\alpha + My \cdot \tan^2\alpha + \left|\frac{Mxy - My \cdot \tan\alpha}{\cos\alpha}\right|$$

$$My^* = \frac{My}{\cos^2\alpha} + \left|\frac{Mxy - My \cdot \tan\alpha}{\cos\alpha}\right|$$

If either moment is positive, they will be defined as:

3. If Mx* > 0

$$Mx^* = 0$$

$$My^* = \frac{1}{\cos^2\alpha}\left(My - \left|\frac{(Mxy - My \cdot \tan\alpha)^2}{Mx - 2Mxy \cdot \tan\alpha + My \cdot \tan^2\alpha}\right|\right)$$

4. If My* > 0

$$Mx^* = Mx - 2Mxy \cdot \tan\alpha + My \cdot \tan^2\alpha + \left|\frac{(Mxy - My \cdot \tan\alpha)^2}{My}\right|$$

$$My^* = 0$$

From these design moments, the required top and bottom reinforcement amounts will be calculated with the same procedure as for beams under bending moments.

5.2.3. Bending Design

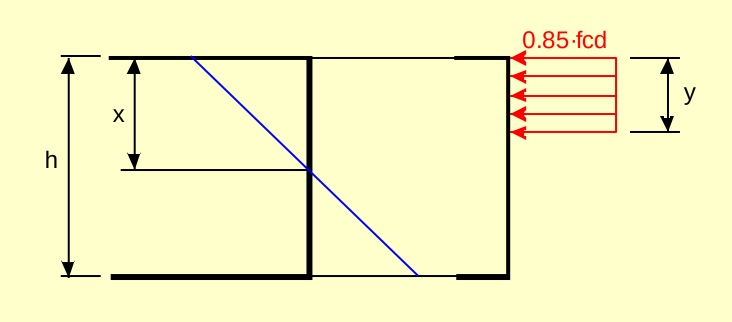
5.2.3.2 Calculation Hypothesis

A rectangular diagram is adopted as the concrete stress-strain diagram. The diagram is formed by a rectangle with a height *y* given by a function of the neutral axis depth *x* and a width equal to 0.85 f_{cd}:

y = 0.8 x for x ≤ 1.25 h

y = h for x > 1.25 h

Where h is the depth of the cross-section.



Concrete rectangular diagram

The steel reinforcement stress-strain diagram is taken as bilinear with the horizontal plastic branch:

$$\sigma_s = E_s \cdot \varepsilon_s \leq f_y$$

The center of gravity of the reinforcement will be placed at a point determined by the mechanical cover defined in each shell vertex.

In the absence of compression reinforcement, the engineering criteria will be taken as the maximum strength of the tensile reinforcement:

$$\sigma_s = f_y$$

5.2.3.2 Calculation Process

Reinforcement design for flexure follows these steps:

1. **Obtaining material strength properties.** These properties are obtained from the material properties associated with each shell vertex, which should be previously defined in CivilFEM database.

2. **Obtaining shell thickness geometrical data.** Shell geometrical data must be defined within the CivilFEM shell structural element.
3. **Obtaining reinforcement data.** The only data concerning flexure design will be the values for the mechanical cover; these must be defined within the CivilFEM shell structural element.
4. **Obtaining internal forces and moments.**

5. **Calculating the limit bending moment.** Depending on the active code, the limit bending moment is calculated as follows:

$$M_{Lim} = 0.85 \cdot f_{cd} \cdot b \cdot \beta \cdot X_{lim} \cdot \left(d - \frac{\beta}{2} X_{lim}\right)$$

Where:

b	Width (one unit length).
d	Effective depth: $d = h - r_c$
h	Shell thickness depth.
r_t	Mechanical cover for the tension reinforcement.
r_c	Mechanical cover for the compression reinforcement.
X_{Lim}	Neutral axis depth for the limit bending moment: $X_{Lim} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{sy}} d$
ε_{cu}	Maximum strain of the extreme compression fiber of the concrete. Depends on the selected code (material EPSmin property).
ε_{sy}	Elongation for the elastic limit: $\varepsilon_{sy} = \frac{f_{yd}}{E_s} d$
β	Compression depth in the concrete rectangular diagram:

Eurocode 2	β = 0.8
EHE	β = 0.8
EB-FIP	β = 0.8
ACI 318 and ACI ACI 349	β = 0.8 if f_{cd} > 4000 psi $\beta = 0.85 - 0.05 \frac{f_{cd} - 4000}{1000} > 0.65 \; if \; f_{cd} > 4000 \; psi$

BS8110 β = 0.8 (values of the bellow figure do not apply for code BS8110)

AS 3600 β = 0.8 if $f_{cd} \leq$ 4000 psi

$$\beta = 0.85 - 0.05\frac{f_{cd} - 4000}{1000} > 0.65\ if\ f_{cd} > 400\ psi$$

GB50010 β = 0.8

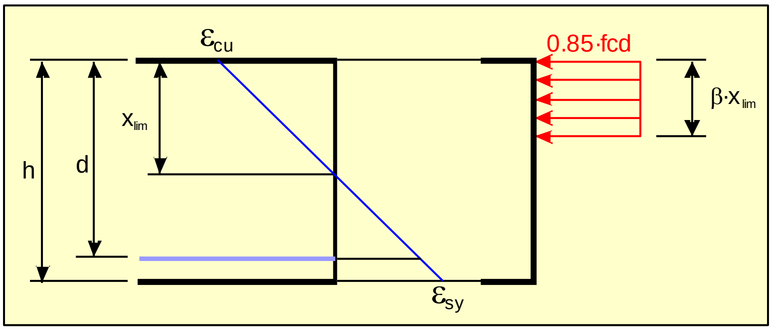
NBR6118 β = 0.8

AASHTO β = 0.8 if f_{cd} ⍰ 4000 psi

$$\beta = 0.85 - 0.05\frac{f_{cd} - 4000}{1000} > 0.65\ if\ f_{cd} > 4000\ psi$$

IS456 β = 0.8

S 52-101 β = 0.8



6. **Calculating the required reinforcement**. If the design bending moment (Md) is greater than the limit bending moment, both the tension and compression reinforcements will be designed. Otherwise, only the tension reinforcement will be designed.

$$Md \leq M_{lim}$$

$$Md = 0.85 \cdot f_{cd} \cdot b \cdot \beta \cdot X_n \left(d - \frac{\beta}{2} X_n\right)$$

From X_n (neutral axis depth), the reinforcements are obtained by:

Tensile reinforcement:

$$Asr = \frac{0.85f_{cd} \cdot b \cdot \beta \cdot X_n}{f_{yd}}$$

Compression reinforcement: Asc=0

Md >M_{lim}

Stress in compression reinforcement is given by:

$$\sigma_{sc} = Es\left(-\varepsilon_{sy} + (\varepsilon_{cu}+\varepsilon_{sy})\frac{d - r_c}{d}\right) < f_{yd}$$

Therefore, the resultant reinforcement is:

Tensile reinforcement:

$$Ast = \frac{M_{lim}}{\left(d - \frac{\beta}{2}X_n\right)f_{yd}} + \frac{Md - M_{lim}}{(d - r_c)f_{yd}}$$

Compression reinforcement:

$$Asc = \frac{Md - M_{lim}}{(d - r_c)\sigma_{sc}}$$

7. **Obtaining design results**. Design results are stored in the CivilFEM results file:

ASTX Reinforcement amount at X top.

ASBX Reinforcement amount at X bottom.

ASTY Reinforcement amount at Y top.

ASBY Reinforcement amount at Y bottom.

5.3. CEB-FIP Method

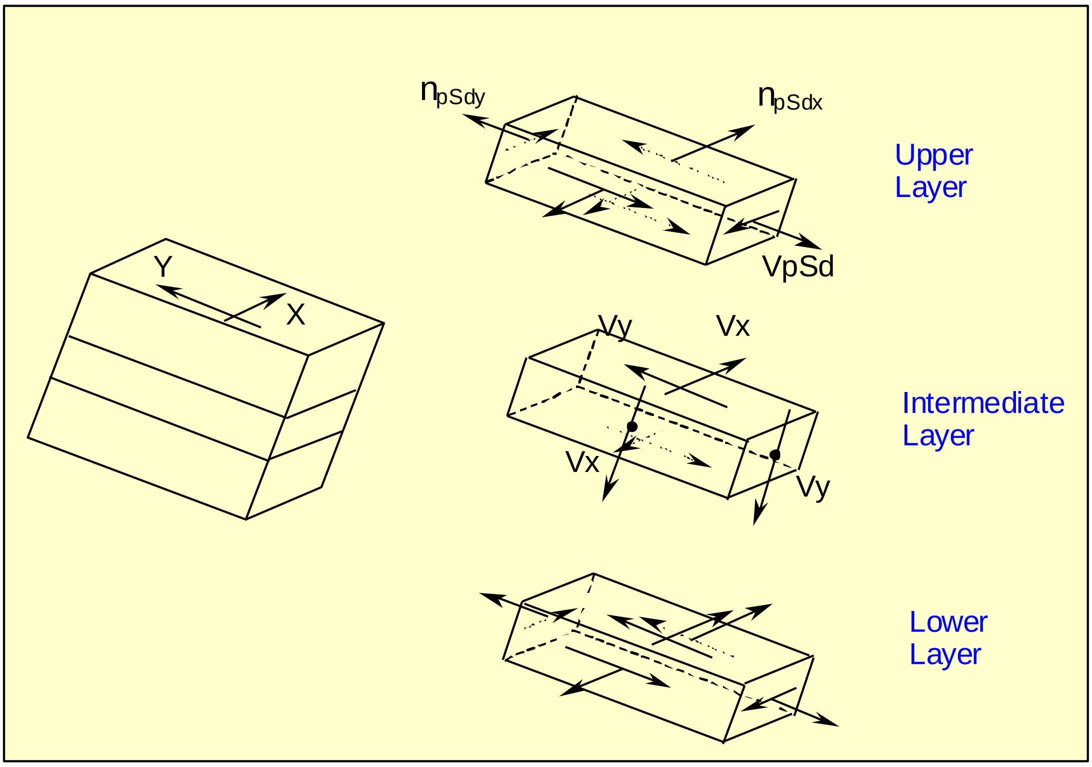
5.3.1. Calculation Hypothesis

Design under Bending Moment and In-Plane Loading:

1. The reinforcement design of shells under bending moment and in plane loading is accomplished by Model Code CEB-FIP 1990.
2. Reinforcements are defined as an orthogonal net (directions of this net are taken as element X and Y axes).

5.3.2. Equivalent Forces and Moments for Reinforcement Calculation

The shell is considered to be divided in three, ideal layers. The outer layers provide resistance to the in-plane effects of both bending and in-plane loading; the inner layer provides for a shear transfer between the outer layers.



From the forces and moments per unit length (m_{Sdx}, m_{Sdy}, m_{Sdxy}, n_{Sdx}, n_{Sdy} and v_{Sd}) that are calculated from the design and obtained from the CivilFEM results file, the following equivalent forces per unit length are obtained:

$$n_{pSdx} = n_{Sdx} \cdot \frac{z_x - y}{z_x} \pm \frac{m_{Sdx}}{z_x}$$

$$n_{pSdy} = n_{Sdy} \cdot \frac{z_y - y}{z_y} \pm \frac{m_{Sdy}}{z_y}$$

$$V_{pSd} = V_{Sd} \cdot \frac{z_V - y}{z_V} \pm \frac{m_{Sdxy}}{z_V}$$

Where:

z_x, z_y, z_v	Lever arms between the axial forces in the X and Y directions respectively and the shear forces.
y	Lever arm between the shear forces (Distance from the mean plane of the slab to the selected force).

Following the Model Code, CivilFEM adopts the values:

$$\frac{z - y}{z} = \frac{1}{2}$$

$$z = \frac{2h}{3}$$

Where h is the overall thickness of the plate.

So, the former equations change now to:

$$n_{pSdx} = \frac{1}{2} n_{Sdx} \pm \frac{3}{2h} m_{Sdx}$$

$$n_{pSdy} = \frac{1}{2} n_{Sdy} \pm \frac{3}{2h} m_{Sdy}$$

$$V_{pSd} = \frac{1}{2} V_{Sd} \pm \frac{3}{2h} m_{Sdxy}$$

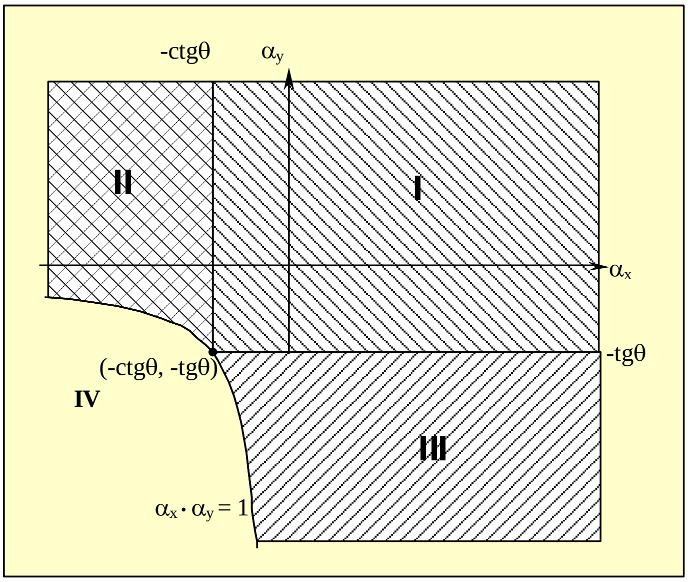
5.3.3. States and Resistance

These parameters are obtained by:

$$\alpha_x = \frac{n_{pSdx}}{|v_{pSd}|}$$

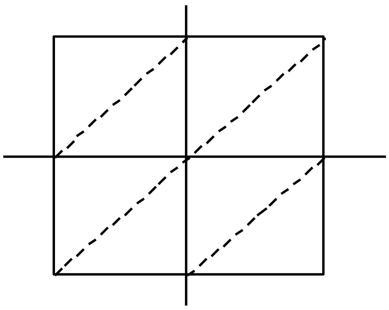
$$\alpha_y = \frac{n_{pSdy}}{|v_{pSd}|}$$

They are also represented in the following figure:

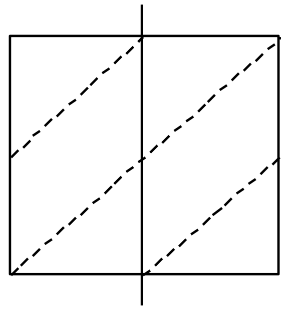


Depending on position of the point ($\alpha_{x,}$ α_y), the applicable procedure is as follows (If $|v_{Sd}| \approx 0$, the program utilizes the sign of n_{Sdx} and n_{Sdy}, to place the point in the correct zone). The internal system providing resistance to in-plane loading may be one of four cases:

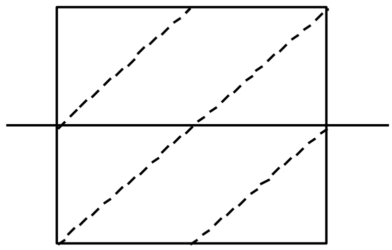
CASE I - Tension in reinforcement in two directions and oblique compression in concrete.



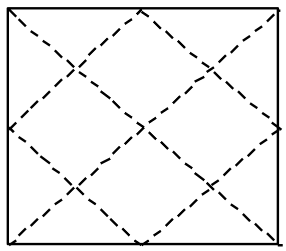
CASE II - Tension in reinforcement in Y direction and oblique compression in concrete.



CASE III - Tension in reinforcement in X direction and oblique compression in concrete.



CASE IV - Biaxial compression in the concrete.



According to the case, resistances for the ultimate limit states are the following:

Case	Reinforcements	Concrete
I	f_{ytd}	f_{cd2}
II	f_{ytd}	f_{cd2}
III	f_{ytd}	f_{cd2}
IV	f_{ytd}	f_{cd1}

Where:

$f_{ytd} = f_{ytk} / \gamma_s$ Design tension strength of steel

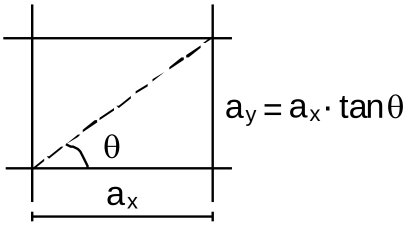
$f_{cd2} = 0.60 \, [1 - f_{ck}/250] \, f_{cd}$ (MPa)

$f_{cd1} = 0.85\ [1 - f_{ck}/250]\ f_{cd}$ (MPa)

5.3.4. Checking Outline

5.3.4.1. Cases

It is assumed that the shell is reinforced with an orthogonal mesh with dimensions of a_x and a_y.



The angle θ is defined between the X-axis and the direction of compression. It can be defined by the user adhering to the condition of $1/3 \geq \tan\theta \geq 3$ (By default, θ = 45º).

Forces and moments that support a cell of a_x x a_y dimensions are:

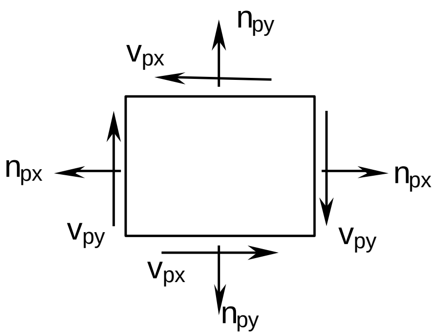
$$n_{px} = a_y \cdot n_{pSdx}$$

$$n_{py} = a_x \cdot n_{pSdy}$$

$$v_{px} = a_x \cdot v_{pSd}$$

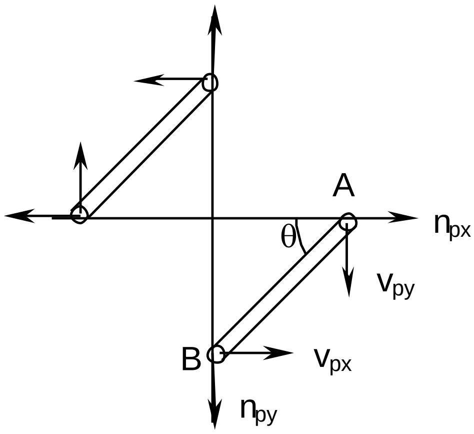
$$v_{py} = a_y \cdot v_{pSd}$$

In general, $v_{px} \neq v_{py}$

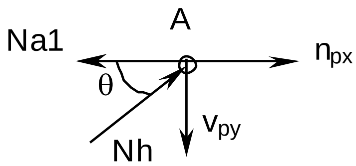


1. CASE 1

The method of struts and ties will be applied to the following truss:

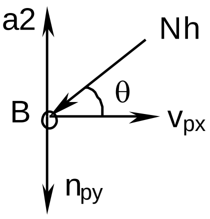


Applying the forces equilibrium in node A:



$$\left.\begin{aligned} Na1 - Nh \cdot \cos\theta = n_{px} \\ Nh \cdot \sin\theta = v_{py} \end{aligned}\right\} Nh = \frac{v_{py}}{\sin\theta};\ Na1 = n_{px} + Nh \cdot \cos\theta$$

From the equilibrium of node B, the result is:



$$\left.\begin{aligned} Na1 - Nh \cdot \sin\theta = n_{py} \\ Nh \cdot \cos\theta = v_{px} \end{aligned}\right\} Nh = \frac{v_{px}}{\cos\theta};\ Na2 = n_{py} + Nh \cdot \sin\theta$$

To check if these forces and moments are feasible, the strength of the concrete is checked.

Concrete area:

$$A_c = \frac{a_y}{2}\cos\theta \cdot \frac{\min(z_x, z_y)}{2}$$

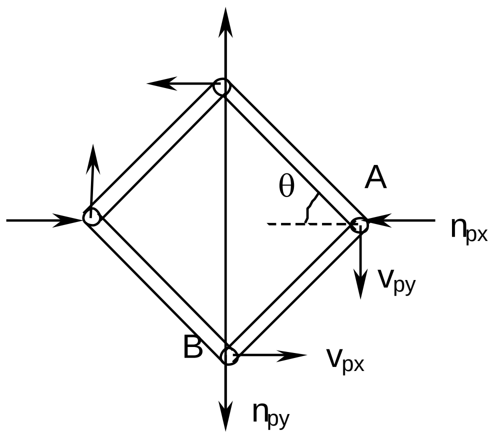
Stress on concrete struts:

$$\mathrm{FCMAX} = \frac{\mathrm{Nh}}{\mathrm{A_c}}$$

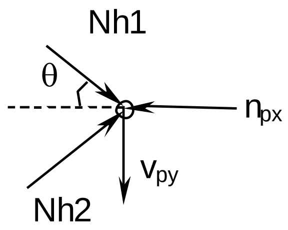
This stress is compared to f_{cd2} to obtain the concrete maximum compression criterion:

$$\mathrm{CRTFC} = \frac{\mathrm{FCMAX}}{\mathrm{f_{cd2}}}$$

2. CASE II



By equilibrium in node A:



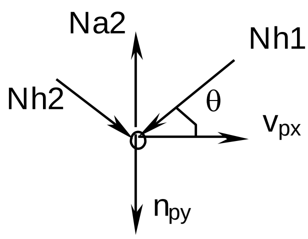
$$\mathrm{Nh1}\cos\theta + \mathrm{Nh2}\cos\theta = n_{px}$$

$$\mathrm{Nh1}\sin\theta - \mathrm{Nh2}\sin\theta = -V_{py}$$

$$\mathrm{Nh} = \frac{1}{2}\left[\frac{n_{px}}{\cos\theta} - \frac{v_{py}}{\sin\theta}\right]$$

$$\mathrm{Nh2} = \frac{1}{2}\left[\frac{n_{px}}{\cos\theta} - \frac{v_{py}}{\sin\theta}\right] > 0$$

By equilibrium in node B:



$$Na2 = \sin\theta\ (Nh1+Nh2) + n_{py}$$

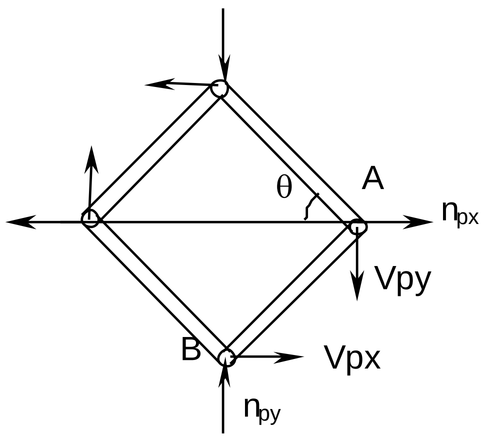
Maximum compression stress on concrete struts:

$$\mathrm{FCMAX} = \frac{\mathrm{Max(Nh1, Nh2)}}{\mathrm{A_c}}$$

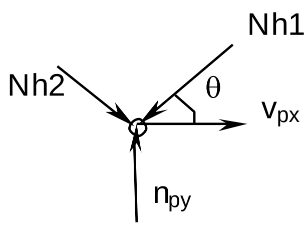
This stress is compared to f_{cd2} to obtain the concrete maximum compression criterion:

$$\mathrm{CRTC} = \frac{\mathrm{FCMAX}}{\mathrm{f_{cd2}}}$$

3. CASE III



By equilibrium in node B:



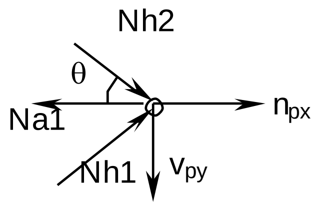
$$Nh1\ \cos\theta - Nh2\ \cos\theta = v_{px}$$

$$Nh1\ \sin\theta + Nh2\ \sin\theta = n_{py}$$

$$Nh1 = \frac{1}{2}\left[\frac{v_{px}}{\cos\theta} - \frac{n_{py}}{\sin\theta}\right]$$

$$Nh2 = \frac{1}{2}\left[\frac{v_{px}}{\cos\theta} - \frac{n_{py}}{\sin\theta}\right] > 0$$

By equilibrium in node A:



$$Na1 = (Nh1 + Nh2)\cos\theta + n_{px}$$

The maximum compression stress on concrete struts:

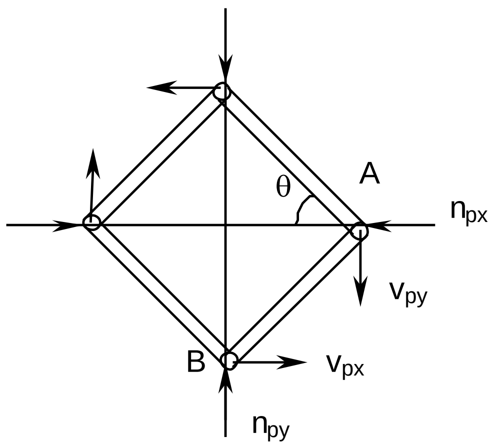
$$FCMAX = \frac{Max(Nh1, Nh2)}{A_c}$$

This stress is compared to f_{cd2} to obtain the maximum compression of the concrete criterion:

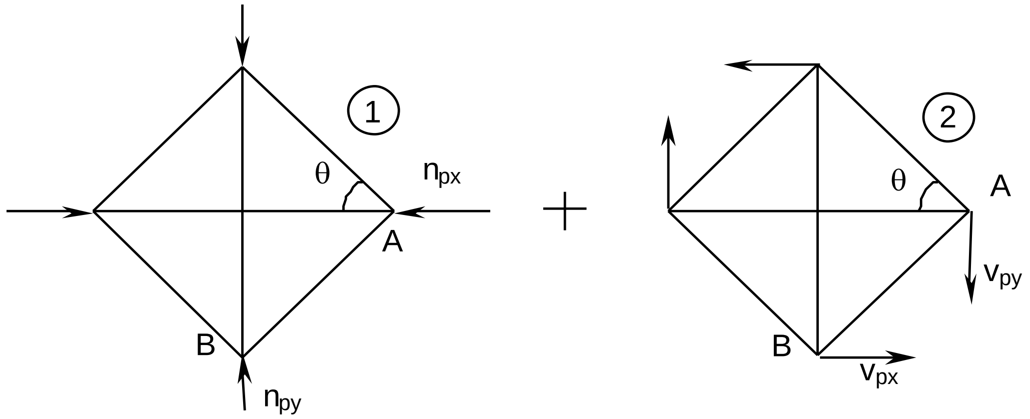
$$CRTFC = \frac{FCMAX}{f_{cd2}}$$

4. CASE IV – Assuming reinforcing bars are braced

In this situation, the struts and tie model will be the following:



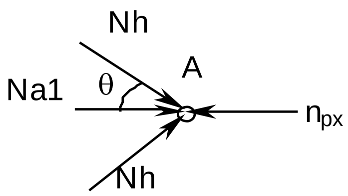
Hyperstatic structure to be separated into two load states.



Both states have simple solutions due to symmetry.

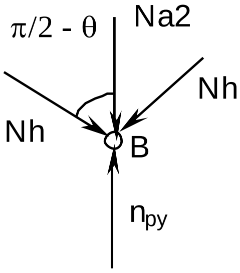
- Solution of Structure 1:

- Node A:



$$2Nh \cdot \cos\theta + Na1 = n_{px}$$

- Node B:



$$2Nh \cdot \sin\theta + Na2 = n_{py}$$

- Movements compatibility

Where:

Ah = Concrete strut area

Aa1 = Horizontal steel amount

Aa2 = Vertical steel amount

Eh = Concrete modulus of elasticity

Ea = Steel modulus of elasticity

a = Cell width (ax)

b = Cell depth (ay), (b/a = tan θ)

$$\beta_1 = \frac{Ah \cdot Eh}{Aa1 \cdot Ea}$$

$$\beta_2 = \frac{Ah \cdot Eh}{Aa2 \cdot Ea}$$

The length of the concrete struts before deformation:

$$L = \sqrt{a^2 + b^2}$$

$$\sin\theta = b/L$$

Differentiating this expression:

$$\Delta L = \frac{2a\Delta a + 2b\Delta b}{2L} = \cos\theta \Delta a + \sin\theta \Delta b$$

However, Δa and Δb must coincide with the strain of steel bars:

$$\Delta a = \frac{Na1}{Aa1 \cdot Ea} \cdot a$$

$$\Delta b = \frac{Na2}{Aa2 \cdot Ea} \cdot b$$

ΔL must coincide with the strain of the concrete struts:

$$\Delta a = \frac{Nh}{Eh \cdot Ah} \cdot L$$

$$\frac{Nh}{Eh \cdot Ah} \cdot L = \cos\theta \cdot \frac{Na1}{Aa1 \cdot Ea} \cdot a + \sin\theta \cdot \frac{Na2}{Aa2 \cdot Ea} \cdot b$$

$$Nh = \beta_1 \cos^2\theta \cdot Na1 + \beta_2 \sin^2\theta \cdot Na2$$

From the obtained equations, the following linear system is created:

$$\begin{bmatrix} 1 & 0 & 2\cos\theta \\ 0 & 1 & 2\sin\theta \\ \beta_1\cos^2\theta & \beta_2\sin^2\theta & -1 \end{bmatrix} \begin{bmatrix} Na1 \\ Na2 \\ Nh \end{bmatrix} = \begin{bmatrix} n_{px} \\ n_{py} \\ 0 \end{bmatrix}$$

Which when solved gives:

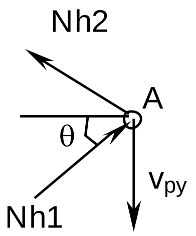
$$Nh^{(1} = \frac{\beta_1\cos^2\theta \cdot n_{px} + \beta_2\sin^2\theta \cdot n_{py}}{1 + 2(\beta_1\cos^3\theta + \beta_2\sin^3\theta)}$$

$$Na1^{(1} = n_{px} - 2\cos\theta \cdot Nh^{(1}$$

$$Na2^{(1} = n_{py} - 2\sin\theta \cdot Nh^{(1}$$

- <u>Solution of Structure 2:</u>

Due to non-symmetrical loads, the central bars (steel) are not applicable; therefore, equation 2 is determinant, and the following expression is obtained:



$$Nh2\ \sin\theta + Nh1\ \sin\theta = v_{py}$$

$$Nh2\ \cos\theta - Nh1\ \cos\theta = 0$$

$$Nh1 = Nh2 = \frac{v_{py}}{2\sin\theta}$$

Therefore:

$$Nh^{(1} = \frac{V_{py}}{2\sin\theta)}$$

$$Na1^{(1} = 0$$

$$Na2^{(1} = 0$$

<u>Total Actions in Case IV</u>

Adding the actions of 1 and 2:

$$Nh = A \cdot n_{px} + B \cdot n_{py} \pm C \cdot v_{py} > 0$$

$$Na1 = (1 - 2A\cos\theta) \cdot n_{px} - 2B\cos\theta \cdot n_{py}$$

$$Na2 = -2A\sin\theta \cdot n_{px} + (1 - 2B\sin\theta) \cdot n_{py}$$

Where:

$$A = \frac{\beta_1 \cos^2\theta}{1 + 2(\beta_1 \cos^3\theta + \beta_2 \sin^3\theta)}$$

$$B = \frac{\beta_2 \sin^2\theta}{1 + 2(\beta_1 \cos^3\theta + \beta_2 \sin^3\theta)}$$

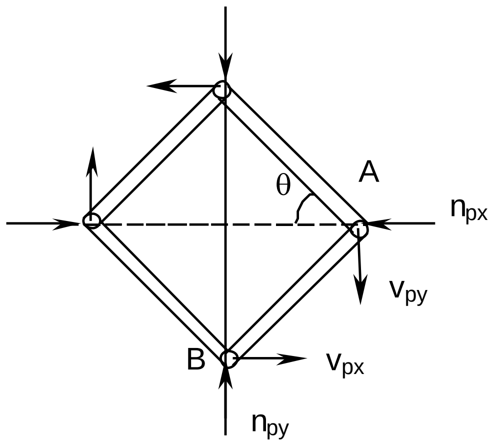
$$C = \frac{1}{2\sin\theta}$$

With the assumption of braced bars, Na1 and Na2 signs correspond to compression for a + sign and tension for a - sign.

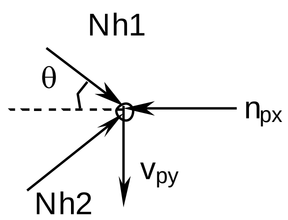
5. CASE IV – Assuming reinforcing bars are not braced

For steel bars without braces, there are two possible determinant truss configurations.

- Case 1



By equilibrium in A node:



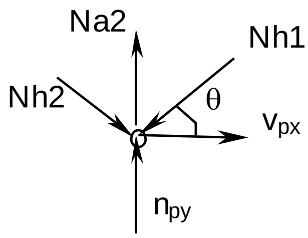
$$Nh1\ \cos\theta + Nh2\ \cos\theta = n_{px}$$

$$Nh1\ \sin\theta - Nh2\ \sin\theta = v_{py}$$

$$Nh1 = \frac{1}{2}\left[\frac{n_{px}}{\cos\theta} + \frac{v_{py}}{\sin\theta}\right]$$

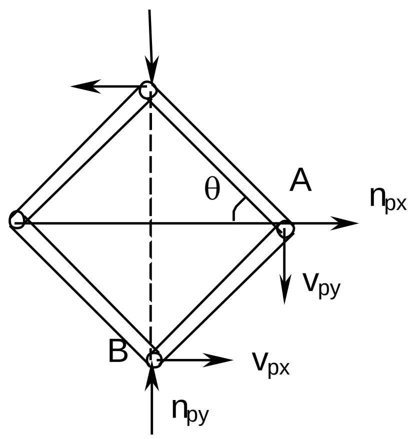
$$Nh2 = \frac{1}{2}\left[\frac{n_{px}}{\cos\theta} - \frac{v_{py}}{\sin\theta}\right] > 0$$

By equilibrium in B node:

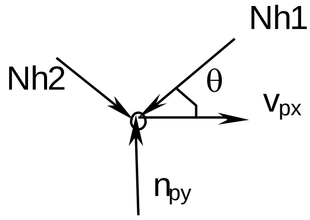


$Na2 = \sin\theta\ (Nh1+Nh2) - n_{py} > 0$

- Case 2



By equilibrium in B node:



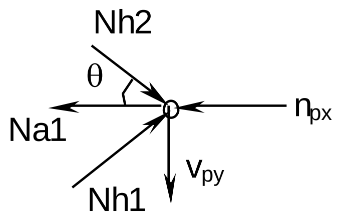
$$Nh1\ \cos\theta - Nh2\ \cos\theta = v_{px}$$

$$Nh1\,\sin\theta + Nh2\,\sin\theta = n_{py}$$

$$Nh1 = \frac{1}{2}\left[\frac{n_{py}}{\sin\theta} + \frac{v_{px}}{\cos\theta}\right]$$

$$Nh2 = \frac{1}{2}\left[\frac{n_{py}}{\sin\theta} - \frac{v_{px}}{\cos\theta}\right] > 0$$

By equilibrium in A node:



$$Na1 = \sin\theta\,(Nh1 + Nh2) - n_{px} > 0$$

- Discussion:

With this situation, CivilFEM will select whichever of the two cases satisfies:

$$Nh1, Nh2 \geq 0 \ \text{ and } \ Na1, Na2 \geq 0$$

If neither case results in appropriate signs, it will be impossible to equilibrate the force and moment states without bracing the steel bars.

The maximum compression stress on the concrete struts is:

$$FCMAX = \frac{Max(Nh1, Nh2)}{A_c}$$

This stress is compared with f_{cd1} to obtain the concrete maximum compression criterion:

$$CRTFC = \frac{FCMAX}{f_{cd1}}$$

5.3.4.2. Steel amount

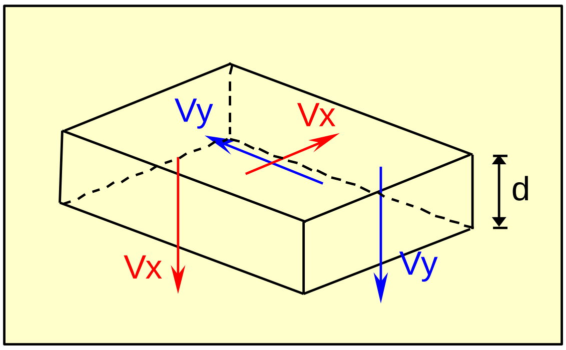
For all the cases, steel reinforcement amounts per unit length of the shell are:

$$Va1 = \frac{Na1}{f_{ybd}} cotan\theta$$

$$Va2 = \frac{Na2}{f_{ybd}}$$

5.3.5. Reinforcement Checking in Intermediate Layer

The checking process described in 6.4.2.5 article of Model Code CEB-FIP1990 will be executed.



The principal shear force is:

$$V1 = \sqrt{Vx^2 + Vy^2}$$

Acting on a surface at an angle ϕ, relative to the Y-axis

$$\phi = \arctan \frac{V_y}{V_x}$$

The following check is to performed: $V1 \leq V_{Rdl}$

$$V_{Rdl} = 0.12 \cdot \xi \cdot (100 \cdot \rho \cdot f_{ck})^{1/3} \cdot d$$

$$\rho = \rho_x \cos^4\theta + \rho_y \sin^4\theta$$

$$\xi = 1 + \sqrt{\frac{200}{d}}$$

Where d is the total depth without the mechanical cover (in mm), and ρx, ρy are the ratios for the reinforcement closest to the face in tension, in the direction perpendicular to the surface that V1 acts on.

5.3.6. Required Parameters

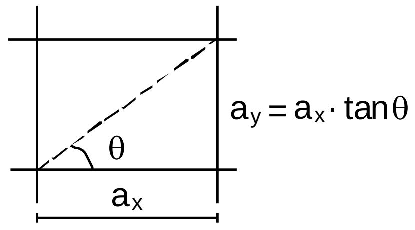
5.3.6.1. General Requirements

- Material properties described in section 9.3.3.
- zx, zy, zv and y parameters which are defined for each element as a fraction of the depth at each point. As previously stated, CivilFEM uses the specifications from section 6.5.4 of the CEB Model Code.

$$z = \frac{2h}{3}$$

$$\frac{z - y}{z} = \frac{1}{2}$$

- The parameter that indicates whether the bars of the element are braced.
- Angle θ between the reinforcement X axis (element X axis) and the direction of compression. By default, θ = 45º (although any angle is valid if $1/3 \geq \tan\theta \geq 3$).



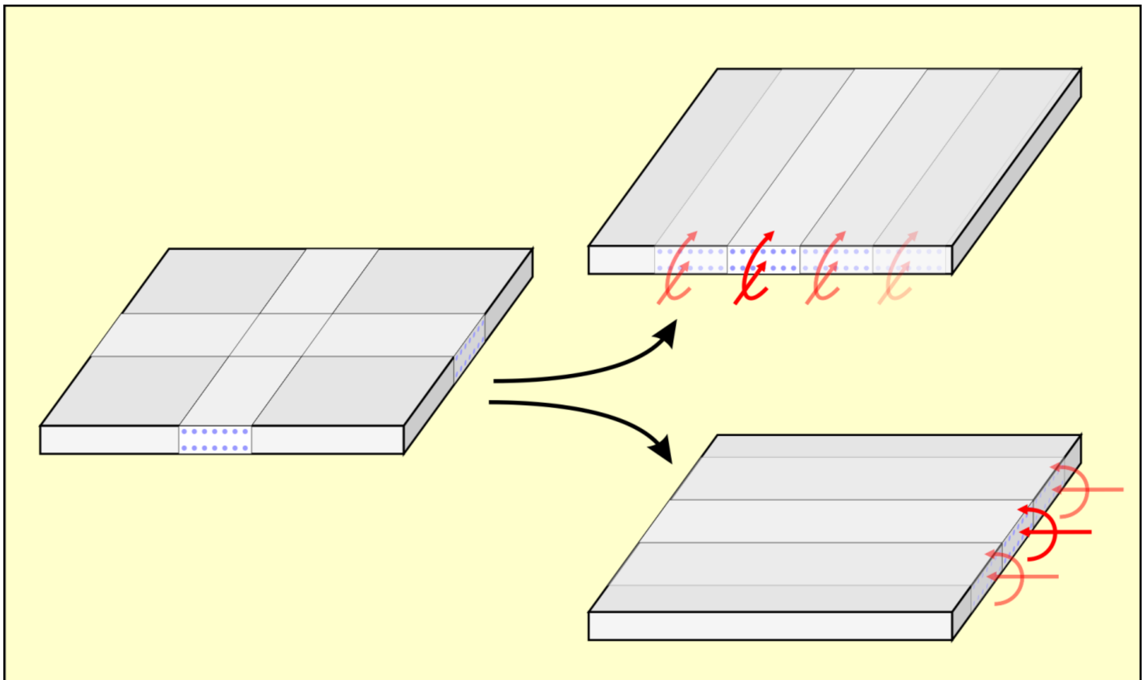
5.3.6.2. Checking Requirements

Va1 and Va2 reinforcement amounts per unit length of the shell.

5.4. Design according to the Orthogonal Directions Method

5.4.1. Calculation Hypothesis

1. The design of reinforcement for bending moments and axial forces is performed independently for each direction.
2. Reinforcements are defined as an orthogonal mesh (directions of this mesh are taken as element X and Y axes).



Orthogonal Directions Method

5.4.2. Design Forces and Moments

The axial forces (T^*_x, T^*_y) and bending moments (M^*_x, M^*_y) used for the design are those obtained for the reinforcement directions as follows:

If torsional moment and membrane shear force are neglected:

$$T^*_x = T_x$$

$$T^*_y = T_y$$

$$M^*_x = M_x$$

$$M^*_y = M_y$$

If torsional moment (M_{xy}) and membrane shear force (T_{xy}) are taken into account, then two processes are performed depending on considering membrane (in-plane) shear as tension and as compression.

1) Torsional moment and membrane shear force in tension:

$$T^*_x = T_x + |T_{xy}| \cdot \mathrm{Sign}(T_x)$$

$$T^*_y = T_y + |T_{xy}| \cdot \mathrm{Sign}(T_y)$$

$$M^*_x = M_x + |M_{xy}| \cdot \mathrm{Sign}(M_x)$$

$$M^*_y = M_y + |M_{xy}| \cdot \mathrm{Sign}(M_y)$$

2) Torsional moment and membrane shear force in compression:

$$T^*_x = T_x - |T_{xy}| \cdot \mathrm{Sign}(T_x)$$

$$T^*_y = T_y - |T_{xy}| \cdot \mathrm{Sign}(T_y)$$

$$M^*_x = M_x + |M_{xy}| \cdot \mathrm{Sign}(M_x)$$

$$M^*_y = M_y + |M_{xy}| \cdot \mathrm{Sign}(M_y)$$

If only torsional moment (M_{xy}) is considered:

$$T^*_x = T_x$$

$$T^*_y = T_y$$

$$M^*_x = M_x + |M_{xy}| \cdot \mathrm{Sign}(M_x)$$

$$M^*_y = M_y + |M_{xy}| \cdot \mathrm{Sign}(M_y)$$

Where X and Y represent the orthogonal directions of bending reinforcement of the shell.

5.4.3. Maximum Allowable Stress/Strain in Reinforcement

Reinforcement is designed using one of the following conditions:

- The maximum strain allowed in tension is defined for the material (EPSMAX). It will be used as pivot A in the interaction diagram. This condition is typically used for Ultimate Limit States.
- The maximum stress allowed is specified. This condition is typically used for Serviceability Limit States in order to control cracking.

5.4.4. Check and Design

Reinforcements design for the Orthogonal Directions method follows these steps:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated with each shell structural element, which should be previously defined in CivilFEM model.
2) **Obtaining shell vertex geometrical data.** Vertex geometrical data must be defined within the CivilFEM model.
3) **Obtaining reinforcement data.** The only data associated with the bending moment design are the mechanical cover values for the reinforcement; these must be defined within the CivilFEM shell structural elements.
4) **Obtaining internal forces and moments.** The acting bending moments and axial forces are those obtained for the X and Y directions of each element (T^*_x, T^*_y, M^*_x, M^*_y).
5) **Check and design**. Depending on the active code, the checking or design is performed using the pivot diagram described for the checking and design of concrete cross sections. For checking, the criteria for axial force and bending moment are obtained as for the pivot diagram for beams for each direction.

   All reinforcements are considered as scalable for design. The obtained reinforcement factor is therefore the value that must be used to multiply the upper and lower reinforcement amount to fulfill the code requirements.

6) **Checking results**. Checking results are stored in the CivilFEM results file:
   Criterion for X direction.

   Criterion for Y direction.

7) **Design results**. Design results are stored in the CivilFEM results file:
   Reinforcement amount for X direction, top surface.

   Reinforcement amount for X direction, bottom surface.

   Reinforcement amount for Y direction, top surface.

   Reinforcement amount for Y direction, bottom surface.

   Design criterion for X direction.

   Design criterion for Y direction.

5.5. Out-of-Plane Shear Load according to EC2 and ITER

Check and Design for Out-of-Plane Shear Loadings according to Eurocode 2 (EN 1992-1-1:2004/AC:2008) and ITER Design Code

5.5.1. Required Input Data

Shear check or design according to Eurocode 2 (EN 1992-1-1:2004/AC:2008) and ITER Design Code requires a series of parameters described below:

1) **Materials strength properties.** These properties are obtained from the material properties associated with each one of the shell vertices and for the active time. Those material properties should be previously defined. The required data are the following:

   f_{ck} characteristic compressive strength of concrete.

   f_{cd} design strength of concrete.

   f_{yk} characteristic yield strength of reinforcement.

   f_{ywd} design strength of reinforcement.

   γ_c partial safety factor for concrete.

   γ_s partial safety factor for reinforcement.

2) **Shell vertex geometrical data:**
   th thickness of the shell vertex (shell structural element).

3) **Geometrical parameters. Required data are the following:**

   c bending reinforcement mechanical cover (shell structural element).

   ρ_{1i} ratio of the longitudinal tensile reinforcement per unit length of the shell:

$$\rho_{1i} = \frac{A_{ssi}}{th - c} < 0.02$$

where:

A_{ss} area of the tensile reinforcement (shell structural element).

θ angle of the compressive struts of concrete with the longitudinal axis of the member, (parameter THETA of shell structural element):

$1.0 \leq \text{cotan}\ \theta \leq 2.5$ Eurocode 2 (EN 1992-1-1:2004/AC:2008)

$1.0 \leq \text{cotan}\ \theta \leq \text{cotan}\ \theta_0$ ITER Design Code

Mean compressive stress $(\sigma_{cp} > 0)$: $\text{cotan}\ \theta_0 = 1.2 + 0.2\ \sigma_{cp}/f_{ctm}$

Mean tensile stress $(\sigma_{cp} < 0)$: $\text{cotan}\ \theta_0 = 1.2 + 0.9\ \sigma_{cp}/f_{ctm} \geq 1$

4) **Shell vertex reinforcement data.** Required data are the following:

A_{ss} area of reinforcement per unit area, (parameter ASS of shell structural element).

The reinforcement ratio may also be obtained with the following data:

sx, sy spacing of the stirrups in each direction of the shell, (parameters SX and SY of shell structural element).

ϕ diameter of bars (parameter PHI of shell structural element).

n_x, n_y number of stirrups per unit length in each direction of the shell (parameters NX and NY of shell structural element).

5) **Shell vertex internal forces**. The shear force (V_{Ed}) acting on the vertex as well as the concomitant axial force (N_{Ed}) are obtained from the CivilFEM results file (.RCF).

$$V_{Ed} = \sqrt{{N_x}^2 + {N_y}^2}$$

which forms an angle with the axis Y

$$\phi_v = \arctan \frac{N_y}{N_x}$$

The value taken for the design compression force (T_{Ed}) is the maximum considering all directions:

$$T_{Ed} = \frac{1}{2}\left(T_x + T_y - \sqrt{\left(T_x + T_y\right)^2 - 4\left(T_x \cdot T_y - {T_{xy}}^2\right)}\right)$$

Design compression force takes into account axial force in x, y directions and in-plane shear using Mohr’s circle stress transformation equations.

The total shear reinforcement ρ_1 is computed from those in each direction, according to equation LL.123 (Annex LL from EN 1992-2:2005):

$$\rho_1 = \rho_x \cdot \cos^2\phi_v + \rho_y \cdot \sin^2\phi_v$$

5.5.2. Out-of-Plane Shear Checking

5.5.2.1. Checking whether shear reinforcement is required

Design shear force V_{Ed} is compared with the design shear resistance ($V_{Rd,c}$):

$$V_{Ed} \leq V_{Rd,c}$$

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100\rho_I \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

With the constraints:

$$V_{Ed} \leq 0.5 b_w \cdot d \cdot \boldsymbol{\nu} \cdot f_{cd}$$

$$V_{Rd,c} \geq \left[V_{min} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

Where:

$C_{Rd,c} =$ $0.18 y_c$

f_{ck} in MPa

$k \quad =$ $1 + \sqrt{\dfrac{200}{d}} \leq 2.0 (\text{d en mm})$

$k_1 \quad =$ 0.15

$\sigma_{cp} \quad =$ $\dfrac{T_{Ed}}{A_c} < 0.2 f_{cd}$ MPa

A_c in mm^2

$\nu \quad =$ $0.6\left(1 - \frac{f_{ck}}{250}\right)$ Eurocode 2 (EN 1992-1-1:2004/AC:2008)

$\begin{cases} 0.6 & f_{ck} \leq 60 \text{ MPa} \\ 0.9 - \frac{f_{ck}}{200} > 0.5 & f_{ck} > 60\ MPa \end{cases}$ ITER Design Code

$v_{min} =$ $0.035\,(k^3 \cdot f_{ck})^{1/2}$

$v_{Rd,c} =$ in N

If shear reinforcement is defined in the section, V_{Ed} must be less than the minimum between the shear reinforcement force:

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot 0.9 \cdot d \cdot f_{ywd} \cdot (\cot\theta + \cot \alpha) \cdot \sin\alpha$$

and the maximum design shear force resisted without crushing of concrete compressive struts:

Eurocode 2 (EN 1992-1-1:2004/AC:2008):

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot 0.9 \cdot d \cdot \nu \cdot f_{cd} \cdot \frac{(\cot\theta + \cot \alpha)}{(1 + \cot^2\theta)}$$

ITER Design Code:

$$V_{Rd,max} = \alpha_{cw} \cdot \nu \cdot f_{cd} \cdot b_w \cdot 0.9 \cdot d \cdot \left(\frac{\cot\theta_0 + \cot\alpha}{(1 + \cot^2\theta_0)}\right)$$

where:

$$\alpha_{cw} = \begin{cases} 1, \text{if the section is not prestressed} \\ 1 - \dfrac{\sigma_{cp}}{f_{cd}}, \quad \text{if } 0 < \sigma_{cp} \le 0.25 f_{cd} \\ 1.25, \text{if} \quad 0.25 f_{cd} < \sigma_{cp} \le 0.5 f_{cd} \\ 2.5\left(1 - \dfrac{\sigma_{cp}}{f_{cd}}\right), \text{if } 0.5 f_{cd} < \sigma_{cp} \le f_{cd} \end{cases}$$

The shear reinforcement must be less than or equal to (Eurocode 2 only):

$$\frac{A_{sw,max}}{s} = 0.5 \cdot \alpha_{cw} \cdot b_w \cdot \boldsymbol{\nu} \cdot \frac{f_{cd}}{f_{yd}}$$

Results are written for each end in the CivilFEM results file:

If there is no shear reinforcement defined, the following results can be obtained:

$\mathrm{VRDC} \quad = \quad V_{Rd,c}$

$\mathrm{VRDS} \quad = \quad V_{Rd,s}$

$\mathrm{VRDMAX} \quad = \quad V_{Rd,max}$

$\mathrm{TENS} \quad = \quad \frac{V_{Ed}}{2} \cot\theta$

Tensile strength for the longitudinal reinforcement

$\mathrm{CRT\_1} \quad = \quad \frac{V_{Ed}}{V_{Rd,c}}$

$\mathrm{CRT\_2} \quad = \quad 0,$ Shear reinforcement not defined

$$CRT\_3 = \begin{cases} \frac{V_{Ed}}{V_{Rd,s}}, & \text{Shear reinforcement defined} \\ 0, & \text{Shear reinforcement not defined} \\ \frac{V_{Ed}}{V_{Rd,max}}, & \text{Shear reinforcement defined} \end{cases}$$

5.5.2.2. *Shear Criterion*

The shear criterion indicates whether the shell vertex is valid for the design forces (if it is less than 1, the vertex satisfies the code provisions; whereas if it exceeds 1, the vertex will not be valid). Furthermore, it includes information about how close the design force is to the ultimate strength. The shear criterion is defined as follows:

$$CRT\_TOT = \begin{cases} \frac{V_{Ed}}{V_{Rd,c}} & \text{If shear reinforcement is not defined.} \\ \min\{\frac{V_{Ed}}{V_{Rd,c}}, \max\{\frac{V_{Ed}}{V_{Rd,s}}, \frac{V_{Ed}}{V_{Rd,máx}}\}\}, & \text{If shear reinforcement is defined.} \end{cases}$$

A value of 2^{100} for this criterion indicates that $V_{Rd,c}$, $V_{Rd,s}$ or $V_{Rd,max}$ are null.

5.5.3. Out-of-Plane Shear Design

5.5.3.1. *Checking whether shear reinforcement is required*

First, a check is made to determine if the design shear force V_{Ed} is less than or equal to the shear design resistance ($V_{Rd,c}$):

$$V_{Ed} \leq V_{Rd,c}$$
$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100\rho_I \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

with constraints:

$$V_{Ed} \leq 0.5 b_w \cdot d \cdot \nu \cdot f_{cd}$$
$$V_{Rd,c} \geq \left[V_{min} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

where:

$C_{Rd,c}$ = $0.18\, y_c$

f_{ck} in MPa

k = $1 + \sqrt{\frac{200}{d} \leq 2.0}$ (d in mm)

k_1 = 0.15

σ_{cp} = $\frac{N_{Ed}}{A_c} < 0.2\, f_{cd}$ MPa

A_C in mm^2

$0.6\left(1 - \frac{f_{ck}}{250}\right)$ Eurocode 2 (EN 1992-1-1:2004/AC:2008)

$\boldsymbol{\nu}$ = $\begin{cases} 0.6 & f_{ck} \leq 60\ \mathrm{MPa} \\ 0.9 - \frac{f_{ck}}{200} > 0.5 & f_{ck} > MPa \end{cases}$

V_{min} = $0.035(k^3 \cdot f_{ck})^{1/2}$

$V_{Rd,c}$ in N

Results are written for each end in the CivilFEM results file as the following parameters:

$$\mathrm{VRDC} = V_{Rd,c}$$

$$\mathrm{CRT\_1} = \frac{V_{Ed}}{V_{Rd,c}}$$

5.5.3.2. Maximum Design Shear Force Resisted Without Crushing of the Concrete Compressive Struts

A check is made to ensure that V_{Ed} does not exceed the maximum design shear force resisted without crushing of the concrete compressive struts.

Eurocode 2 (EN 1992-1-1:2004/AC:2008):

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot 0.9 \cdot d \cdot \nu \cdot f_{cd} \cdot \frac{(\cot\theta + \cot\alpha)}{(1 + \cot^2\theta)}$$

ITER Design Code:

$$V_{Rd\ max} = \alpha_{cw} \cdot \nu \cdot f_{cd} \cdot b_w \cdot 0.9 \cdot d \cdot \left(\frac{\cot\theta + \cot\alpha}{1 + \cot^2\theta_0}\right)$$

where:

$$\alpha_{cw} = \begin{cases} 1, \text{if section is not prestressing} \\ 1 + \dfrac{\sigma_{cp}}{f_{cd}}, \text{if } 0 < 1 + \sigma_{cp} \leq 0.25 f_{cd} \\ 1.25, \text{if } 0.25 f_{cd} < \sigma_{cp} \leq 0.5 f_{cd} \\ 2.5\left(1 - \dfrac{\sigma_{cp}}{f_{cd}}\right), \text{if } 0.5 f_{cd} < \sigma_{cp} \leq f_{cd} \end{cases}$$

The following results will be saved:

$$VRDMAX = V_{Rd,max}$$

$$CRT\_3 = \frac{V_{Ed}}{V_{Rd,max}}$$

If the design shear force is greater than the shear force required to crush the concrete compressive struts, the reinforcement design will not be feasible; as a result, the reinforcement parameter will be defined as 2^{100}.

In this case, the element will be marked as not designed.

5.5.3.3. *Ratio of Reinforcement Required*

The required strength of the reinforcement is given by:

$$V_{Rd,s} = \begin{cases} V_{Ed} & \text{if } V_{Ed} < V_{Rd,max} \\ V_{Rd,max} & \text{if} V_{Ed} \geq V_{Rd,max} \end{cases}$$

The amount of reinforcement per length unit is given by:

$$\frac{A_{sw}}{s} = \frac{V_{Rd,s}}{0.9 \cdot d \cdot f_{ywd} \cdot \text{cotan}\theta}$$

The following is also verified (Eurocode 2 only):

$$\frac{A_{sw}}{s} \leq 0.5 \cdot \alpha_{cw} \cdot b_w \cdot \nu \cdot \frac{f_{cd}}{f_{ywd}} / \sin\alpha$$

If design shear force is greater than the shear force due to crushing of concrete compressive struts, the reinforcement design will not be feasible; therefore, the parameter containing this datum will be marked with 2^{100}. In this case, the element will be marked as not designed.

ASST and ASSB parameters store the amount of top and bottom reinforcement required due to the additional tensile force DF_{td}, in the longitudinal reinforcement due to shear V_{Ed}.

$$\Delta F_{td} = 0{,}5\ V_{Ed}\ (\cot\text{an}\theta\ )$$

$$\Delta A_{sl} = \frac{0{,}5V_{Ed}\cot\text{an}\theta}{f_{ywd}}$$

ASST= ΔAsl for negative Bending Moments

ASSB= ΔAsl for positive Bending Moments

Results are written for each element end in the CivilFEM results file as the parameters:

$$VRDS = V_{Rd,s}$$

$$ASSH = \frac{A_{sw}}{s}$$

DSG_CRT design criterion

5.6. Out-of-Plane Shear Load according to Structural code (Spanish code)

Check and Design for Out-of-Plane Shear Loadings according to Structural Code (Annex 19)

5.6.1. Required Input Data

Shear check or design according to Structural Code (Annex 19) requires a series of parameters described below:

4) **Materials strength properties.** These properties are obtained from the material properties associated with each one of the shell vertices and for the active time. Those material properties should be previously defined. The required data are the following:

f_{ck} characteristic compressive strength of concrete.

f_{cd} design strength of concrete.

f_{yk} characteristic yield strength of reinforcement.

f_{ywd} design strength of reinforcement.

γ_c partial safety factor for concrete.

γ_s partial safety factor for reinforcement.

5) **Shell vertex geometrical data:**
th thickness of the shell vertex (shell structural element).

6) **Geometrical parameters. Required data are the following:**

c bending reinforcement mechanical cover (shell structural element).
ρ_{1i} ratio of the longitudinal tensile reinforcement per unit length of the shell:

$$\rho_{1i} = \frac{A_{ssi}}{th - c} < 0.02$$

where:

A_{ss} area of the tensile reinforcement (shell structural element).

θ angle of the compressive struts of concrete with the longitudinal axis of the member, (parameter THETA of shell structural element):

$$0.5 \leq \text{cotan}\,\theta \leq 2$$

4) **Shell vertex reinforcement data.** Required data are the following:

A_{ss} area of reinforcement per unit area, (parameter ASS of shell structural element).

The reinforcement ratio may also be obtained with the following data:

sx, sy spacing of the stirrups in each direction of the shell, (parameters SX and SY of shell structural element).

ϕ diameter of bars (parameter PHI of shell structural element).

n_x, n_y number of stirrups per unit length in each direction of the shell (parameters NX and NY of shell structural element).

5) **Shell vertex internal forces**. The shear force (V_{Ed}) acting on the vertex as well as the concomitant axial force (N_{Ed}) are obtained from the CivilFEM results file (.RCF).

$$V_{Ed} = \sqrt{{N_x}^2 + {N_y}^2}$$

which forms an angle with the axis Y

$$\phi_v = \arctan \frac{N_y}{N_x}$$

The value taken for the design compression force (T_{Ed}) is the maximum considering all directions:

$$T_{Ed} = \frac{1}{2}\left(T_x + T_y - \sqrt{(T_x + T_y)^2 - 4(T_x \cdot T_y - {T_{xy}}^2)}\right)$$

Design compression force takes into account axial force in x, y directions and in-plane shear using Mohr's circle stress transformation equations.

The total shear reinforcement ρ_1 is computed from those in each direction, according:

$$\rho_1 = \rho_x \cdot \cos^2\phi_v + \rho_y \cdot \sin^2\phi_v$$

5.6.2. Out-of-Plane Shear Checking

5.5.2.3. Checking whether shear reinforcement is required

Design shear force V_{Ed} is compared with the design shear resistance ($V_{Rd,c}$):

$$V_{Ed} \leq V_{Rd,c}$$

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100\rho_I \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

With the constraints:

$$V_{Ed} \leq 0.5 b_w \cdot d \cdot \boldsymbol{\nu} \cdot f_{cd}$$

$$V_{Rd,c} \geq \left[V_{min} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

Where:

$C_{Rd,c} =$ $0.18y_c$

f_{ck} in MPa

$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 (d \text{ en mm})$

$k_1 = 0.15$

$\sigma_{cp} = \frac{T_{Ed}}{A_c} < 0.2 f_{cd}$ MPa

A_c in mm^2

$\nu = 0.6\left(1 - \frac{f_{ck}}{250}\right)$

$v_{min} = 0.035\,(k^3 \cdot f_{ck})^{1/2}$

$v_{Rd,c} =$ in N

If shear reinforcement is defined in the section, V_{Ed} must be less than the minimum between the shear reinforcement force:

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot 0.9 \cdot d \cdot f_{ywd} \cdot (\cot\theta + \cot\alpha) \cdot \sin\alpha$$

and the maximum design shear force resisted without crushing of concrete compressive struts:

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot 0.9 \cdot d \cdot \nu \cdot f_{cd} \cdot \frac{(\text{cotan}\theta + \text{cotan}\,\alpha)}{(1 + \text{cotan}^2\theta)}$$

where:

$$\alpha_{cw} = \begin{cases} 1, \text{if the section is not prestressed} \\ 1 - \frac{\sigma_{cp}}{f_{cd}}, \quad \text{if } 0 < \sigma_{cp} \le 0.25f_{cd} \\ 1.25, \text{if} \quad 0.25f_{cd} < \sigma_{cp} \le 0.5f_{cd} \\ 2.5\left(1 - \frac{\sigma_{cp}}{f_{cd}}\right), \text{if } 0.5f_{cd} < \sigma_{cp} \le f_{cd} \end{cases}$$

The shear reinforcement must be less than or equal to :

$$\frac{A_{sw,max}}{s} = 0.5 \cdot \alpha_{cw} \cdot b_w \cdot \boldsymbol{\nu} \cdot \frac{f_{cd}}{f_{yd}}$$

Results are written for each end in the CivilFEM results file:

If there is no shear reinforcement defined, the following results can be obtained:

$VRDC = V_{Rd,c}$

$VRDS = V_{Rd,s}$

$VRDMAX = V_{Rd,max}$

$TENS = \frac{V_{Ed}}{2}\cot\theta$

Tensile strength for the longitudinal reinforcement

$CRT\_1 = \frac{V_{Ed}}{V_{Rd,c}}$

$CRT\_2 = 0,$ Shear reinforcement not defined

$\frac{V_{Ed}}{V_{Rd,s}},$ Shear reinforcement defined

$CRT\_3 = 0,$ Shear reinforcement not defined

$\frac{V_{Ed}}{V_{Rd,max}},$ Shear reinforcement defined

5.5.2.4. Shear Criterion

The shear criterion indicates whether the shell vertex is valid for the design forces (if it is less than 1, the vertex satisfies the code provisions; whereas if it exceeds 1, the vertex will not be valid). Furthermore, it includes information about how close the design force is to the ultimate strength. The shear criterion is defined as follows:

$CRT\_TOT = \dfrac{V_{Ed}}{V_{Rd,c}}$ If shear reinforcement is not defined.

$\min\{\frac{V_{Ed}}{V_{Rd,c}}, \max\{\frac{V_{Ed}}{V_{Rd,s}} \dfrac{V_{Ed}}{V_{Rd,máx}}\}\},$ If shear reinforcement is defined.

A value of 2^{100} for this criterion indicates that $V_{Rd,c}$, $V_{Rd,s}$ or $V_{Rd,max}$ are null.

5.6.3. Out-of-Plane Shear Design

5.5.3.4. Checking whether shear reinforcement is required

First, a check is made to determine if the design shear force V_{Ed} is less than or equal to the shear design resistance ($V_{Rd,c}$):

$$V_{Ed} \leq V_{Rd,c}$$

$$V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100\rho_I \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d$$

with constraints:

$$V_{Ed} \leq 0.5 b_w \cdot d \cdot \nu \cdot f_{cd}$$

$$V_{Rd,c} \geq [V_{min} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d$$

where:

$C_{Rd,c}$	=	$0.18\, y_c$
f_{ck}		in MPa
k	=	$1 + \sqrt{\frac{200}{d}} \leq 2.0$ (d in mm)
k_1	=	0.15
σ_{cp}	=	$\frac{N_{Ed}}{A_c} < 0.2\, f_{cd}$ MPa
A_C		in mm^2
ν	=	$0.6\left(1 - \frac{f_{ck}}{250}\right)$
V_{min}	=	$0.035(k^3 \cdot f_{ck})^{1/2}$

$V_{Rd,c}$ in N

Results are written for each end in the CivilFEM results file as the following parameters:

$$VRDC = V_{Rd,c}$$

$$CRT\_1 = \frac{V_{Ed}}{V_{Rd,c}}$$

5.5.3.5. Maximum Design Shear Force Resisted Without Crushing of the Concrete Compressive Struts

A check is made to ensure that V_{Ed} does not exceed the maximum design shear force resisted without crushing of the concrete compressive struts.

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot 0.9 \cdot d \cdot \nu \cdot f_{cd} \cdot \frac{(\cot\theta + \cot\alpha)}{(1 + \cot^2\theta)}$$

where:

$$\alpha_{cw} = \begin{cases} 1, \text{if section is not prestressing} \\ 1 + \frac{\sigma_{cp}}{f_{cd}}, \text{if } 0 < 1 + \sigma_{cp} \leq 0.25f_{cd} \\ 1.25, \text{if } 0.25f_{cd} < \sigma_{cp} \leq 0.5f_{cd} \\ 2.5\left(1 - \frac{\sigma_{cp}}{f_{cd}}\right), \text{if } 0.5f_{cd} < \sigma_{cp} \leq f_{cd} \end{cases}$$

The following results will be saved:

$$VRDMAX = V_{Rd,max}$$

$$CRT\_3 = \frac{V_{Ed}}{V_{Rd,max}}$$

If the design shear force is greater than the shear force required to crush the concrete compressive struts, the reinforcement design will not be feasible; as a result, the reinforcement parameter will be defined as 2^{100}.

In this case, the element will be marked as not designed.

5.5.3.6. Ratio of Reinforcement Required

The required strength of the reinforcement is given by:

$$V_{Rd,s} = \begin{cases} V_{Ed} & \text{if } V_{Ed} < V_{Rd,max} \\ V_{Rd,max} & \text{if} V_{Ed} \geq V_{Rd,max} \end{cases}$$

The amount of reinforcement per length unit is given by:

$$\frac{A_{sw}}{s} = \frac{V_{Rd,s}}{0.9 \cdot d \cdot f_{ywd} \cdot \text{cotan}\theta}$$

The following is also verified:

$$\frac{A_{sw}}{s} \leq 0.5 \cdot \alpha_{cw} \cdot b_w \cdot \nu \cdot \frac{f_{cd}}{f_{ywd}} / \sin\alpha$$

If design shear force is greater than the shear force due to crushing of concrete compressive struts, the reinforcement design will not be feasible; therefore, the parameter containing this datum will be marked with 2^{100}. In this case, the element will be marked as not designed.

ASST and ASSB parameters store the amount of top and bottom reinforcement required due to the additional tensile force ΔF_{td}, in the longitudinal reinforcement due to shear V_{Ed}.

$$\Delta F_{td} = 0,5\, V_{Ed}\, (\text{cotan}\theta)$$

$$\Delta A_{sl} = \frac{0,5 V_{Ed} \text{cotan}\theta}{f_{ywd}}$$

ASST= ΔAsl for negative Bending Moments

ASSB= ΔAsl for positive Bending Moments

Results are written for each element end in the CivilFEM results file as the parameters:

$$\text{VRDS} = V_{Rd,s}$$

$$\text{ASSH} = \frac{A_{sw}}{s}$$

DSG_CRT design criterion

5.7. Out-of-Plane Shear Load according to ACI-318-05

5.7.1. Required Input Data

Shear checking or design according to ACI 318-05 requires the data described below:

1. **Material strength properties.** Material properties are assigned to each shell structural element. These material properties must be defined prior to the check and design process. The required properties are:

   f'_c specified compressive strength of concrete.

   f_y specified yield strength of reinforcement.

2. **Shell vertex data**:

   th thickness of the shell vertex (shell structural element).

Required properties are:

3. **Shell vertex reinforcement data.**

   c bending reinforcement mechanical cover (shell structural element).

   A_{ss} the area of bending reinforcement per unit length. This parameter is used for checking (parameters of shell structural element).

4. **Shell vertex shear reinforcement data.**

   A_{ss} area of shear reinforcement per unit of area. This parameter is used for checking (parameter of shell structural element).

   The shear reinforcement ratio may also be obtained from:

   A_{ssX}, A_{ssY} area of shear reinforcement per unit of area in each direction of the shell. (parameters of shell structural element)

   s_x, s_y spacing of the stirrups in each direction of the shell, (parameters of shell structural element).

   diameter of bars in mm (shell structural element).

   N_x, N_y number of stirrups per unit length in each direction of the shell (parameters of shell structural element).

5. **Shell vertex internal forces**. The shear force acting on the vertex as well as the concomitant membrane force are obtained from the CivilFEM results file (.RCF). For each direction of the shell vertex:

Force	**Description**
V_u	Design out-of-plane shear force
N_u	Axial force (positive for compression).

5.7.2. Out-of-Plane Shear Checking

5.6.2.1. Shear Strength Provided by Concrete

The shear strength provided by concrete (V_c) is calculated with the following expression:

$$V_c = 2 \cdot \sqrt{f'_c} \cdot b_w \cdot (th - c)$$

(ACI 318-05 Eqn:11-3)

where:

b_w = 1 (unit length)

$\sqrt{f'_c}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force,

$$V_c = 2 \cdot \left(1 + \frac{N_u}{2000 \cdot b_w \cdot th}\right) \cdot \sqrt{f'_c} \cdot b_w \cdot (th - c)$$

(ACI 318-05 Eqn:11-4)

where:

$N_u/(th \cdot b_w)$expressed in psi.

If section is subjected to a tensile force so that the tensile stress is less than 500 psi:

$$V_c = 2 \cdot \left(1 + \frac{N_u}{500 \cdot b_w \cdot th}\right) \cdot \sqrt{f'_c} \cdot b_w \cdot (th - c)$$

(ACI 318-05 Eqn:11-8)

If the shell is subjected to a tensile force so that the tensile stress exceeds 500 psi, it is assumed V_c=0.

The calculation result for all elements is stored in the CivilFEM results file as the parameter VC (# is the direction of the shell, X or Y):

VC_# Shear strength provided by concrete.

$$VC = V_c$$

5.6.2.2. Shear Strength Provided by Shear Reinforcement

The strength provided by the shear reinforcement (V_s) is calculated with the following expression:

$$V_s = A_{ss} \cdot f_y \cdot (th - c)$$

(ACI 318-05 Eqn.11-15)

The calculation result for all elements is stored in the CivilFEM results file as the parameter VS (# is the direction of the shell, X or Y direction):

VS_# Shear strength provided by transverse reinforcement.

$$VS = V_s$$

The following condition must be satisfied:

$$VS\_X + VS\_Y \leq 8\sqrt{f_c'} \cdot (th - c)$$

(ACI 318-01 Eqn.11.5.6.8)

This condition is reflected in the total criterion.

5.6.2.3. Nominal Shear Strength

The nominal shear strength (V_n) is the sum of the provided by concrete and by the shear reinforcement:

$$V_n = V_c + V_s$$

This nominal strength, is stored in the CivilFEM results file as the parameter VN (# is the direction of the shell, X or Y):

VN_# Nominal shear strength.

$$VN = V_n$$

5.6.2.4. Minimum Reinforcement

If reinforcement is required, the minimum allowable value is:

$$Ass_{min} = 50\frac{b_w}{f_y}$$

(ACI 318-05 Eqn.11-13)

5.6.2.5. Shear Criterion

The shell vertex will be valid for shear if the following condition is satisfied and if the reinforcement is greater than the minimum required:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

(ACI 318-05 Eqn.11-1 and 11-2)

Where ϕ is strength reduction factor (ϕ = 0.75 according to chapter 9.3.2.3 of code requirements). Therefore, the shear criterion for the validity of the shell vertex is as follows:

$$\text{Max}\left\{\frac{V_u}{\phi V_n}\right\}$$

For each element, this value is stored in the CivilFEM results file as the parameter CRT_#.

If the strength provided by concrete is null and the shear reinforcement is not defined in the shell vertex, and the criterion is equal to 2^{100}.

The $\phi \cdot V_n$ value is stored in CivilFEM results file as the parameter VFI_#.

The total checking criterion is defined as:

$$CRT\_TOT = \text{Max}\left\{CRT\_X;\ CRT\_Y;\frac{VS\_X + VS\_Y}{8 \cdot \sqrt{f_c'} \cdot b_w \cdot (th - c)};\frac{Ass_{min}}{Ass}\right\}$$

5.7.3. Out-of-Plane Shear Design

5.6.3.1. Shear Strength Provided by Concrete

The shear strength provided by the concrete (V_c) is calculated by:

$$V_c = 2 \cdot \sqrt{f_c'} \cdot b_w \cdot (th - c)$$

(ACI 318-05 Eqn.11-8)

where:

b_w = 1 (unit length)

$\sqrt{f_c'}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force,

$$V_c = 2 \cdot \left(1 + \frac{N_u}{2000 \cdot b_w \cdot th}\right) \cdot \sqrt{f_c'} \cdot b_w \cdot (th - c)$$

(ACI 318-05 Eqn.11-4)

where:

$N_u/(th \cdot b_w)$ expressed in units of psi.

If the section is subjected to a tensile force such that the tensile stress is less than 500 psi,

$$V_c = 2 \cdot \left(1 + \frac{N_u}{500 \cdot b_w \cdot th}\right) \cdot \sqrt{f_c'} \cdot b_w \cdot (th - c)$$

(ACI 318-05 Eqn.11-8)

If the shell is subjected to a tensile force such that the tensile stress exceeds 500 psi, it is assumed that V_c=0.

The calculation result for all element ends is stored in the CivilFEM results file as the parameter VC (# is the direction of the shell, X or Y):

VC_# Shear strength provided by concrete.

$$VC = V_c$$

5.6.3.2. Required Reinforcement Contribution

The shell must satisfy the following condition to resist the shear force:

$$V_u \le \phi V_n = \phi(V_c + V_s)$$

(ACI 318-05 Eqn.11-1 and 11-2)

where: ϕ is the strength reduction factor (defined in **Environment Configuration**).

Therefore, the required shear strength of the reinforcement is:

$$V_s = \frac{V_u}{\phi} - V_c$$

Calculated results are stored in the CivilFEM results file for both element ends as the parameter VS (# is the direction of the shell, X or Y):

VS_# Shear resistance provided by the transverse reinforcement.

$$VS = V_s$$

The following condition must be satisfied:

$$VS\_X + VS\_Y \le 8 \cdot \sqrt{f_c'} \cdot (th - c)$$

(ACI 318-01 Eqn.11.5.6.8)

If the required shear strength of the reinforcement does not satisfy the expression above, the shell vertex cannot be designed; therefore, the reinforcement parameter will be set as 2^{100}.

$$ASSH = A_{ss} = 2^{100}$$

In this case, the element will be labeled as not designed.

5.6.3.3. Required Reinforcement

Once the required shear strength of the reinforcement has been determined, the reinforcement is calculated as the maximum of the following expressions (for both X and Y directions):

$$A_{ss} = \frac{V_s}{f_y \cdot (th - c)} \text{ for both X and Y directions}$$

(ACI 318-05 Eqn.11-15)

These reinforcement areas will be proportionally increased, if needed, to reach the minimum required ratio:

$$Ass_x + Ass_y \geq 50 \frac{b_w}{f_y}$$

(ACI 318-05 Eqn.11-13)

The area of the designed reinforcement per unit length is stored in the CivilFEM results file as:

$$ASSH\_X = Ass \text{ for } X \text{ direction}$$
$$ASSH\_Y = Ass \text{ for } Y \text{ direction}$$
$$ASSH\_X = ASSH\_X + ASSH\_Y$$

In this case, the element will be labeled as designed (providing the design process is correct for all element ends).

5.8. Out-of-Plane Shear Load according to ACI-318-14

5.8.1. Required Input Data

Shear checking or design according to ACI 318-14 requires the data described below:

1. **Material strength properties.** Material properties are assigned to each shell structural element. These material properties must be defined prior to the check and design process. The required properties are:

f'_c specified compressive strength of concrete.

f_y specified yield strength of reinforcement.

λ modification factor for lightweight concrete ($\lambda = 1.0$ default value)

2. **Shell vertex data**:

th thickness of the shell vertex (shell structural element).

Required properties are:

3. Shell vertex reinforcement data.

c bending reinforcement mechanical cover (shell structural element).

A_{ss} the area of bending reinforcement per unit length. This parameter is used for checking (parameters of shell structural element).

4. Shell vertex shear reinforcement data.

A_{ss} area of shear reinforcement per unit of area. This parameter is used for checking (parameter of shell structural element).

The shear reinforcement ratio may also be obtained from:

A_{ssX}, A_{ssY} area of shear reinforcement per unit of area in each direction of the shell. (parameters of shell structural element)

s_x, s_y spacing of the stirrups in each direction of the shell, (parameters of shell structural element).

diameter of bars in mm (shell structural element).

N_x, N_y number of stirrups per unit length in each direction of the shell (parameters of shell structural element).

5. **Shell vertex internal forces**. The shear force acting on the vertex as well as the concomitant membrane force are obtained from the CivilFEM results file (.RCF). For each direction of the shell vertex:

Force	**Description**
V_u	Design out-of-plane shear force
N_u	Axial force (positive for compression).

5.8.2. Out-of-Plane Shear Checking

5.7.2.1. Shear Strength Provided by Concrete

The shear strength provided by concrete (V_c) is calculated with the following expression:

$$V_c = 2 \cdot \lambda \cdot \sqrt{f_c'} \cdot b_w \cdot (th - c)$$

(ACI 318-14 Eqn:11-3)

where:

b_w = 1 (unit length)

$\sqrt{f_c'}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force,

$$V_c = 2 \cdot \left(1 + \frac{N_u}{2000 \cdot b_w \cdot th}\right) \cdot \lambda \cdot \sqrt{f_c'} \cdot b_w \cdot (th - c)$$

(ACI 318-14 Eqn:11-4)

where:

$N_u/(th \cdot b_w)$expressed in psi.

If section is subjected to a tensile force so that the tensile stress is less than 500 psi:

$$V_c = 2 \cdot \left(1 + \frac{N_u}{500 \cdot b_w \cdot th}\right) \cdot \lambda \cdot \sqrt{f_c'} \cdot b_w \cdot (th - c)$$

(ACI 318-14 Eqn:11-8)

If the shell is subjected to a tensile force so that the tensile stress exceeds 500 psi, it is assumed V_c=0.

The calculation result for all elements is stored in the CivilFEM results file as the parameter VC (# is the direction of the shell, X or Y):

VC_# Shear strength provided by concrete.

$$VC = V_c$$

5.7.2.2. Shear Strength Provided by Shear Reinforcement

The strength provided by the shear reinforcement (V_s) is calculated with the following expression:

$$V_s = A_{ss} \cdot f_y \cdot (th - c)$$

(ACI 318-14 Eqn.11-15)

The calculation result for all elements is stored in the CivilFEM results file as the parameter VS (# is the direction of the shell, X or Y direction):

VS_# Shear strength provided by transverse reinforcement.

$$VS = V_s$$

The following condition must be satisfied:

$$VS\_X + VS\_Y \leq 8\sqrt{f'_c} \cdot (th - c)$$

(ACI 318-01 Eqn.11.5.6.8)

This condition is reflected in the total criterion.

5.7.2.3. Nominal Shear Strength

The nominal shear strength (V_n) is the sum of the provided by concrete and by the shear reinforcement:

$$V_n = V_c + V_s$$

This nominal strength, is stored in the CivilFEM results file as the parameter VN (# is the direction of the shell, X or Y):

VN_# Nominal shear strength.

$$VN = V_n$$

5.7.2.4. Minimum Reinforcement

If reinforcement is required, the minimum allowable value is:

$$Ass_{min} = 50\frac{b_w}{f_y}$$

(ACI 318-14 Eqn.11-13)

5.7.2.5. Shear Criterion

The shell vertex will be valid for shear if the following condition is satisfied and if the reinforcement is greater than the minimum required:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

(ACI 318-14 Eqn.11-1 and 11-2)

Where ϕ is strength reduction factor (ϕ = 0.75 according to chapter 21.2 of code requirements). Therefore, the shear criterion for the validity of the shell vertex is as follows:

$$\mathrm{Max}\left\{\frac{V_u}{\phi V_n}\right\}$$

For each element, this value is stored in the CivilFEM results file as the parameter CRT_#.

If the strength provided by concrete is null and the shear reinforcement is not defined in the shell vertex, and the criterion is equal to 2^{100}.

The $\phi \cdot V_n$ value is stored in CivilFEM results file as the parameter VFI_#.

The total checking criterion is defined as:

$$CRT\_TOT = \mathrm{Max}\left\{CRT\_X;\ CRT\_Y; \frac{VS\_X + VS\_Y}{8 \cdot \sqrt{f'_c} \cdot b_w \cdot (th - c)}; \frac{Ass_{min}}{Ass}\right\}$$

5.8.3. Out-of-Plane Shear Design

5.7.3.1. Shear Strength Provided by Concrete

The shear strength provided by the concrete (V_c) is calculated by:

$$V_c = 2 \cdot \lambda \cdot \sqrt{f'_c} \cdot b_w \cdot (th - c)$$

(ACI 318-14 Eqn.11-8)

where:

b_w = 1 (unit length)

$\sqrt{f'_c}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force,

$$V_c = 2 \cdot \left(1 + \frac{N_u}{2000 \cdot b_w \cdot th}\right) \cdot \lambda \cdot \sqrt{f'_c} \cdot b_w \cdot (th - c)$$

(ACI 318-14 Eqn.11-4)

where:

$N_u/(th \cdot b_w)$ expressed in units of psi.

If the section is subjected to a tensile force such that the tensile stress is less than 500 psi,

$$V_c = 2 \cdot \left(1 + \frac{N_u}{500 \cdot b_w \cdot th}\right) \cdot \lambda \cdot \sqrt{f'_c} \cdot b_w \cdot (th - c)$$

(ACI 318-14 Eqn.11-8)

If the shell is subjected to a tensile force such that the tensile stress exceeds 500 psi, it is assumed that V_c=0.

The calculation result for all element ends is stored in the CivilFEM results file as the parameter VC (# is the direction of the shell, X or Y):

VC_# Shear strength provided by concrete.

$$VC = V_c$$

5.7.3.2. Required Reinforcement Contribution

The shell must satisfy the following condition to resist the shear force:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

(ACI 318-14 Eqn.11-1 and 11-2)

where: ϕ is the strength reduction factor (defined in **Environment Configuration**).

Therefore, the required shear strength of the reinforcement is:

$$V_s = \frac{V_u}{\phi} - V_c$$

Calculated results are stored in the CivilFEM results file for both element ends as the parameter VS (# is the direction of the shell, X or Y):

VS_# Shear resistance provided by the transverse reinforcement.

$$VS = V_s$$

The following condition must be satisfied:

$$VS\_X + VS\_Y \leq 8 \cdot \sqrt{f'_c} \cdot (th - c)$$

(ACI 318-01 Eqn.11.5.6.8)

If the required shear strength of the reinforcement does not satisfy the expression above, the shell vertex cannot be designed; therefore, the reinforcement parameter will be set as 2^{100}.

$$\mathrm{ASSH} = \mathrm{A_{ss}} = 2^{100}$$

In this case, the element will be labeled as not designed.

5.7.3.3. Required Reinforcement

Once the required shear strength of the reinforcement has been determined, the reinforcement is calculated as the maximum of the following expressions (for both X and Y directions):

$$\mathrm{A}_{ss} = \frac{\mathrm{v_s}}{\mathrm{f_y}(th - c)}$$

for both X and Y directions

(ACI 318-14 Eqn.11-15)

These reinforcement areas will be proportionally increased, if needed, to reach the minimum required ratio:

$$\mathrm{Ass_x} + \mathrm{Ass_y} \geq 50\frac{\mathrm{b_w}}{\mathrm{f_y}}$$

(ACI 318-14 Eqn.11-13)

The area of the designed reinforcement per unit length is stored in the CivilFEM results file as:

$$\mathrm{ASSH\_X} = \mathrm{Ass\ for\ \ X\ direction}$$
$$\mathrm{ASSH\_Y} = \mathrm{Ass\ for\ \ Y\ direction}$$
$$\mathrm{ASSH\_X} = \mathrm{ASSH\_X} + \mathrm{ASSH\_Y}$$

In this case, the element will be labeled as designed (providing the design process is correct for all element ends).

5.9. Out-of-Plane Shear Load according to ACI-349-01

5.9.1. Required Input Data

Shear checking or design according to ACI 349-01 requires a series of parameters that are described below. The formulas listed in this section utilize U.S. (British) units: inch (in), pound (lb), and second (s).

1. **Material strength properties.** Material properties are assigned to each active shell vertex. These material properties must be defined prior to checking and design. The required properties are:

   f'_c specified compressive strength of concrete.

   f_y specified yield strength of reinforcement.

2. **Shell vertex data:**

   th thickness of the shell vertex.

Required properties are:

3. **Shell vertex reinforcement data.**

   c bending reinforcement mechanical cover.

   A_{ss} the area of bending reinforcement per unit length. This parameter is used for checking (parameters of shell structural element).

4. **Shell vertex shear reinforcement data.**

   A_{ss} area of shear reinforcement per unit of area. This parameter is used for checking (parameters of shell structural element).

   The shear reinforcement ratio may also be obtained from:

   A_{ssX}, A_{ssY} area of shear reinforcement per unit of area in each direction of the shell. Parameters of shell structural element.

   s_x, s_y spacing of the stirrups in each direction of the shell, parameters of shell structural element.

   diameter of bars in mm.

   N_x, N_y number of stirrups per unit length in each direction of the shell.

5. **Shell vertex internal forces**. The shear force acting on the vertex as well as the concomitant membrane force are obtained from the CivilFEM results file (.RCF). For each direction of the shell vertex:

Force	Description
V_u	Design out-of-plane shear force
N_u	Axial force (positive for compression).

5.9.2. Out-of-Plane Shear Checking

5.8.2.1. Shear Strength Provided by Concrete

The shear strength provided by concrete (V_c) is calculated by:

$$V_c = 2 \cdot \sqrt{f_c'} \cdot b_w \cdot (th - c)$$

(ACI 349-01 Eqn:11-3)

where:

$b_w = 1$ (unit length)

$\sqrt{f_c'}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to an axial compressive force,

$$V_c = 2 \cdot \left(1 + \frac{N_u}{2000 \cdot b_w \cdot th}\right) \cdot \sqrt{f_c'} \cdot b_w \cdot (th - c)$$

(ACI 349-01 Eqn:11-4)

where:

$N_u/(th \cdot b_w)$expressed in units of psi.

If the section is subjected to a tensile force such that the tensile stress is less than 500 psi then,

$$V_c = 2 \cdot \left(1 + \frac{N_u}{500 \cdot b_w \cdot th}\right) \cdot \sqrt{f_c'} \cdot b_w \cdot (th - c)$$

(ACI 349-01 Eqn:11-8)

If the shell is subjected to a tensile force so that the tensile stress exceeds 500 psi, it is assumed V_c=0.

The calculation result for all elements is stored in the CivilFEM results file as the parameter VC (# is the direction of the shell, X or Y):

VC_# Shear strength provided by concrete.

$$VC = V_c$$

5.8.2.2. *Shear strength provided by shear reinforcement*

The strength provided by shear reinforcement (V_s) is calculated with the following expression:

$$V_s = A_{ss} \cdot f_y \cdot (th - c)$$

(ACI 349-01 Eqn.11-15)

The calculated result for all elements is stored in the CivilFEM results file as the parameter VS (# is the direction of the shell, X or Y):

VS_# Shear strength provided by transverse reinforcement.

$$VS = V_s$$

The following condition must be satisfied:

$$VS\_X + VS\_Y \leq 8 \cdot \sqrt{f_c'} \cdot (th - c)$$

(ACI 349-01 Eqn.11.5.6.8)

This condition is reflected in the total criterion.

5.8.2.3. *Nominal shear strength*

The nominal shear strength (V_n) is the sum of the concrete and shear reinforcement components calculated previously:

$$V_n = V_c + V_s$$

This nominal strength is stored in the CivilFEM results file as the parameter VN (# is the direction of the shell, X or Y):

VN_# Nominal shear strength.

$$VN = V_n$$

5.8.2.4. *Minimum reinforcement*

If reinforcement is required, the minimum allowable value is:

$$Ass_{min} = 50 \frac{b_w}{f_y}$$

(ACI 349-01 Eqn.11-13)

5.8.2.5. *Shear criterion*

The shell vertex will be valid for shear if the following condition is satisfied:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

(ACI 349-01 Eqn.11-1 and 11-2)

where ϕ is strength reduction factor (ϕ = 0.85 according to chapter 9.3.2.3 of code requirements) and if the reinforcement is greater than the minimum required. Therefore, the validity shear criterion is defined as follows:

$$\mathrm{Max}\left\{\frac{V_u}{\phi V_n}\right\}$$

For each element, this value is stored in the CivilFEM results file as the parameter CRT_#.

If the strength provided by concrete is null and the shear reinforcement is not defined in the shell vertex, the criterion is set equal to 2^{100}.

The $\phi \cdot V_n$ value is stored in the CivilFEM results file as the parameter VFI_#.

The total checking criterion is defined as:

$$\mathrm{CRT\_TOT} = \mathrm{Max}\left\{\mathrm{CRT\_X};\ \mathrm{CRT\_Y}; \frac{\mathrm{VS\_X} + \mathrm{VS\_Y}}{8 \cdot \sqrt{f'_c} \cdot b_w \cdot (\mathrm{th} - c)}; \frac{\mathrm{Ass}_{min}}{\mathrm{Ass}}\right\}$$

5.9.3. Out-of-Plane Shear Design

5.8.3.1. Shear strength provided by concrete

The shear strength provided by concrete (V_c) is calculated with the following expression:

$$V_c = 2 \cdot \sqrt{f'_c} \cdot b_w \cdot (\mathrm{th} - c)$$

(ACI 349-01 Eqn.11-3)

where:

b_w = 1 (unit length)

$\sqrt{f'_c}$ square root of specified compressive strength of concrete, in psi (it is always taken as less than 100 psi).

For sections subject to a compressive axial force,

$$V_c = 2 \cdot \left(1 + \frac{N_u}{2000 \cdot b_w \cdot \mathrm{th}}\right) \cdot \sqrt{f'_c} \cdot b_w \cdot (\mathrm{th} - c)$$

(ACI 349-01 Eqn.11-4)

Where:

$N_u/(\mathrm{th} \cdot b_w)$ is expressed in psi.

If the section is subjected to a tensile force such that the tensile stress is less than 500 psi:

$$V_c = 2 \cdot \left(1 + \frac{N_u}{500 \cdot b_w \cdot th}\right) \cdot \sqrt{f_c'} \cdot b_w \cdot (th - c)$$

(ACI 349-01 Eqn.11-8)

If the shell is subjected to a tensile force such that the tensile stress exceeds 500 psi, it is assumed that V_c=0.

The calculation result for all elements is stored in the CivilFEM results file as the parameter VC (# is the direction of the shell, X or Y):

VC_# Shear strength provided by concrete.

$$VC = V_c$$

5.8.3.2. Required reinforcement contribution

The shell must satisfy the following condition to resist the shear force:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

(ACI 349-01 Eqn.11-1 and 11-2)

Where ϕ is the strength reduction factor (defined in **Environment Configuration**).

Therefore, the shear force the reinforcement must support is:

$$V_s = \frac{V_u}{\phi} - V_c$$

Calculation results are stored in the CivilFEM results file for all elements as the parameter VS (# is the direction of the shell, X or Y):

VS_# Shear resistance provided by the transverse reinforcement.

$$VS = V_s$$

The following condition must be satisfied:

$$VS\_X + VS\_Y \leq 8 \cdot \sqrt{f_c'} \cdot (th - c)$$

(ACI 349-01 Eqn.11.5.6.8)

If the shear force the reinforcement must support does not satisfy the expression above, the shell vertex cannot be designed, so the parameters where the reinforcement is stored are set to 2^{100}. Then:

$$ASSH = A_{ss} = 2^{100}$$

In this case, the element will labeled as not designed.

5.8.3.3. Required reinforcement

Once the shear force that the shear reinforcement must support has been obtained, the reinforcement is calculated as follows:

$$A_{ss} = \frac{V_s}{f_y \cdot (th - c)}$$

(ACI 349-01 Eqn.11-15)

for each direction X and Y

These reinforcement areas will be increased proportionally, if needed, to reach the minimum required ratio:

$$Ass_x + Ass_y \geq 50 \frac{b_w}{f_y}$$

(ACI 349-01 Eqn.11-13)

The area of the designed reinforcement per unit of area is stored in the CivilFEM results file as:

ASSH_X = Shear reinforcement in X direction.

ASSH_Y = Shear reinforcement in Y direction.

ASSH = ASSH_X + ASSH_Y

In this case, the element will be marked as designed (providing the design process is correct for all element directions).

5.10. Out-of-Plane Shear Load according to EHE-08

5.10.1. Required Input Data

Shear checking or design according to EHE-08 requires the following parameters:

1) **Materials strength properties.** These properties are obtained from the material properties associated with each one of shell structural element and for the active time. Those material properties should be previously defined. The required data are the following:

f_{ck} characteristic compressive strength of concrete.

f_{yk} characteristic yield strength of reinforcement.

$f_{ct,m}$ mean tensile strength of concrete.

γ_c partial safety factor for concrete.

γ_s partial safety factor for reinforcement.

2) **Shell vertex geometrical data:**

th thickness of the shell structural element.

3) **Geometrical parameters.** Required data are the following:

c Bending reinforcement mechanical cover.

ρ_1 ratio of the longitudinal tensile reinforcement per unit length of shell:

$$\rho_1 = \frac{A_{ssi}}{th - c} < 0.02$$

where:

A_{ss} the area of the tensile reinforcement.

θ angle of the compressive struts of concrete with the longitudinal axis of member.

$$0.5 < \cot\theta < 2.0$$

4) **Shell vertex reinforcement data.** Data concerning reinforcements of the shell vertex must be included within CivilFEM database. Required data are the following:

A_{ss} area of reinforcement per unit area.

The reinforcement ratio may also be obtained with:

sx, sy spacing of the stirrups in each direction of the shell.

ϕ diameter of bars.

n_x, n_y number of stirrups per unit length in each direction of the shell.

5) **Shell vertex internal forces.** The shear force acting on the vertex as well as the concomitant axial force are obtained from the CivilFEM results file (.RCF).

Design shear force (V_{rd}) is obtained from the shear forces in the X and Y directions:

$$V_{rd} = \sqrt{{N_x}^2 + {N_y}^2}$$

Which forms an angle with the axis Y:

$$\phi_v = \arctan\frac{N_y}{N_x}$$

The value taken for the design compression force (T_{Ed}) is the maximum considering all directions:

$$T_{Ed} = \frac{1}{2}\left(T_x + T_y - \sqrt{\left(T_x + T_y\right)^2 - 4\left(T_x \cdot T_y - {T_{xy}}^2\right)}\right)$$

Design compression force takes into account axial force in x, y directions and in-plane shear using Mohr's circle stress transformation equations.

The total shear reinforcement ρ_1 is computed from those in each direction, according to equation LL.123 (Annex LL from EN 1992-2:2005):

$$\rho_1 = \rho_x \cdot \cos^2\phi_v + \rho_y \cdot \sin^2\phi_v$$

5.10.2. Out-of-Plane Shear Checking

5.9.2.1. Checking Failure by Compression

The design shear force (V_{rd}) is compared to the oblique compression resistance of concrete (V_{u1}):

$$V_{rd} \leq V_{u1}$$

$$V_{u1} = K \cdot f_{1cd} \cdot (th - c) \cdot \frac{\cot\theta}{1 + \cot^2\theta}$$

where:

f_{1cd} design compressive strength of concrete.

$$f_{1cd} = \begin{cases} 0.60 \cdot f_{cd} & f_{ck} < 60\ MPa \\ \left(0.90 - \frac{f_{ck}[\mathrm{MPa}]}{200}\right) f_{cd} \geq 0.5 \cdot f_{cd} & f_{ck} \geq 60\ \mathrm{MPa} \end{cases}$$

K reduction factor by axial forces effect

$$K = \begin{cases} 1 & \sigma'_{cd} = 0 \\ 1 + \frac{\sigma'_{cd}}{f_{cd}} & 0 < \sigma'_{cd} \leq 0.25 \cdot f_{cd} \\ 1.25 & 0.25 \cdot f_{cd} < \sigma'_{cd} \leq 0.50 \cdot f_{cd} \\ 2.5\left(1 - \frac{\sigma'_{cd}}{f_{cd}}\right) & 0.50 \cdot f_{cd} < \sigma'_{cd} \leq f_{cd} \end{cases}$$

σ'_{cd} effective axial stress in concrete (compression positive) accounting for the axial stress taken by reinforcement in compression.

For each element end, calculation results are written in the CivilFEM results file:

VU1 Ultimate shear strength due to oblique compression of the concrete.

$$VU1 = V_{u1}$$

CRTVU1 Ratio of the design shear (V_{rd}) to the resistance V_{u1}.

$$CRTVU1 = \frac{V_{rd}}{V_{u1}}$$

5.9.2.2. Checking Failure by Tension in the Web

The design shear force (V_{rd}) must be less than or equal to the shear force due to tension in the web (V_{u2}):

$$V_{rd} \leq V_{u2}$$

$$V_{u2} = V_{su} + V_{cu}$$

V_{su} contribution of transverse shear reinforcement in the web to the shear strength.

V_{cu} contribution of concrete to the shear strength.

Members Without Shear Reinforcement

$$V_{su} = 0$$

$$V_{u2} = V_{cu} = \left[\frac{0.18}{\gamma_c}\xi(100\ \rho_1 f_{ck})^{1/3} + 0.15\sigma'_{cd}\right](th - c)$$

$$V_{u2} = \left[\frac{0.075}{\gamma_c}\xi^{3/2}\sqrt{f_{ck}} + 0.15\sigma'_{cd}\right](th - c)$$

where:

$$\sigma'_{cd} = \frac{T_d}{A_c} < 0.30 \cdot f_{cd} \leq 12 \text{ MPa (Compression positive)}$$

$$\xi = 1 + \sqrt{\frac{200}{d}} < 2, \text{ d in mm}$$

$$f_{ck} \text{ limited to } 60 \text{ MPa}$$

Member With Shear Reinforcement

$$V_{su} = \frac{0.9 \cdot (th - c)}{\tan\theta} \cdot A_{ss} \cdot f_{yd}$$

Where A_s/s is the shear reinforcement area per unit length

In this case, the concrete contribution to shear strength is:

$$V_{cu} = \left[\frac{0.15}{\gamma_c}\xi(100\,\rho_1 f_{ck})^{1/3} + 0.15\sigma'_{cd}\right](th - c)\beta$$

where:

$$\beta = \frac{2\cot\theta - 1}{2\cot\theta_e - 1} \quad \text{if} \quad 0.5 \leq \cot\theta < \cot\theta_e$$

$$\beta = \frac{\cot\theta - 2}{\cot\theta_e - 2} \quad \text{if} \quad \cot\theta_e \leq \cot\theta \leq 2.0$$

θ_e inclination angle of cracks, obtained from:

$$\cot\theta_e = \frac{\sqrt{f_{ct,m}^2 - f_{ct,m}(\sigma_{xd} + \sigma_{yd}) + \sigma_{xd}\sigma_{yd}}}{f_{ct,m} - \sigma_{yd}} \quad \begin{cases} \geq 0.5 \\ \leq 2.0 \end{cases}$$

σ_{xd}, σ_{yd} design normal stresses, at the center of gravity, parallel to the longitudinal axis of member and to the shear force V_d respectively (tension positive)

Taking $\sigma_{yd} = 0 \quad \rightarrow \quad \cot\theta_e = \sqrt{1 - \frac{\sigma_{xd}}{f_{ct,m}}}$

In addition, the increment in tensile force due to shear force is calculated with the following equation:

$$\Delta T = V_{rd} \cdot \cot\theta - \frac{V_{su}}{2} \cdot \cot\theta$$

For each end, calculation results are written in the CivilFEM results file:

VSU Contribution of the shear reinforcement to the shear strength.

$$VSU = V_{su}$$

VCU Contribution of concrete to the shear strength.

$$VCU = V_{cu}$$

VU2 Ultimate shear strength by tension in the web.

$$VU2 = V_{u2} = V_{su} + V_{cu}$$

CRTVU2 Ratio of the design shear force (V_{rd}) to the resistance V_{u2} .

$$CRTVU2 = \frac{V_{rd}}{V_{u2}}$$

If V_{u2} = 0, the CTRVU2 criterion is assigned the value of 2^{100}.

The increase in longitudinal reinforcement due to shear is stored in ASST and ASSB parameters (for top and bottom surfaces of the shell respectively).

5.9.2.3. Shear Criterion

The shear criterion indicates whether the shell vertex is valid for the design forces (if it is less than 1, the vertex satisfies the code provisions; whereas if it exceeds 1, the vertex will not be valid). Furthermore, it includes information about how close the design force is to the ultimate strength. The shear criterion is defined as follows:

$$CRT\_TOT = Max\left(\frac{V_{rd}}{V_{u1}}, \frac{V_{rd}}{V_{u2}}\right) \leq 1$$

For each end, this value is stored in the CivilFEM results file as the parameter CRT_TOT.

A value of 2^{100} for this criterion indicates V_{u2} is equal to zero.

5.10.3. Out-of-Plane Shear Design

5.9.3.1. Checking Compression Failure in the Web

The design shear force (V_{rd}) is compared to the oblique compression resistance of concrete (V_{u1}):

$$V_{rd} = V_{u1}$$

$$V_{u1} = K \cdot f_{1cd} \cdot (th - c) \cdot \frac{\cot\theta}{1 + \cot^2\theta}$$

where:

f_{1cd} design compressive strength of concrete

$$f_{1cd} = \begin{cases} 0.60 \cdot f_{cd} & f_{ck} < 60\ MPa \\ \left(0.90 - \frac{f_{ck}[MPa]}{200}\right) f_{cd} \geq 0.5 \cdot f_{cd} & f_{ck} \geq 60MPa \end{cases}$$

K reduction coefficient by axial force effect

$$K = \begin{cases} 1 & \sigma'_{cd} = 0 \\ 1 + \dfrac{\sigma'_{cd}}{f_{cd}} & 0 < \sigma'_{cd} \leq 0.25 \cdot f_{cd} \\ 1.25 & 0.25 \cdot f_{cd} < \sigma'_{cd} \leq 0.50 \cdot f_{cd} \\ 2.5\left(1 - \dfrac{\sigma'_{cd}}{f_{cd}}\right) & 0.50 \cdot f_{cd} < \sigma'_{cd} \leq f_{cd} \end{cases}$$

σ'_{cd} effective axial stress in concrete (compression positive) accounting for the axial stress taken by the reinforcement in compression.

For each element end, calculation results are written in the CivilFEM results file:

VU1 Ultimate shear strength due to oblique compression of the concrete in web.

$$VU1 = V_{u1}$$

CRTVU1 Ratio of the design shear force (V_{rd}) to the resistance V_{u1}.

$$CRTVU1 = \frac{V_{rd}}{V_{u1}}$$

If design shear force is greater than shear force that causes the failure by oblique compression of concrete in the web, the reinforcement design is not feasible. Therefore, the reinforcement parameter will be defined as 2^{100}.

$$ASSH = A_{ss} = 2^{100}$$

In this case, the element is labeled as not designed.

If there is no failure due to oblique compression, the calculation process continues.

5.9.3.2. Checking If the Shell Requires Shear Reinforcement

First, a check is made to ensure the design shear force V_d is less than the strength provided by concrete in members without shear reinforcement (V_{cu}):

$$V_{rd} = V_{u2}$$

$$V_{u2} = V_{cu} = \left[\frac{0.18}{\gamma_c}\xi(100\,\rho_1 f_{ck})^{1/3} + 0.15\sigma'_{cd}\right](th - c)$$

$$V_{u2} = \left[\frac{0.075}{\gamma_c}\xi^{3/2}\sqrt{f_{ck}} + 0.15\sigma'_{cd}\right](th - c)$$

Where:

$$\sigma'_{cd} = \frac{T_d}{A_c} < 30 \cdot f_{cd} \leq 12\text{ MPa (Compression positive)}$$

$$\xi = 1 + \sqrt{\frac{200}{d}} < 2, \text{ d in mm}$$

f_{ck} limited to 60 MPa

If the shell does not require shear reinforcement, the following parameters are defined:

VCU Contribution of concrete to the shear strength.

$$VCU = V_{cu}$$

VU2 Ultimate shear strength by tension.

$$VU2 = V_{cu}$$

VSU Contribution of the shear reinforcement to the shear strength.

$$VSU = 0$$

ASSH Required amount of shear reinforcement.

$$ASSH = A_{ss} = 0$$

5.9.3.3. Contribution of the Required Transverse Reinforcement

If the shell requires shear reinforcement, sections under shear force will be valid if they satisfy the following condition:

$$V_{rd} \leq V_{u2}$$

$$V_{u2} = V_{su} + V_{cu}$$

V_{su} contribution of shear transverse reinforcement in the web to shear strength.

V_{cu} contribution of concrete to shear strength.

$$V_{cu} = \left[\frac{0.15}{\gamma_c}\xi(100\rho_1 f_{ck})^{1/3} + 0.15\sigma'_{cd}\right](th - c)\beta$$

where:

$$\beta = \frac{2\cot\theta - 1}{2\cot\theta_e - 1} \quad \text{if} \quad 0.5 \leq \cot\theta < \cot\theta_e$$

$$\beta = \frac{\cot\theta - 2}{\cot\theta_e - 2} \quad \text{if} \quad \cot\theta_e \leq \cot\theta \leq 2.0$$

θ_e inclination angle of cracks, obtained from:

$$\cot\theta_e = \frac{\sqrt{f_{ct,m}^2 - f_{ct,m}(\sigma_{xd} + \sigma_{yd}) + \sigma_{xd}\sigma_{yd}}}{f_{ct,m} - \sigma_{yd}} \begin{cases} \geq 0.5 \\ \leq 2.0 \end{cases}$$

σ_{xd}, σ_{yd} design normal stresses at the gravity center, parallel to the longitudinal axis of the member and to the shear force V_d, respectively (tension positive)

Taking $\sigma_{yd} = 0 \rightarrow \cot\theta_e = \sqrt{1 - \frac{\sigma_{xd}}{f_{ct,m}}}$

Therefore, the shear reinforcement contribution is given by the equation below:

$$V_{su} = V_{u2} - V_{cu} = V_{rd} - V_{cu}$$

For each element end, the value of V_{cu} and V_{su} is stored in the CivilFEM results file as the following parameters:

VCU Contribution of concrete to the shear strength.

$$VCU = V_{cu}$$

VU2 Ultimate shear strength by tension.

$$VU2 = V_{rd}$$

VSU Contribution of the shear reinforcement to the shear strength.

$$VSU = V_{su}$$

5.9.3.4. Required Reinforcement Ratio

Once the required shear strength of the reinforcement has been obtained, the reinforcement can be calculated from the equation below:

$$A_{ss} = \frac{V_{su}}{f_{yd} \cdot 0.9 \cdot (th - c) \cdot (\cot\theta)}$$

The area of designed reinforcement per unit of shell area is stored in the CivilFEM results file as the parameter:

$$ASSH = A_{ss}$$

In this case, the element will be labeled as designed (provided the design process is correct for all element shell vertices).

If the design is not possible, the reinforcement will be marked as 2^{100} and the element will not be designed.

ASST and ASSB parameters store the amount of top and bottom reinforcement required due to the additional tensile force DF_{td}, in the longitudinal reinforcement due to shear V_{Ed}.

$$\Delta F_{td} = V_{rd} \cot an\theta - \frac{Vsu}{2} \cot g\,\theta$$

$$\Delta A_{sl} = \frac{V_{rd}\,\text{cotan}\theta - \frac{Vsu}{2}\,\text{cotg}\,\theta}{f_{yd}}$$

ASST= ΔAsl for negative Bending Moments

ASSB= ΔAsl for positive Bending Moments

5.11. In-Plane Shear Load according to ACI 349-01

5.11.1. Required Input Data

Shear checking or design according to ACI 349 require the parameters described below. The formulas listed in this section utilize U.S. (British) units: inch (in), pound (lb), and second (s).

1. **Material strength properties.** This data is obtained from the material properties assigned to each active shell vertex. These material properties must be defined prior to check and design. The required properties are:

   f'_c specified compressive strength of concrete.

   f_y specified yield strength of reinforcement.

2. **Shell vertex data**:

   th thickness of the shell vertex.

Required properties are:

3. **Shell vertex reinforcement data.**

   c bending reinforcement mechanical cover.

   A_{ss} the area of bending reinforcement per unit length.

4. **Shell vertex shear reinforcement data.**

   A_{ssipX}, A_{ssipX} area of in plane shear reinforcement per unit of length in each direction of the shell.

5. **Shell vertex internal forces.** The shear force that acts on the vertex as well as the concomitant membrane force are obtained from the CivilFEM results file (.RCF). For each direction of the shell vertex:

Force	**Description**
V_u	Design in plane shear force
N_u	Membrane force (positive for compression) perpendicular to V_u

6. **Type of check/design.** In-plane shear check/design according to ACI 349-01 is divided into the three following types:

   - Walls with non-seismic loads. Covers chapter 14 of ACI 349-01 for walls.
   - Walls with seismic loads. Covers chapters 14 and 21 of ACI 349-01 for walls.
   - Slabs with seismic loads. Covers chapter 21 of ACI 349-01 for slabs.

5.11.2. In-Plane Shear Checking for Walls

5.10.2.1. Shear strength provided by concrete

For sections subjected to an axial compressive force, the shear strength provided by concrete (V_c) is calculated as:

$$V_c = 2 \cdot \sqrt{f_c'} \cdot d \cdot th$$

$$d = 0.8 \cdot I_w = 0.8 \cdot 1(\text{unit length})$$

(ACI 349-01 11.10.4 and 11.10.5)

Where:

$\sqrt{f'_c}$ square root of specified compressive strength of concrete, in psi (always taken less than 100 psi).

If the section is subjected to a tensile force such that the tensile stress is less than 500 psi then,

$$V_c = 2 \cdot \left(1 + \frac{N_u}{500 \cdot b_w \cdot th}\right) \cdot \sqrt{f_c'} \cdot d \cdot th$$

$$d = 0.8 \cdot I_w = 0.8 \cdot 1(\text{unit length})$$

(ACI 349-01 11.10.5, 11.3.2.3)

If the shell is subjected to a tensile force such that the tensile stress exceeds 500 psi, it is assumed V_c=0.

The calculation result for all elements is stored in the CivilFEM results file as the parameter VC (# is the direction of the shell, X or Y):

VC_# Shear strength provided by concrete.

$$VC = V_c$$

5.10.2.2. Shear strength provided by shear reinforcement

The strength provided by shear reinforcement (V_s) is calculated with the following expression:

$$V_s = \frac{A_{ssip} \cdot f_y \cdot d}{s}$$

$$d = 0.8 \cdot I_w = 0.8 \cdot 1(\text{unit length})$$

$$s = 1(\text{unit length})$$

(ACI 349-01 11.10.4 Eqn: 11-33)

The calculated result for all elements is stored in the CivilFEM results file as the parameter VS (# is the direction of the shell, X or Y):

VS_# Shear strength provided by reinforcement.

$$VS = V_s$$

5.10.2.3. In-plane shear criterion – Non seismic loads

The nominal shear strength (V_n) is the sum of the concrete and shear reinforcement components calculated previously:

$$V_n = V_c + V_s$$

The shell vertex will be valid for shear if the following conditions are satisfied:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

(ACI 349-01 Eqn: 11-1 and 11-2)

$$V_u \leq \phi 10\sqrt{f'_c} \cdot d \cdot th$$

$$d = 0.8 \cdot I_w = 0.8 \cdot 1(\text{unit length})$$

(ACI 349-01 11.10.3 and 11.10.4)

$$\frac{A_{ssip}}{th} \geq 0.0025$$

(ACI 349-01 11.10.9.2)

Where ϕ is the strength reduction factor.

The shear criteria are calculated as:

$$CRT\_1\_\# = \frac{V_u}{\phi V_n}$$

$$CRT\_2\_\# = \frac{V_u}{\phi 10\sqrt{f'_c} \cdot 0.8 \cdot th}$$

$$CRT\_3\_\# = \frac{0.0025 \cdot th}{A_{ssip}}$$

(# is the direction of the shell, X or Y)

Therefore, the validity shear criterion is defined as follows:

$$CRT\_TOT = max\{CRT\_1\_X, CRT\_1\_Y, CRT\_2\_X, CRT2\_Y, CRT\_3\_X, CRT\_3\_Y\} \leq 1$$

These values are stored for all elements in the CivilFEM results file as the parameters $CRT\_1\_X, CRT\_1\_Y, CRT\_2\_X, CRT\_2\_Y, CRT\_3\_X, CRT\_3\_Y$ and CRT_TOT.

If the strength provided by concrete is null and the shear reinforcement is not defined in the shell vertex, then V_n=0, and the criterion CRT_1 will be set equal to 2^{100}.

If the shear reinforcement is not defined in the shell vertex, then the criterion CRT_3 is set equal to 2^{100}.

The $\phi \cdot V_n$ value is stored in the CivilFEM results file as the parameter VPHI_# (# is the direction of the shell, X or Y).

5.10.2.4. *In-plane shear criterion – Seismic loads*

The nominal shear strength (V_n) is the sum of the concrete and shear reinforcement components:

$$V_n = V_c + V_s$$

But also limited by:

$$V_n = A_{cv} 2\sqrt{f_c'} + A_{ssip} \cdot f_y$$

(ACI 349-01 21.6.5.2)

where A_{cv} is the area of concrete:

$$A_{cv} = (\text{unit width}) \cdot (\text{Thickness of the wall})$$

The shell vertex will be valid for shear if the following conditions are satisfied:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

$$V_u \leq \phi\left(A_{cv} 2\sqrt{f_c'} + A_{ssip} \cdot f_y\right)$$

(ACI 349-01 21.6.5.2)

$$V_u \leq \phi 8\sqrt{f_c'} \cdot th \cdot I_w$$
$$I_w = 1(\text{unit length})$$

(ACI 349-01 21.6.5.6)

$$\frac{A_{ssip}}{th} \geq 0.0025$$

(ACI 349-01 21.6.2.1)

Where ϕ is the strength reduction factor.

The shear criteria are calculated as:

$$CRT\_1\_\# = \frac{V_u}{\phi \min\left(V_n, A_{cv} 2\sqrt{f_c'} + A_{ssip} \cdot f_y\right)}$$

$$CRT\_2\_\# = \frac{V_u}{\phi 8\sqrt{f_c'} \cdot th}$$

$$CRT\_3\_\# = \frac{0.0025 \cdot th}{A_{ssip}}$$

(# is the direction of the shell, X or Y)

Therefore, the validity shear criterion is defined as follows:

$$CRT\_TOT = \max\{CRT\_1\_X, CRT\_1\_Y, CRT\_2\_X, CRT\_2\_Y, CRT\_3\_X, CRT\_3\_Y\} \leq 1$$

These values are stored for all elements in the CivilFEM results file as the parameters $CRT\_1\_X, CRT\_1\_Y, CRT\_2\_X, CRT\_2\_Y, CRT\_3\_X, CRT\_3\_Y$ and CRT_TOT.

In case the strength provided by concrete is null and the shear reinforcement is not defined in the shell vertex, then V_n=0, and the criterion CRT_1 is set equal to 2^{100}.

If shear reinforcement is not defined in the shell vertex, then the criterion CRT_3 is set equal to 2^{100}.

The $\phi \cdot V_n$ value is stored in the CivilFEM results file as the parameter VPHI_# (# is the direction of the shell, X or Y).

5.11.3. In-Plane Shear Design for Walls

5.10.3.1. Shear strength provided by concrete

For sections subject to an axial compressive force, the shear strength provided by concrete (V_c) is calculated by:

$$V_c = 2 \cdot \sqrt{f_c'} \cdot d \cdot th$$

$$d = 0.8 \cdot I_w = 0.8 \cdot 1(\text{unit length})$$

(ACI 349-01 11.10.4 and 11.10.5)

Where:

$\sqrt{f_c'}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

If the section is subjected to a tensile force such that the tensile stress is less than 500 psi:

$$V_c = 2 \cdot \left(1 + \frac{N_u}{500 \cdot b_w \cdot th}\right) \cdot \sqrt{f_c'} \cdot d \cdot th$$

$$d = 0.8 \cdot I_w = 0.8 \cdot 1(\text{unit length})$$

(ACI 349-01 11.10.5, 11.3.2.3)

If the shell is subjected to a tensile force so that the tensile stress exceeds 500 psi, it is assumed V_c=0.

The calculated result for each element is stored in the CivilFEM results file as the parameter VC (# is the direction of the shell, X or Y):

VC_# Shear strength provided by concrete.

$$VC = V_c$$

5.10.3.2. Required reinforcement contribution

The shell must satisfy the following condition to resist the shear force:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

(ACI 349-01 11-1 and 11-2)

Where ϕ is the strength reduction factor.

Required shear strength of the reinforcement:

$$V_s = \frac{V_u}{\phi} - V_c$$

The calculated result for each element is stored in the CivilFEM results file as the parameter VS (# is the direction of the shell, X or Y):

VS_# Shear strength provided by reinforcement.

$$VS = V_s$$

5.10.3.3. Required reinforcement – Non seismic loads

The reinforcement amount is obtained by inserting the value of V_s, determined above, into the following equation:

$$V_s = \frac{A_{ssip} \cdot f_y \cdot d}{s}$$

$$d = 0.8 \cdot I_w = 0.8 \cdot 1(\text{unit length})$$

$$s = (\text{unit length})$$

(ACI 349-01 11.10.4 Eqn: 11-33)

The reinforcement amount has a minimum requirement of:

$$\frac{A_{ipss}}{th} \geq 0.0025$$

(ACI 349-01 11.10.9.2)

Therefore:

$$A_{ssip} = \max\left\{\frac{V_s}{f_y \cdot 0.8}, 0.0025 \cdot th\right\} \text{ for each direction X and Y}$$

The area of the designed reinforcement per unit length is stored in the CivilFEM results file as:

$$ASSIP\_X = A_{ssip} \text{ for x direction}$$

$$ASSIP\_Y = A_{ssip} \text{ for Y direction}$$

Also, the following condition must be satisfied:

$$V_u \leq \phi 10\sqrt{f_c'} \cdot d \cdot th$$

$$d = 0.8 \cdot I_w = 0.8 \cdot 1(\text{unit length})$$

(ACI 349-01 11.10.3 and 11.10.4)

This criterion is calculated as:

$$CRT\_2\_\# = \frac{V_u}{\phi 10\sqrt{f'_c} \cdot 0.8 \cdot th}$$

(# is the direction of the shell, X or Y)

If CRT_2_# is greater than 1.0, the condition will not be satisfied, and therefore, the element will not be designed. ASSIP_# will be set to 2^{100} and the element will be labeled as not designed.

The criterion below compares the calculated reinforcement with the minimum reinforcement requirement:

$$CRT\_3\_\# = \frac{0.0025 \cdot th}{A_{ssip}}$$

(# is the direction of the shell, X or Y)

5.10.3.4. Required reinforcement – Seismic loads

The shell vertex will be valid for shear if the following conditions are satisfied:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

(ACI 349-01 Eqn: 11-1 and 11-2)

With

$$V_s = \frac{A_{ssip} \cdot f_y \cdot d}{s}$$

$$d = 0.8 \cdot I_w = 0.8 \cdot 1(\text{unit length})$$

$$s = 1(\text{unit length})$$

(ACI 349-01 Eqn: 11-33)

and

$$V_u \leq \phi\left(A_{cv} 2\sqrt{f'_c} + A_{ssip} \cdot f_y\right)$$

(ACI 349-01 21.6.5.2)

where A_{cv} is the area of concrete:

$$A_{cc} = (\text{unit with}) \cdot (\text{Thickness of the wall})$$

Therefore the reinforcement amount is the minimum value that satisfies both expressions:

$$A_{ssip} = \max\left\{\frac{V_u - \phi V_c}{\phi \cdot 0.8 \cdot f_y}, \frac{V_u - \phi A_{cv} 2\sqrt{f'_c}}{\phi f_y}\right\}$$

The reinforcement amount has a minimum requirement of:

$$\frac{A_{ssip}}{th} \geq 0.0025$$

(ACI 349-01 21.6.2.1)

Therefore:

$$A_{ssip} = \max\left\{\frac{V_u - \phi V_c}{\phi \cdot 0.8 \cdot f_y}, \frac{V_u - \phi A_{cv} 2\sqrt{f'_c}}{\phi f_y}, 0.0025 \cdot th\right\} \text{ for each direction X and Y}$$

The area of the designed reinforcement per unit length is stored in the CivilFEM results file as:

$$ASSIP\_X = A_{ssip} \text{ for x direction}$$

$$ASSIP\_Y = A_{ssip} \text{ for Y direction}$$

Also, the following condition must be satisfied:

$$V_u \leq \phi 8\sqrt{f'_c} \cdot I_w \cdot th$$

$$I_w = 1 (\text{unit length})$$

(ACI 349-01 21.6.5.6)

This criterion is calculated as:

$$CRT\_2\_\# = \frac{V_u}{\phi 8\sqrt{f'_c} \cdot th}$$

(# is the direction of the shell, X or Y)

If CRT_2_# is greater than 1.0, the condition will not be satisfied, and therefore, the element will not be designed. ASSIP_# will then be set to 2^{100} and the element will be labeled as not designed.

The criterion below compares the calculated reinforcement with the minimum required reinforcement:

$$CRT\_3\_\# = \frac{0.0025 \cdot th}{A_{ipss}}$$

(# is the direction of the shell, X or Y)

5.11.4. In-Plane Shear Checking for Slabs (Seismic Loads)

5.10.4.1. In plane shear criterion

The nominal shear strength (V_n) is limited by:

$$V_n = A_{cv} 2\sqrt{f'_c} + A_{ssip} \cdot f_y$$

(ACI 349-01 21.6.5.2)

Where A_{cv} is the area of concrete:

$$A_{cv} = (\text{unit width}) \cdot (\text{Thickness of the slab})$$

The shell vertex will be valid for shear if the following conditions are satisfied:

$$V_u \leq \phi\left(A_{cv} 2\sqrt{f'_c} + A_{ssip} \cdot f_y\right)$$

(ACI 349-01 21.6.5.2)

$$V_u \leq \phi 8\sqrt{f'_c} \cdot I_w \cdot th$$

$$I_w = 1(\text{unit length})$$

(ACI 349-01 21.6.5.6)

$$\frac{A_{ssip}}{th} \geq 2 \cdot 0.0012 = 0.0024 \quad (th < 48\ in.)$$

(ACI 349-01 21.6.2.1, 7.12.2)

Note: A minimum reinforcement amount is not calculated for a thickness greater or equal than 48 in.

Where ϕ is the strength reduction factor.

The shear criteria are calculated as:

$$CRT\_1\_\# = \frac{V_u}{\phi\left(A_{cv} 2\sqrt{f'_c} + A_{ssip} \cdot f_y\right)}$$

$$CRT\_2\_\# = \frac{V_u}{\phi 8\sqrt{f'_c} \cdot th}$$

$$CRT\_3\_\# = \frac{0.0024 \cdot th}{A_{ssip}} \quad (th < 48in.)$$

$$CRT\_3\_\# = 0(th < 48in.)$$

(# is the direction of the shell, X or Y)

Therefore, the validity shear criterion is defined as follows:

$$CRT\_TOT = \max\{CRT\_1\_X, CRT\_1\_Y, CRT\_2\_X, CRT\_2\_Y, CRT\_3\_X, CRT\_3\_Y\} \leq 1$$

These values are stored for each element in the CivilFEM results file as the parameters CRT_1_X, CRT_1_Y, CRT_2_X, CRT_2_Y, CRT_3_X, CRT_3_Y and CRT_TOT.

If the strength provided by concrete is null and shear reinforcement is not defined in the shell vertex, then V_n=0, and the criterion CRT_1 is set equal to 2^{100}.

If shear reinforcement is not defined in the shell vertex, then the criterion CRT_3 is set equal to 2^{100}.

The $\phi \cdot V_n$ value is stored in the CivilFEM results file as the parameter VPHI_# (# is the direction of the shell, X or Y).

5.11.5. In-Plane Shear Design for Slabs (Seismic Loads)

5.10.5.1. *Required reinforcement*

The shell vertex will be valid for shear if the following condition is satisfied:

$$V_u \leq \phi\left(A_{cv} 2\sqrt{f'_c} + A_{ssip} \cdot f_y\right)$$

(ACI 349-01 21.6.5.2)

where A_{cv} is the area of concrete:

$$A_{cv} = (\text{unit width}) \cdot (\text{Thickness of the slab})$$

The reinforcement amount has a minimum requirement of:

$$\frac{A_{ssip}}{th} \geq 2 \cdot 0.0012 = 0.0024 \;\; (th < 48\; in.)$$

(ACI 349-01 21.6.2.1, 7.12.2)

Note: A minimum reinforcement amount is not calculated for a thickness greater or equal than 48 in.

Therefore the reinforcement amount is the minimum value that satisfies the following expressions for both X and Y directions:

$$A_{ssip} = \max\left\{\frac{V_u - \phi A_{cv} 2\sqrt{f_c'}}{\phi f_y}, 0.0024 \cdot th\right\} (th < 48\ in.)$$

$$A_{ssip} = \max\left\{\frac{V_u - \phi A_{cv} 2\sqrt{f_c'}}{\phi f_y}\right\} \quad (th \geq 48\ \mathrm{in.})$$

The area of the designed reinforcement per unit length is stored in the CivilFEM results file as:

$$\mathrm{ASSIP\_X} = A_{ssip}\ \text{for X direction}$$

$$\mathrm{ASSIP\_Y} = A_{ssip}\ \text{for Y direction}$$

Also, the following condition must be satisfied:

$$V_u = \leq \phi 8\sqrt{f_c'} \cdot I_w \cdot th$$

$$I_w = 1(\text{unit length})$$

(ACI 349-01 21.6.5.6)

This criterion is calculated as:

$$\mathrm{CRT\_2\_\#} = \frac{V_u}{\phi 8\sqrt{f_c'} \cdot th}$$

(# is the direction of the shell, X or Y)

If CRT_2_# is greater than 1.0, the condition above will not be satisfied and therefore the element cannot be designed. ASSIP_# will be set to 2^{100} and the element will be labeled as not designed.

To determine if a minimum reinforcement amount has been defined, the CRT_3_# criterion is defined as:

$$\mathrm{CRT\_3\_\#} = \frac{0.0024 \cdot th}{A_{ssip}} \quad (th < 48in.)$$

$$\mathrm{CRT\_3\_\#} = 0\ (th \geq 48\mathrm{in.})$$

5.12. Cracking Checking according to Eurocode 2 (EN 1992-1-1:2004/AC:2008)

5.12.1. Cracking Checking

The cracking check calculates the crack width and checks the following condition:

$$W_k \leq W_{max}$$

where:

W_k Design crack width.

W_{max} Maximum crack width (option in the Checking menu)

The design crack width is obtained from the following expression (Art. 7.3.4):

$$W_k = S_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$$

$S_{r,max}$ Maximum spacing between cracks.

ε_{sm} Mean strain in the reinforcement.

ε_{cm} Mean strain in the concrete between bars.

$$S_{r,max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \frac{\phi}{\rho_{p,eff}}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}}(1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

ϕ Reinforcement bar size in mm (cross section code property).

$\rho_{p,eff} = \frac{A_s + \xi_1^2 \cdot A_p'}{A_{c,eff}}$ Effective reinforcement ratio, where $A_{c,eff}$ is the effective area of concrete in tension, A_s is the area of reinforcement contained within the effective concrete area and A_p' is the area of pre- or post-tensioned tendons within $A_{c,eff}$ CivilFEM calculates this value with $A_p' = 0$ and $A_{c,eff} = \min(2.5(h-d), \frac{h-x}{3}, h/2)$

k_1 Coefficient accounting for the influence of the bond properties of the bonded reinforcement (option in the Checking menu).

k_2 Coefficient accounting for the influence of the form of the strain distribution:

$$k_2 = \frac{\varepsilon_1 + \varepsilon_2}{2\varepsilon_1}$$

Where ε_1 is the larger tensile strain and ε_2 is the smaller tensile strain at the boundary of a section subjected to eccentric tension.

k_3, k_4 Constants defined in the National Annexes (option in the Checking menu).

c Cover to the longitudinal reinforcement. (Cross section code property).

σ_s Stress in the tensile reinforcement calculated for a cracked section.

E_s Elastic modulus of the longitudinal reinforcement.

k_t Coefficient accounting for the influence of the duration of the loading (option in the Checking menu).

α_e Ratio between steel-concrete elastic modulus (E_s/E_{cm}).

5.12.2. Reinforcement Stress Calculation

During the calculation process, it is necessary to determine the reinforcement stress under service loads (σs) with the assumption the section is cracked.

The calculation of these stresses is an iterative process in which CivilFEM searches for the deformation plane that causes a stress state that is in equilibrium with the external loads. The reinforcement stress is obtained from this deformation plane and from the reinforcement position.

The design loads are taken as external loads for the case of serviceability stress calculation. For the stress calculation at the instant the crack appears, the external loads are taken as homothetic to the design loads that cause a stress equivalent to the concrete tensile strength in the fiber under the greatest amount of tension.

If the loads acting on the cross section cause collapse under axial plus bending checking, the cross section and the associated element are labeled as non-checked.

5.12.3. Checking results

Checking results are stored in the corresponding CivilFEM result file.

The following results are available:

CRT_TOT_X Cracking criterion.

SIGMA_X Maximum tensile stress.

WK_X Design crack width

SRMAX_X Maximum spacing between cracks.

EM_X Difference between the mean strain in the reinforcement and the mean strain in concrete $\varepsilon_{sm} - \varepsilon_{cm}$.

POS_X Cracking position inside the section

1 Upper fiber.

-1 Lower fiber.

0 Upper and lower fibers.

CRT_TOT_Y Cracking criterion.

SIGMA_Y Maximum tensile stress.

WK_Y Design crack width

SRMAX_Y Maximum spacing between cracks.

EM_Y Difference between the mean strain in the reinforcement and the mean strain in concrete $\varepsilon_{sm} - \varepsilon_{cm}$.

POS_Y Cracking position inside the section.

1 Upper fiber.

-1 Lower fiber.

0 Upper and lower fibers.

For the cracking check ($w_{max} > 0$) the total criterion is defined as:

$$\mathrm{CRT\_TOT} = \frac{W_k}{W_{max}}$$

For decompression checking ($w_{max} = 0$) the total criterion is defined as:

$$CRT\_TOT = \frac{f_{cd} + \sigma_{max}}{f_{cd}}$$

where

f_{cd} concrete design compressive strength

σ_{max} Maximum section stress (positive tension). It corresponds to the SIGMA result. (If CRT_TOT is negative, it's taken as zero)

Therefore, values for the total criterion larger than one indicate that the section does not pass as valid for this code.

5.13. Cracking Checking according to Structural Code (Spanish code)

5.13.1. Cracking Checking

The cracking check calculates the crack width and checks the following condition:

$$W_k \leq W_{max}$$

where:

W_k Design crack width.

W_{max} Maximum crack width (option in the Checking menu)

The design crack width is obtained from the following expression (Art. 7.3.4):

$$W_k = S_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$$

$S_{r,max}$ Maximum spacing between cracks.

ε_{sm} Mean strain in the reinforcement.

ε_{cm} Mean strain in the concrete between bars.

$$S_{r,max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \frac{\phi}{\rho_{p,eff}}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}}(1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

ϕ Reinforcement bar size in mm (cross section code property).

$\rho_{p,eff} = \frac{A_s + \xi_1^2 \cdot A_p'}{A_{c,eff}}$ Effective reinforcement ratio, where $A_{c,eff}$ is the effective area of concrete in tension, A_s is the area of reinforcement contained within the effective concrete area and A_p' is the area of pre- or post-tensioned tendons within $A_{c,eff}$ CivilFEM calculates this value with $A_p' = 0$ and $A_{c,eff} = \min(2.5(h-d), \frac{h-x}{3}, h/2)$

k_1 Coefficient accounting for the influence of the bond properties of the bonded reinforcement (option in the Checking menu).

k_2 Coefficient accounting for the influence of the form of the strain distribution:

$$k_2 = \frac{\varepsilon_1 + \varepsilon_2}{2\varepsilon_1}$$

Where ε_1 is the larger tensile strain and ε_2 is the smaller tensile strain at the boundary of a section subjected to eccentric tension.

k_3, k_4 Constants defined in the National Annexes (option in the Checking menu).

c Cover to the longitudinal reinforcement. (Cross section code property).

σ_s Stress in the tensile reinforcement calculated for a cracked section.

E_s Elastic modulus of the longitudinal reinforcement.

k_t Coefficient accounting for the influence of the duration of the loading (option in the Checking menu).

α_e Ratio between steel-concrete elastic modulus (E_s/E_{cm}).

5.13.2. Reinforcement Stress Calculation

During the calculation process, it is necessary to determine the reinforcement stress under service loads (σs) with the assumption the section is cracked.

The calculation of these stresses is an iterative process in which CivilFEM searches for the deformation plane that causes a stress state that is in equilibrium with the external loads. The reinforcement stress is obtained from this deformation plane and from the reinforcement position.

The design loads are taken as external loads for the case of serviceability stress calculation. For the stress calculation at the instant the crack appears, the external loads are taken as homothetic to the design loads that cause a stress equivalent to the concrete tensile strength in the fiber under the greatest amount of tension.

If the loads acting on the cross section cause collapse under axial plus bending checking, the cross section and the associated element are labeled as non-checked.

5.13.3. Checking results

Checking results are stored in the corresponding CivilFEM result file.

The following results are available:

CRT_TOT_X Cracking criterion.

SIGMA_X Maximum tensile stress.

WK_X Design crack width

SRMAX_X Maximum spacing between cracks.

EM_X Difference between the mean strain in the reinforcement and the mean strain in concrete $\varepsilon_{sm} - \varepsilon_{cm}$.

POS_X Cracking position inside the section

1 Upper fiber.

-1 Lower fiber.

0 Upper and lower fibers.

CRT_TOT_Y Cracking criterion.

SIGMA_Y Maximum tensile stress.

WK_Y Design crack width

SRMAX_Y Maximum spacing between cracks.

EM_Y Difference between the mean strain in the reinforcement and the mean strain in concrete $\varepsilon_{sm} - \varepsilon_{cm}$.

POS_Y Cracking position inside the section.

1 Upper fiber.

-1 Lower fiber.

0 Upper and lower fibers.

For the cracking check ($w_{max} > 0$) the total criterion is defined as:

$$\mathrm{CRT\_TOT} = \frac{W_k}{W_{max}}$$

For decompression checking ($w_{max} = 0$) the total criterion is defined as:

$$CRT\_TOT = \frac{f_{cd} + \sigma_{max}}{f_{cd}}$$

where

f_{cd} concrete design compressive strength

σ_{max} Maximum section stress (positive tension). It corresponds to the SIGMA result. (If CRT_TOT is negative, it's taken as zero)

Therefore, values for the total criterion larger than one indicate that the section does not pass as valid for this code.

5.14. Cracking Checking according to ACI 318-05 and ACI 318-14

5.14.1. Cracking Checking

Checking of the Cracking Limit State according to ACI 318-05 and ACI 318-14 consists of the following condition:

$$S_d \leq S$$

Where:

S_d Reinforcement spacing closest to the fiber in tension (option in the Checking menu)

S Design reinforcement spacing

CivilFEM checks this condition by applying the general calculation method for the reinforcement spacing (Art. 10.6.4):

$$S = 15\left(\frac{40}{f_s}\right) - 2.5 \cdot C_c$$

$$S \leq 12\frac{40}{f_s}$$

where:

f_s Calculated stress in reinforcement at service loads (in ksi).

C_c Geometrical cover (cross section code property) (in inches).

5.14.2. Reinforcement Stress Calculation

During the calculation process, it’s necessary to determine the reinforcement stress under service loads (fs).

The calculation of the stresses is an iterative process in which the program searches for the deformation plane that causes a stress state that is in equilibrium with the external loads. The reinforcement stress is obtained from this deformation plane and from the reinforcement position.

The design loads are taken as external loads.

If the loads acting on the cross section cause collapse under axial plus bending checking, the cross section and the element to which it belongs are marked as non checked.

5.14.3. Checking results

Checking results are stored in the corresponding CivilFEM results file.

The following results are available:

CRT_TOT_X Cracking criterion in X direction.

S_X Design reinforcement spacing in X direction.

FS_X Reinforcement stress in X direction.

SIGMA_X Maximum tensile stress in X direction.

POS_X Cracking position inside the section in X direction.

- 1 Upper fiber.
- -1 Lower fiber.
- 0 Upper and lower fibers.

CRT_TOT_Y Cracking criterion in Y direction.

S_Y Design reinforcement spacing in Y direction.

FS_Y Reinforcement stress in Y direction.

SIGMA_Y Maximum tensile stress in Y direction.

POS_Y Cracking position inside the section in Y direction.

- 1 Upper fiber.
- -1 Lower fiber.
- 0 Upper and lower fibers.

For the cracking check ($s_d > 0$) the total criterion is defined as:

$$CRT\_TOT = \frac{S_d}{S}$$

For decompression checking ($s_d = 0$) the total criterion is defined as:

$$CRT\_TOT = \frac{f_c + \sigma_{max}}{f_c}$$

where

f_c concrete design compressive strength.

σ_{max} Maximum section stress (positive tension), corresponding to the SIGMA result. (If CRT_TOT is negative, it is taken as zero)

Therefore, the values for the total criterion larger than one indicate that the section is not considered valid for this code.

Chapter 6
Reinforced Concrete Sections

6.1. Introduction

Checking and reinforcement designing of reinforced concrete beams in CivilFEM is available for structures formed by 2D and 3D beam elements under axial loading plus biaxial bending, axial loading plus bending (particular case), shear, torsion and combined shear and torsion.

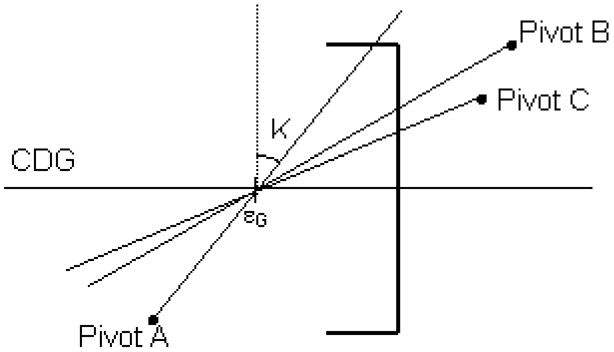
The check and design process of reinforced concrete beams under axial loading plus biaxial bending is based on the 3D interaction diagram of the analysed transverse section. This 3D interaction diagram contains forces and moments (FX, MY, MZ) corresponding to the sections ultimate strength states. Using this diagram, the program is able to check and design the section accounting for forces and moments previously obtained that act on the section. This process considers both generic sections and sections formed by different concretes and reinforcement steels.

The codes CivilFEM considers for the checking and design of reinforced beams subjected to axial force and biaxial bending are: ACI 318, EHE, Eurocode 2, ITER Design code, British Standard 8110, Australian Standard 3600, CEB-FIP 1990 model code, the Chinese code GB50010, NBR6118, AASHTO Standard Specifications for Highway Bridges, Russian Code СП 52-101-03, Indian Standard IS 456 and ACI 349.

6.2. 2D Interaction Diagram

6.2.1. Pivots Diagram

The interaction diagram is a graphical summary that contains the forces and moments (FX, MY) or (FX, MZ) corresponding to the section ultimate strength states. In CivilFEM the ultimate strength states are determined through the pivots diagram.



The “Pivot” concept is related to the limit behavior of the cross section with respect to steel and concrete material characteristics.

A pivot is a strain limit associated with a material and its position in the section. If the strain in a section’s pivot exceeds the limit for that pivot, the section will be considered as cracked. Thus, pivots establish the positions of the strain plane. In an ultimate strength state, the strain plane supports at least one pivot of the section.

In CivilFEM, pivots are defined as material properties and these properties (pivots) are extrapolated to all the section’s points, taken into account the material of each point. Therefore, for the section’s strain plane determination, the following pivots and their corresponding material properties will be considered:

A Pivot	EPSmax. Maximum allowable strain in tension at any point of the section (largest value of the maximum strains allowable for each point of the section if there are different materials in the section).
B Pivot	EPSmin. Maximum allowable strain in compression at any point of the

section (largest value of the maximum strains allowable for each point of the section).

C Pivot — EPSint. Maximum allowable strain in compression at the interior points of the section.

Navier's hypothesis is assumed for the determination of the strains plane. The strain's plane is determined according to the following equation:

$$\varepsilon(y,z) = \varepsilon_g + K_z y + K_y z$$

where:

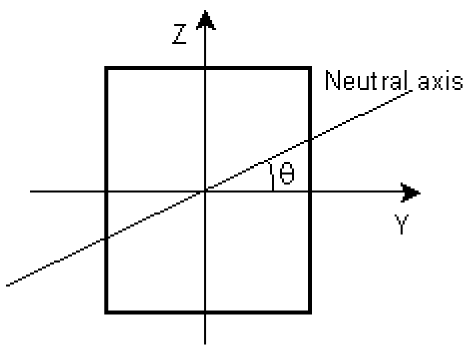
ε (y,z)	Strain of a section point as a function of the Y, Z axes of the section.
ε_g	Strain in the origin of the section (center of gravity).
K_z	Curvature in Z axis.
K_y	Curvature in Y axis.

In CivilFEM, the three elements ε_g, Kz, Ky are substituted by the elements ε_g, θ, K to determine the strain plane. The relationship between (Kz, Ky) and (θ, K) is the following:

$$K_y = K\cdot\cos(\theta)$$

$$K_z = K\cdot\sin(\theta)$$

θ = Angle of the neutral axis with respect to the section's Y axis



6.2.2. Diagram Construction Process

As stated in the previous section, CivilFEM uses the elements $(\varepsilon_{g,}\theta, K)$to determine the strains plane (ultimate strength plane) of the section.
ε_g and θ are used as independent variables. The process is composed of the following steps:

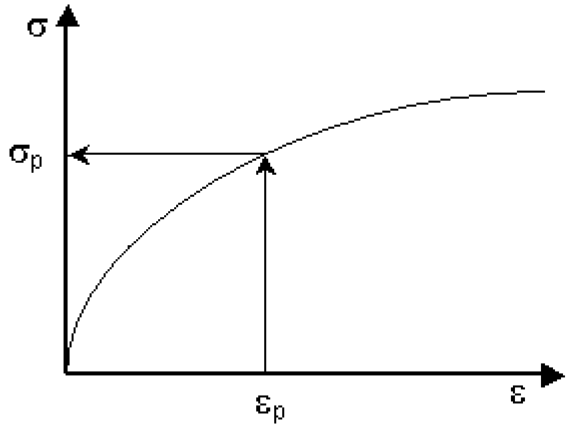
1. Values of ε_g and θ are chosen arbitrarily inside the extreme values allowed for these variables, which are:

   EPSmin (B pivot) $< \varepsilon_g <$EPSmax (A pivot)

   -180º $< \theta <$ +180º

   If there is no A pivot, (if there is no reinforcement steel or if ACI, AS3600 or BS8110 codes are used) the tension limit does not exist and is considered infinite.

2. From the angle θ, the program can identify which points are inside and outside the nucleus of the section.
3. Once the interior and exterior points are known, the two extreme admissible strains, EPSmin and EPSmax, are defined in each of the points (for each point based on its material).
4. For each point of the section, the minimum ultimate strength curvature (K) is calculated.
5. The K curvature will be adopted as the minimum of all the curvatures of all the section points, according to the condition K $\geq$ 0.
6. From the obtained K curvature and ε_g (strain imposed in the section's center of gravity), the deformation corresponding to each of the section points ε (x, y), is determined using the equations shown previously.
7. From the ε (x, y) strain, the stress corresponding to each point of the section (σp) is calculated and entered into the stress-strain diagram for that point. Through this method, the stress distribution inside the section is determined.



8. Thus, as the elements $(\varepsilon_{g,}\theta, K)$ are determined, the ultimate forces and moments (FX, MY, MZ) corresponding to the ε_g strain and the θ angle defined in step 1 are obtained by the summation of stresses at each of the section's points multiplied by its corresponding weight.

$$FX = \sum_{1}^{NP} WFx_p \cdot \sigma_p$$

$$MY = \sum_{1}^{NP} WMy_p \cdot \sigma_p$$

$$MZ = \sum_{1}^{NP} WMz_p \cdot \sigma_p$$

Where: NP = number of points of the section

WF_x, WM_y, WM_z = weights at each point of the section.

Note: For the design process, two components of forces and moments will be calculated: the component relating to the fixed points (corresponding to the reinforcement defined as fixed and to the concrete) and the component relating to the scalable points (corresponding to the part of the section reinforcement defined as scalable, see Chapter 4.6.). The final forces and moments will be equal to the sum of the forces and moments of both components. The forces and moments due to the component for scalable points will be multiplied by the reinforcement factor (ω).

$$\mathbf{(FX, MY, MZ)_{real} = (FX, MY, MZ)_{fixed} + \omega.(FX, MY, MZ)_{scalable}}$$

9. Steps 1 to 8 are repeated, adjusting the εg and θ values and calculating the corresponding ultimate force and moments (FX, MY, MZ). Each defined couple (ε_g and θ) represents a point in the 3D interaction diagram of the section. The greater the number of ε_g and θ values used (inside the interval specified in step 1), the larger of the number of points in the diagram, and therefore the accuracy of the diagram will increase.

With all of the 2D points previously obtained, the program constructs the interaction diagram by calculating the convex hull of these points. Once the convex hull is calculated, the “convexity criterion” of the diagram is determined; this criterion is the minimum of the criteria calculated for all the points of the diagram. The ideal value of the convexity criterion of the diagram is 1. In CivilFEM, it is not recommended to perform the check and design described above with interaction diagrams whose convexity criterion is less than 0.95.

It has been proven that the interaction diagram of sections composed by materials whose stress-strain law (for sections analysis) presents a descending branch has a very low convexity criterion. The check and design process with the diagram of these sections may lead to unsafe solutions. Therefore, it is NOT RECOMMENDED to use materials with this characteristic.

6.2.3. Determination of the Diagram Center

Normal interaction diagrams contain the coordinates’ origin in their interior, but in some cases the origin may be a point belonging to the surface or even a point outside the diagram (such as for prestressed concrete sections). In this situation, the section is cracked for null forces and moments.

To avoid these situations, CivilFEM changes the axes, placing the origin of the coordinate system inside the geometric center of the diagram. In this case, the calculation of the safety criterion is executed according to the new coordinate’s origin instead of the real origin.

If these changes are not made, safety forces and moments (in the diagram interior) could have a safety factor less than 1.00 and vice versa. If the coordinate’s origin is close to the diagram’s surface (although still inside), it will also be necessary to change the origin coordinates. In these cases, although the safety factors maintain values greater than 1.0 for safe sections and less than 1.0 for unsafe ones, they may adopt arbitrary values not very related to the section’s real safety factor.

Therefore, CivilFEM establishes a criterion to determine whether to use the real coordinate system origin or a modified one as a reference. Thus, if the following condition is fulfilled, the origin of the coordinates will be modified, moving the diagram's real center to its geometric center.

$$\text{Distance} \leq \text{Delta} \cdot \frac{\text{Diameter}}{2}$$

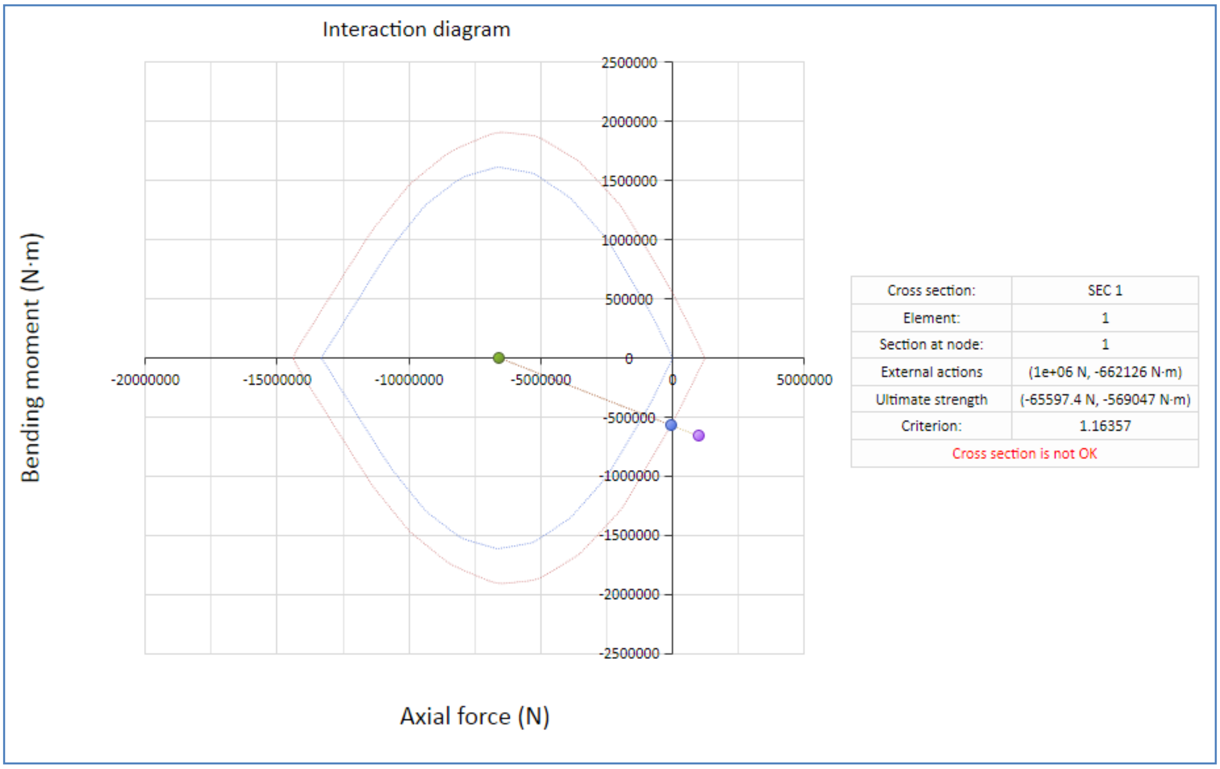
Where:

Distance	Minimum distance from any point of the diagram to the real coordinate system origin.
Delta	Variable parameter which may be defined inside the [0,1] range. By default Delta=0.05.
Diameter	Diagonal of the rectangle which involves the diagram surface points.

6.2.4. Considerations

- The selection of the strains values at the origin of the section (εg) inside the interval (EPSmin, EPSmax) for each adopted angle of the neutral axis (θ) is made uniformly spaced for sections with reinforcements below the center of gravity (bottom reinforcement). Half of it is distributed in the tension zone and the other half in the compression zone, avoiding a concentration of points in the ultimate tension zone and obtaining an even distribution of points.
- If the section does not have bottom reinforcement for each θ or the reinforcement does not have pivot (EPSmax) (as in the case of the ACI or BS8110 codes), the distribution of the tension zone is hyperbolic. The compression zone will continue to have uniform distribution. By default the number of the values adopted by ε_g is 30. The number of values must be a multiple of 2.
- At the same time, the selection of the θ values is also uniform, inside the interval (-180º, +180º). The number of values must be a multiple of 4 in order to embrace the 4 quadrants of the section. By default, the number of values adopted by the program is 28.
- Although the number of the values of ε_g and θ used for the construction of the diagram can be defined by the user, it is recommended to choose numbers close to the default values. These values have been chosen in consideration of the calculation time and precision. If a number of values for either variable is a great deal higher than the default value, the processing time increases significantly.

On the other hand, if the number of values of ε_g and θ is reduced significantly, the precision in the calculation of the diagram may be affected.


Interaction diagram
Bending moment (N·m)
Axial force (N)
2500000
2000000
1500000
1000000
500000
0
-500000
-1000000
-1500000
-2000000
-2500000
-20000000
-15000000
-10000000
-5000000
0
5000000
Cross section: SEC 1
Element: 1
Section at node: 1
External actions (1e+06 N, -662126 N·m)
Ultimate strength (-65597.4 N, -569047 N·m)
Criterion: 1.16357
Cross section is not OK

6.3. Axial Load and Biaxial Bending Checking

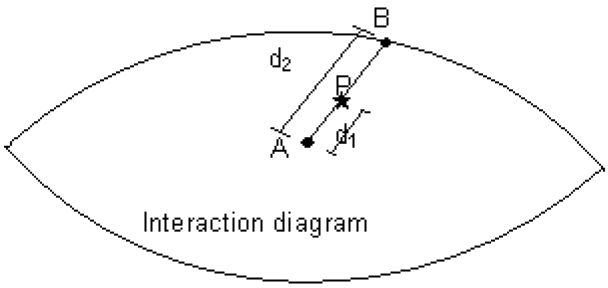
6.3.1. Calculation Hypothesis

- This checking procedure only verifies the section's strength requirements; thus, requirements relating to serviceability conditions, minimum reinforcement amounts or reinforcement distribution for each code and structural typology will be not be considered.
- Navier's hypothesis is always assumed as valid; therefore, the deformed section will remain plane. The longitudinal strain of concrete and steel will be proportional to the distance from the neutral axis.

6.3.2. Calculation Process

Checking elements for axial force and biaxial bending adheres to the following steps:

1. **Obtaining the acting forces and moments of the section (FXd, MYd, MZd).** The acting forces and moments are obtained, following a calculation, directly from the CivilFEM results file (file .RCF).
2. **Constructing the interaction diagram of the section.** The ultimate strain state is determined such that the ultimate forces and moments are homothetic to the acting forces and moments with respect to the diagram center.



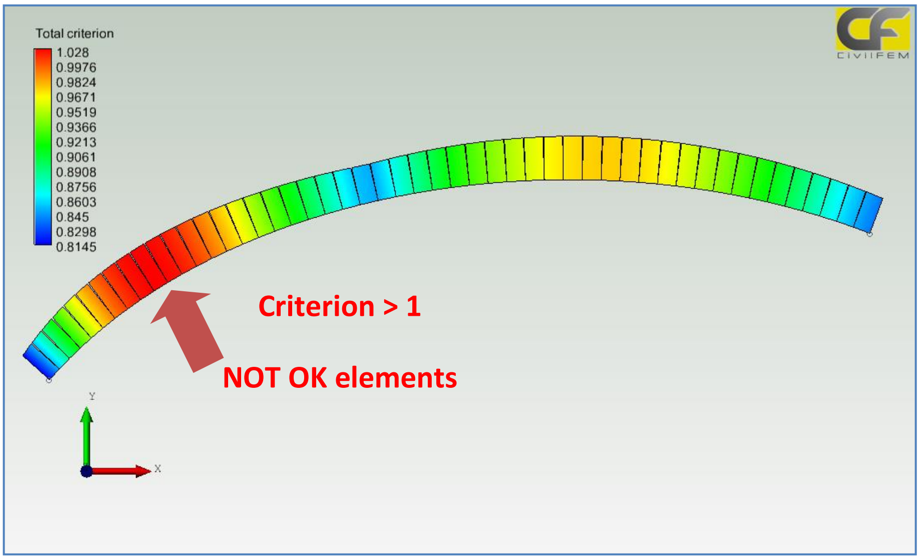
Obtaining the strength criterion of the section. This criterion is defined as the ratio between two distances. As shown above, the distance to the "center" of the diagram (point A of the figure) from the point representing the acting forces and moments (point P of the figure) is labeled as d_1 and the distance to the center from the point representing the homothetic ultimate forces and moments (point B) is d_2.

$$\text{Criterion} = \frac{d_1}{d_2}$$

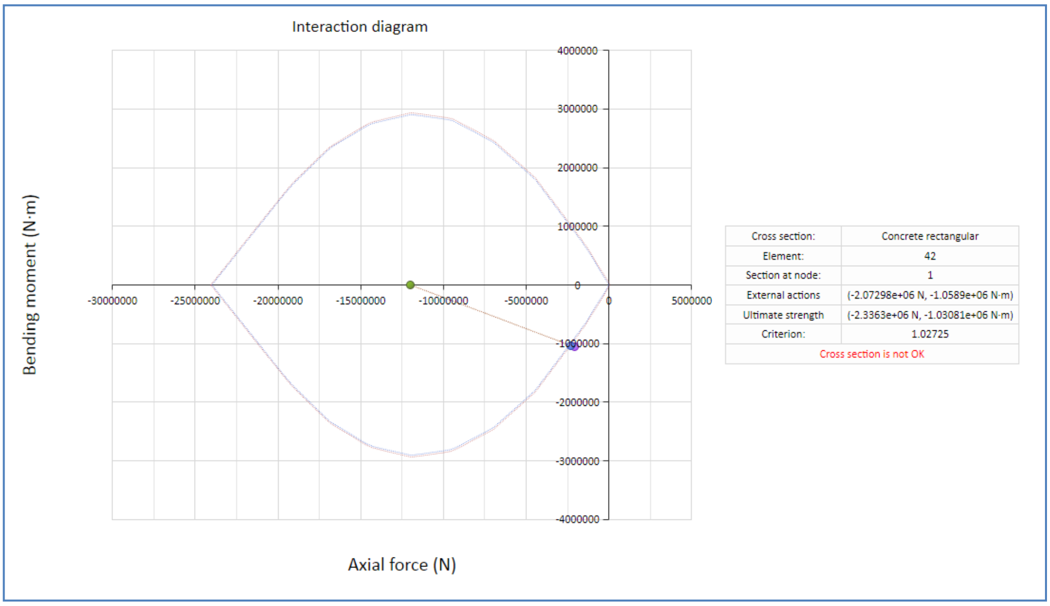
If this criterion is less than 1.00, the forces and moments acting on the section will be inferior to its ultimate strength, and the section will be safe. On the contrary, for criterion larger than 1.00, the section will not be considered as valid.

6.3.3. Check Results

- **Total Criterion**, if this criterion is less than 1.0, in such a way that the forces and moments acting on the section are inferior to its ultimate strength, the section is safe (element is OK). On the contrary, for criterion higher than 1.0, the section will be considered as not valid (element is NOT OK).



- **Interaction Diagram**, it includes all the necessary information for checking as well as design. Effects of actions, ultimate strength, safety information, as well as strength with and without reinforcement can be seen. The criterion provided is the ratio between the distances of the center of the diagram to the design loads point and the center of the diagram to the ultimate strength.


Interaction diagram
Bending moment (N·m)
Axial force (N)
4000000
3000000
2000000
1000000
0
-1000000
-2000000
-3000000
-4000000
-30000000
-25000000
-20000000
-15000000
-10000000
-5000000
0
5000000
Cross section: Concrete rectangular
Element: 42
Section at node: 1
External actions (-2.07298e+06 N, -1.0589e+06 N·m)
Ultimate strength (-2.3363e+06 N, -1.03081e+06 N·m)
Criterion: 1.02725
Cross section is not OK

6.4. Axial Load and Biaxial Bending Design

6.4.1. Calculation Hypothesis

For the design of sections under axial loading and biaxial bending, the same hypothesis for the axial load and biaxial bending check is adopted.

6.4.2. Calculation Process

For the design, an optimization process is carried out through successive iterations; within this process, the safety factor of the section (or its criterion) must be strict (≈1.00). These values are determined by the following steps:

1. Obtaining the minimum and maximum reinforcement factors. The maximum and minimum reinforcement factors (ω_{max} ,ω_{min}) are introduced by the user**.** The designed reinforcement of the section will always be more than ω_{min} times and less than ω_{max} times the initial distribution.
2. Obtaining the reinforcement data of the section. The reinforcements of the section to be designed must be defined by the class (only reinforcements defined as scalable are modified), type, position and initial amount (see **Chapter 4.4).** The designed reinforcement will be homothetic to the one defined in the section, in such a way that it complies with the strength requirements of the section. If the reinforcement amount is null, the program will not perform the design.
3. Obtaining the forces and moments acting on the section. Forces and moments (F_x, M_y, M_z) acting on the section are obtained directly from the CivilFEM results file (file .RCF).
4. Constructing the 2D interaction diagram. The diagram of the section is constructed for reinforcement corresponding to ω_{min} times the initial distribution to determine the ultimate forces and moments of the section with this configuration.

$$(\mathrm{FX,MY,MZ})_{\mathrm{real}} = (\mathrm{FX,MY,MZ})_{\mathrm{fixed}} + \omega_{\mathrm{min}}.(\mathrm{FX,MY,MZ})_{\mathrm{scalable}}$$

5. From the interaction diagram of the previous step the ultimate strain state homothetic to the acting forces and moments can be determined with respect to the diagram center.
6. Obtaining the strength criterion of the section. This criterion is determined following the same process as described in the checking section.

7. If the value of the criterion is less than 1.00 (the forces and moments acting on the section are inferior to its ultimate strength), the section will be assigned reinforcement equal to ω_{min} times the initial distribution and the calculation will be terminated.
8. Repetition of steps 4, 5 and 6 for a reinforcement corresponding to ω_{max} times the initial reinforcement distribution.

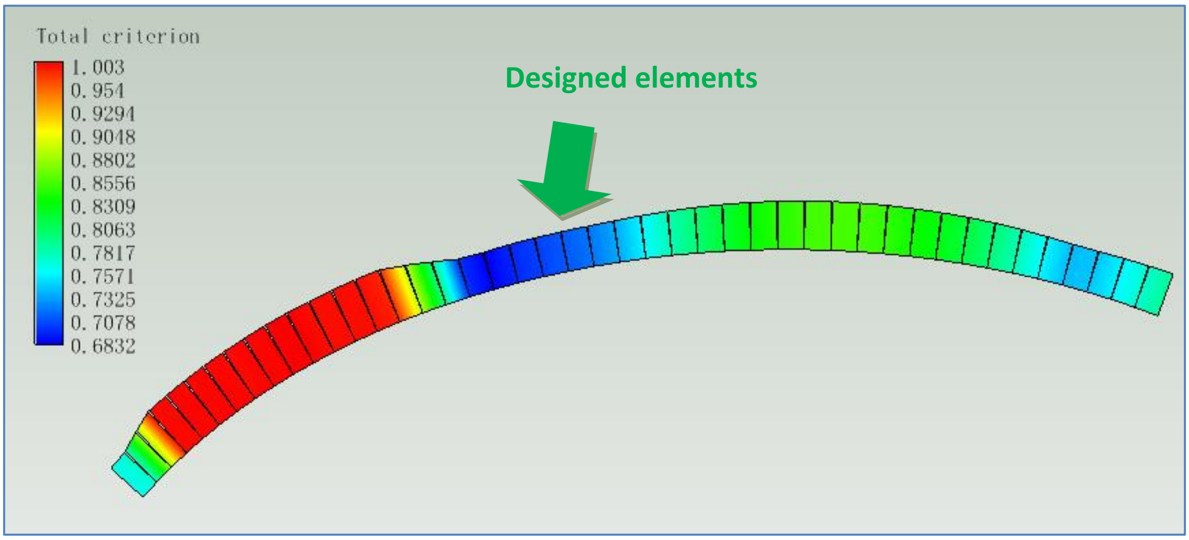
$$(FX, MY, MZ)_{real} = (FX, MY, MZ)_{stationary} + \omega_{max}.(FX, MY, MZ)_{scalable}$$

9. If the value of the strength criterion of the section is more than 1.00 (acting forces are larger than the ultimate strength of the section), the program will indicate it is not possible to design the section and will not assign reinforcement nor will it continue calculating.
10. Optimization of the section reinforcement through successive iterations. From the forces and moments previously determined $(FX, MY, MZ)_{fixed}$ and $(FX, MY, MZ)_{scalable}$, a search is done to obtain a reinforcement factor ω that will produce a value of the section criterion between 0.99 and 1.01. The program will then assign reinforcement equal to ω times the initial distribution of the section.

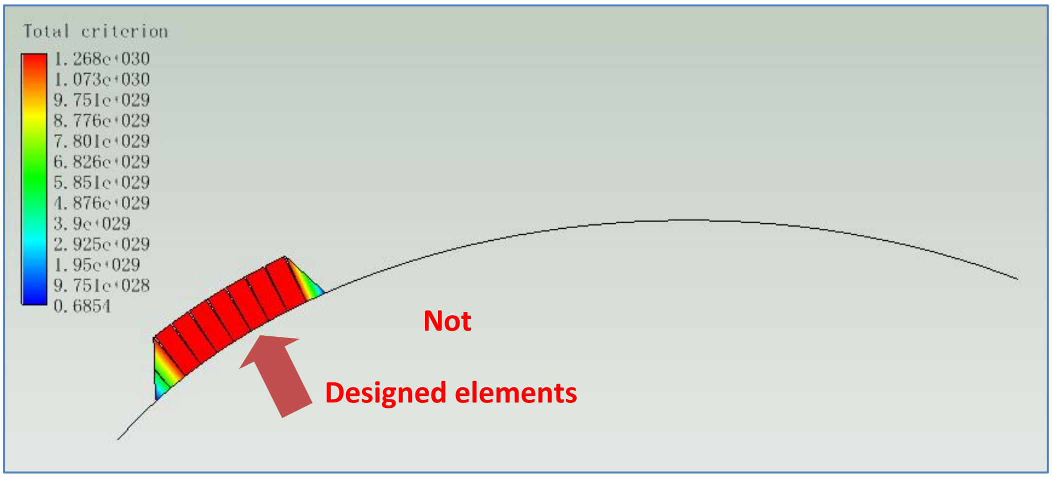
6.4.3. Design results

CivilFEM can obtain the needed reinforcement (design) in order to fulfill the code requirements. CivilFEM uses the interaction diagram of each section, taking into account the design stress-strain curve for each of the materials of the section. Moments in two directions are applied to the section. Scalable reinforcement will be increased/decreased until the section reaches a safety factor of 1.0 according to the code.

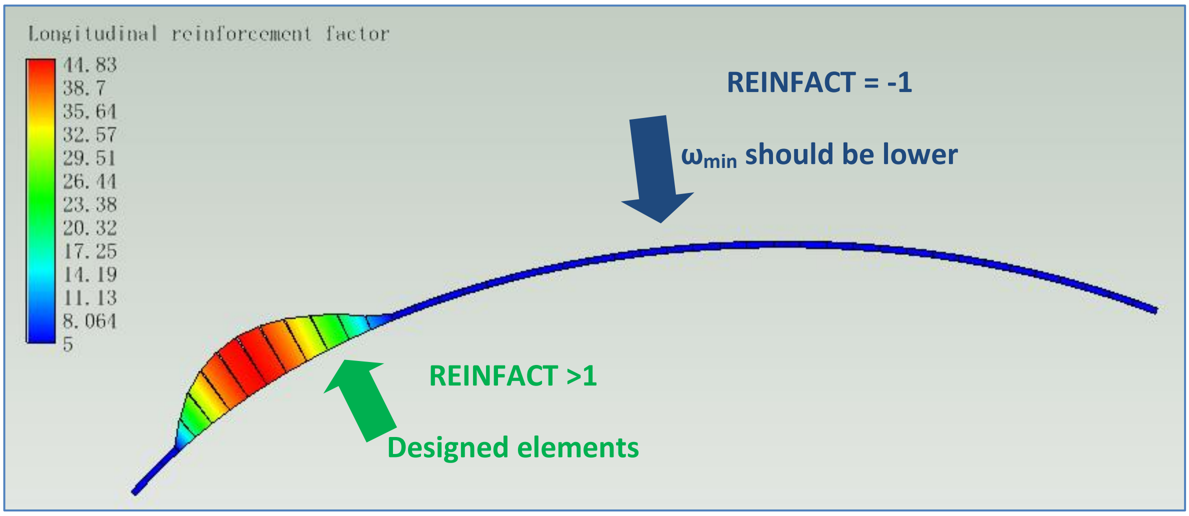
- Design Total Criterion, elements with values equal to unity means that those elements were designed and a reinforcement factor was found within the provided range of ω_{min} and ω_{max}.

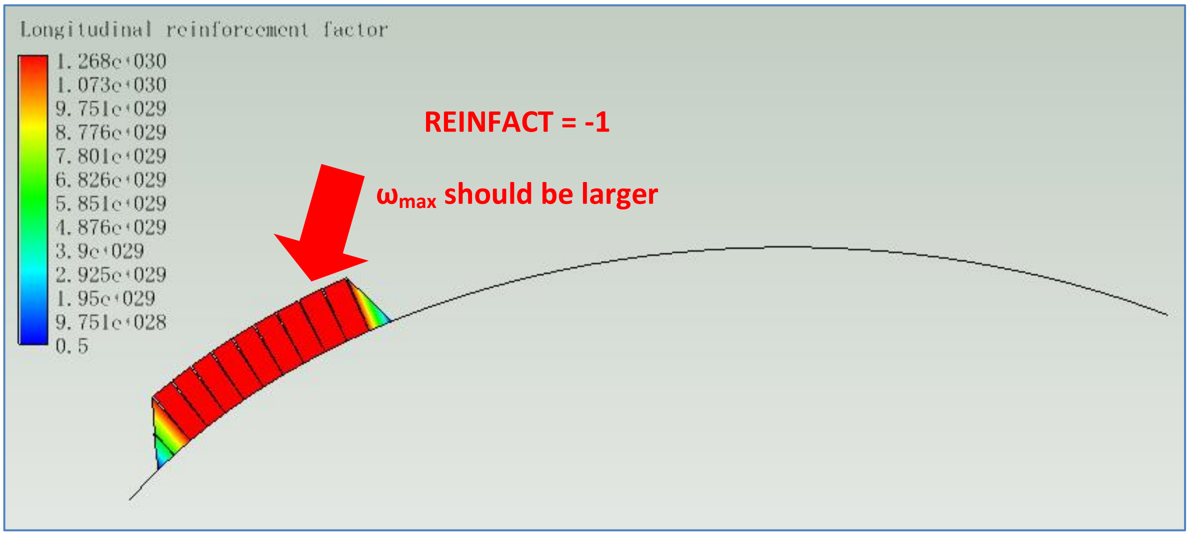


If the result is out of range then 2^{100} values will appear.



- Reinforcement factor, depending on the range of ω_{min} and ω_{max} provided, different results appear:
  1) Obtained reinforcement factor is **inside** (ω_{min} ,ω_{max}), then REINFACT value times the defined reinforcement amount gives the needed reinforcement.
  2) **ω_{max} is smaller** than the reinforcement factor obtained, then REINFACT = 2^{100} for those elements.
  3) **ω_{min} is greater** than the reinforcement factor obtained, then REINFACT = ω_{min} for those elements.





If CivilFEM is not able to design reinforcement with considered section, materials and initial reinforcement amount, then 2^{100} values will appear.

- Total scalable reinforcement (SCALAREA): This option gives the the product of the REINFACT previously obtained and the total **scalable** area of all reinforcement groups defined as scalable:

  $$SCALAREA = REINFACT \times Total\ scalable\ reinforment\ area$$

  The user should note that reinforcement defined as Fixed will not be included in the calculation. If REINFACT is 2^{100} , SCALAREA will also be assigned the value 2^{100} .

6.5. Axial Force and Biaxial Bending Calculation Codes

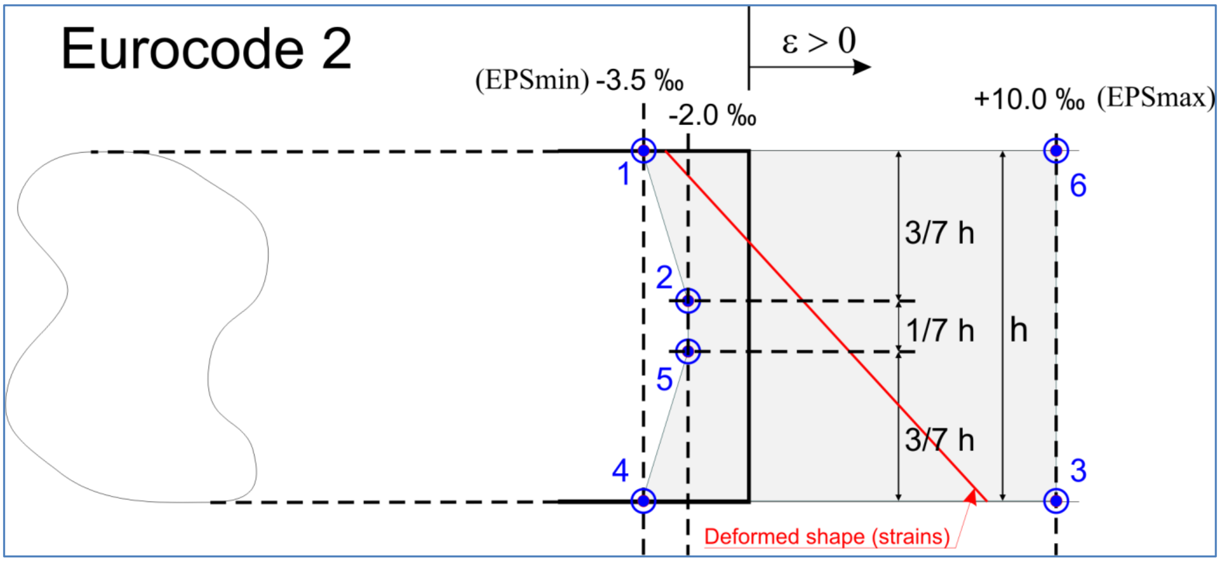
For the check and design of reinforced concrete beams with different codes, the only variation will be the consideration of the pivots relative to the concrete (corresponding to EPSmin) and to the steel (corresponding to EPSmax). Therefore the pivots diagram for each code will differ in the construction of the section interaction diagram.

Codes provided by CivilFEM for the check and design of reinforced concrete beams under axial load and biaxial bending include: Eurocode 2, ITER Design code, Spanish code EHE, American codes ACI 318 and ACI 349, British Standard 8110, Australian Standard 3600, CEB-FIP model code, Chinese code GB50010, Brazilian code NBR6118, AASHTO Standard Specifications for Highway Bridges and ITER Structural Design Code for Buildings.

The strain limits defined hereafter are default values, but can be changed for each of the materials defined in the model.

6.5.1. Eurocode 2, ITER Design Code and Structural Code (Spanish code)

If the active code is Eurocode 2 ,ITER Structural Design Code for Buildings or Structural Code (Spanish code), the strain states relative to concrete and reinforcement steel are those defined in the following figure:



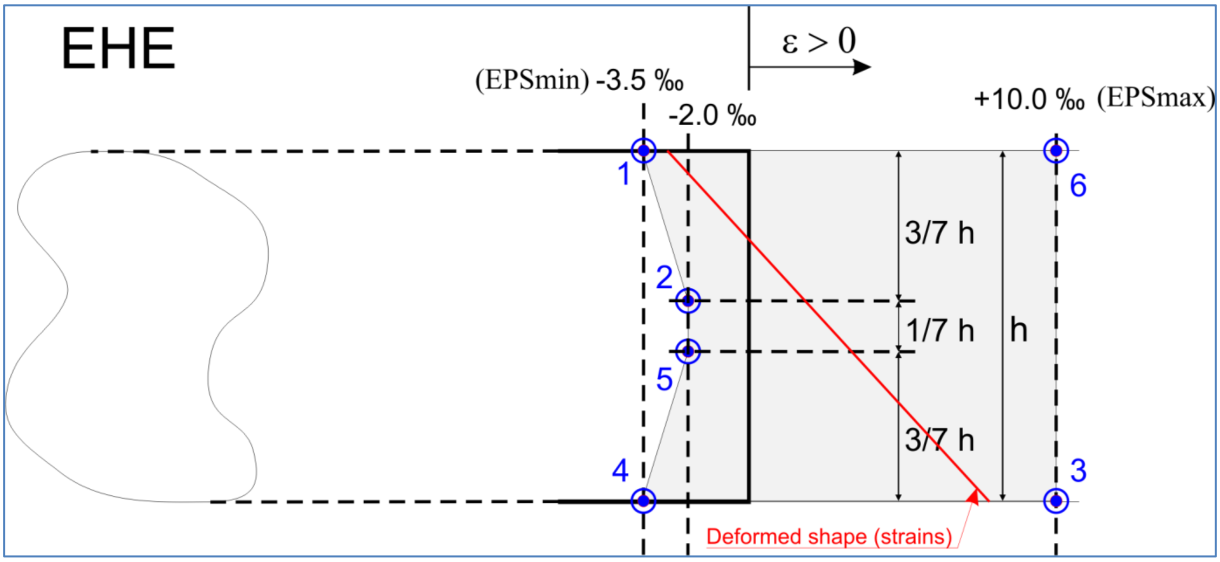
If concrete has $f_{ck} > 50\ MPa$, the concrete strain limits are the following:

EPSmin (‰) = $-(2.6 + 35[(90 - f_{ck})/100]^4)(\text{with } f_{ck} \text{ in MPa})$

EPSint (‰) = $-(2.0 + 0.085(f_{ck} - 50)^{0.53})(\text{with } f_{ck} \text{ in MPa})$

6.5.2. EHE Spanish Code

If the active code is EHE, the strain states relative to concrete and reinforcement steel are those defined in the following figure:



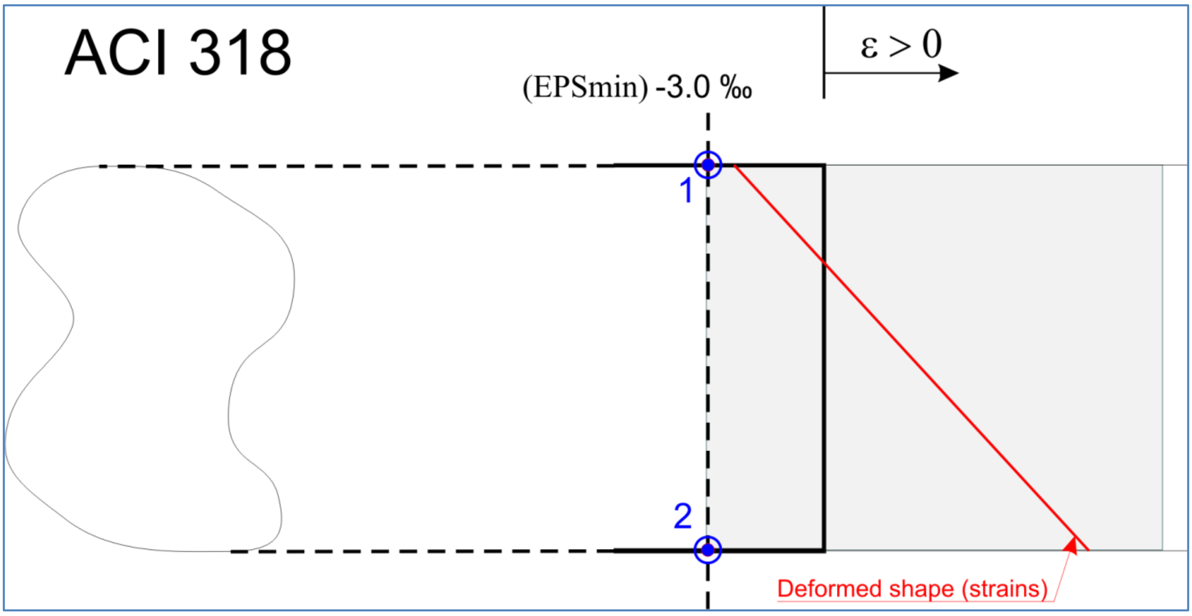
If concrete has f_{ck} > 50 MPa, the concrete strain limits are the following:

EPSmin (‰) = $-(2.6 + 14.4[(100 - f_{ck})/100]^4)(\text{with } f_{ck} \text{ in MPa})$

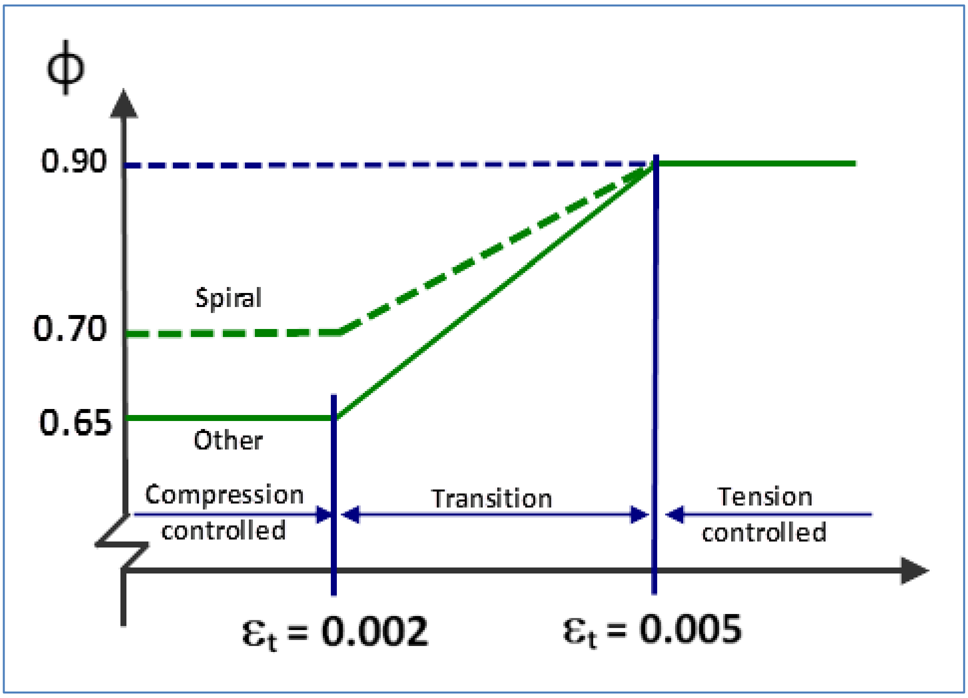
EPSint (‰) = $-(2.0 + 0.085(f_{ck} - 50)^{0.5})(\text{with } f_{ck} \text{ in MPa})$.

6.5.3. ACI 318-05

If the active code is ACI 318-05, the strain states relative to concrete and reinforcement steel are those defined in the following figure. It can be noted that there is no pivot relative to steel (EPSmax), considering the section does not fail due to reinforcement steel deformation.



The theoretical values of the interaction diagram are affected by the strength reduction factor ϕ according to Chapter 9.3 of *Building Code Requirements for Structural Concrete Structures (ACI 318-05)* document:



$$\Phi = \Phi_c \qquad \varepsilon_t \leq 0.002$$

$$\Phi = \Phi_c + (\varepsilon_t - 0.002)(0.90 - \Phi_c)\frac{1000}{3} \qquad 0.002 < \varepsilon_t < 0.005$$

$\Phi = 0.90$ $\varepsilon_t \geq 0.005$

Where ε_t is the maximum strain obtained at the reinforcement and ϕ_c is the strength reduction factor for compression controlled sections:

For **ACI 318-05** (according to chapter 9.3.2 from code requirements)

- Member with spiral reinforcement Φ_c=0.70
- Other reinforcement members Φ_c=0.65 (default value)

Furthermore, according to Chapter 10.3.6 of *Building Code Requirements for Structural Concrete Structures (ACI 318-05)* document, design axial strength ϕP_n of compression members must not be greater than:

1. For member with spiral reinforcement:

$$\Phi P_{n,max} = 0.85\phi[0.85f'_c\,(A_g - A_{st}) + f_y\,A_{st}]$$

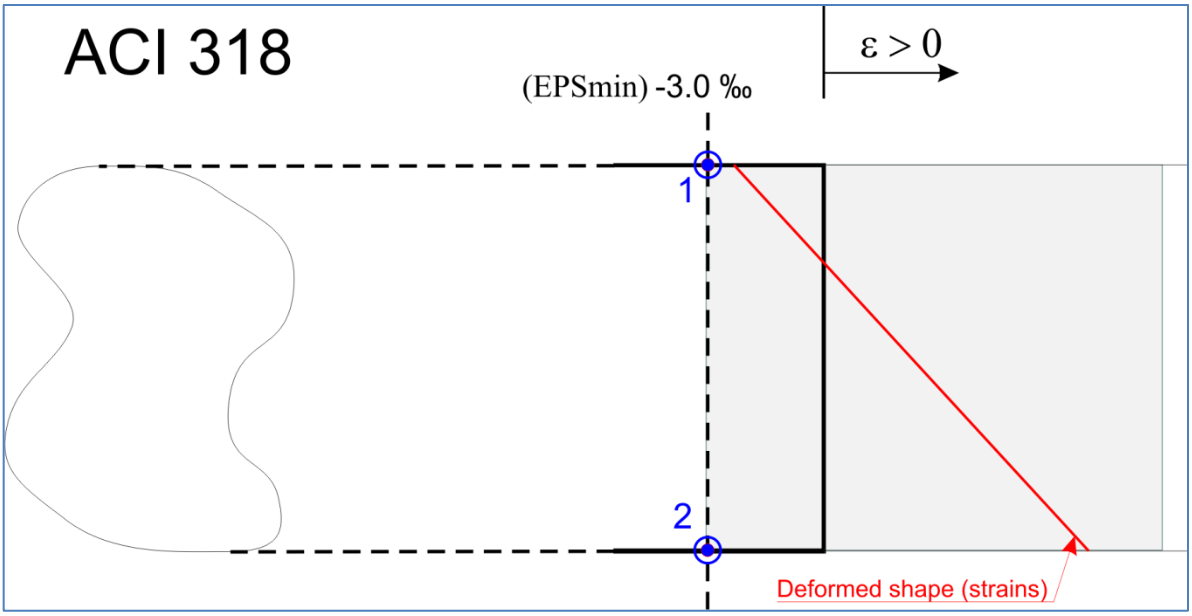
2. For other reinforcement:

$$\Phi P_{n,max} = 0.80\phi[0.85f'_c\,(A_g - A_{st}) + f_y\,A_{st}]$$

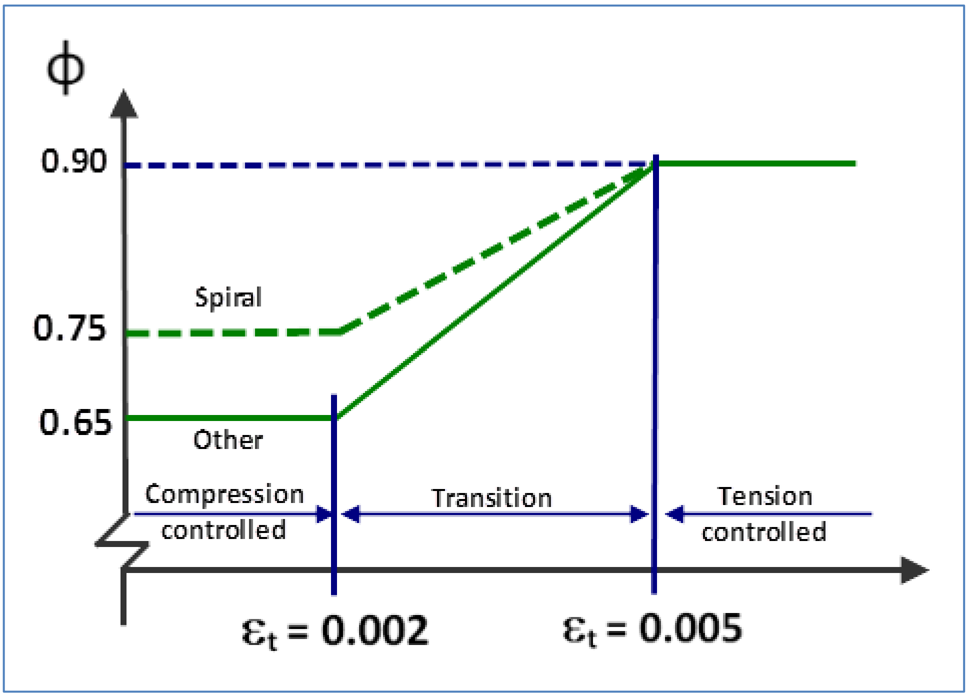
Where A_g is the gross area of concrete section and A_{st} is the total area of longitudinal reinforcement.

6.5.4. ACI 318-14

If the active code is ACI 318-14, the strain states relative to concrete and reinforcement steel are those defined in the following figure. It can be noted that there is no pivot relative to steel (EPSmax), considering the section does not fail due to reinforcement steel deformation.



The theoretical values of the interaction diagram are affected by the strength reduction factor ϕ according to Chapter 21.2.2 of *Building Code Requirements for Structural Concrete Structures (ACI 318-14)* document



$$\Phi = \Phi_c \qquad \varepsilon_t \leq 0.002$$

$$\Phi = \Phi_c + (\varepsilon_t - 0.002)(0.90 - \Phi_c)\frac{1000}{3} \qquad 0.002 < \varepsilon_t < 0.005$$

$\Phi = 0.90$ $\varepsilon_t \geq 0.005$

Where ε_t is the maximum strain obtained at the reinforcement and ϕ_c is the strength reduction factor for compression controlled sections:

For **ACI 318-14** (according to chapter 21.2.2 from code requirements)

- Member with spiral reinforcement Φ_c=0.75
- Other reinforcement members Φ_c=0.65 (default value)

Furthermore, according to Chapter 22.4.2 of *Building Code Requirements for Structural Concrete Structures (ACI 318-14)* document, design axial strength ϕP_n of compression members must not be greater than:

1. For member with spiral reinforcement:

$$\Phi P_{n,max} = 0.85\phi[0.85f'_c\,(A_g - A_{st}) + f_y\ A_{st}]$$

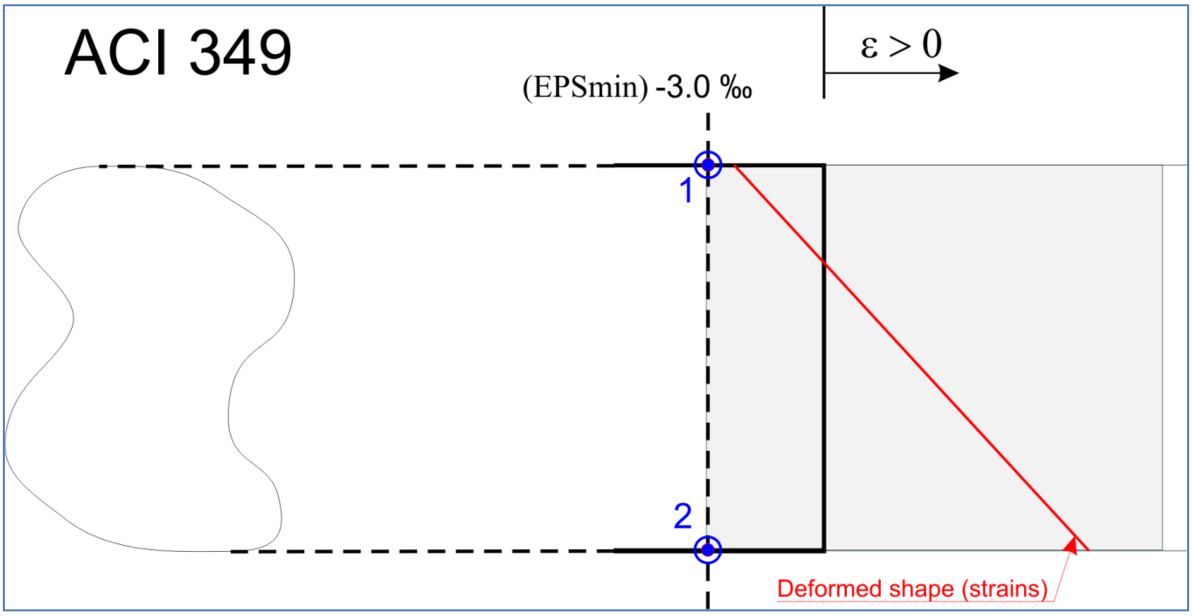
2. For other reinforcement:

$$\Phi P_{n,max} = 0.80\phi[0.85f'_c\,(A_g - A_{st}) + f_y\ A_{st}]$$

Where A_g is the gross area of concrete section and A_{st} is the total area of longitudinal reinforcement.

6.5.5. ACI 349-01

If the active code is ACI 349-01, the strain states relative to concrete and reinforcement steel are the ones defined in the following figure. It can be noted that there is no pivot relative to steel (EPSmax), considering the section does not fail due to reinforcement steel deformation.



The theoretical values of the interaction diagram are affected by the strength reduction factor ϕ according to Chapter 9.3 of *Code Requirements for Nuclear Safety Related Concrete Structures (ACI 349-01)* document:

$$\Phi = 0.90 \qquad P_n > 0$$

$$\Phi = 0.90 - (0.90 - \Phi_c) \cdot \frac{P_n}{P_a} \qquad 0 \le P_n \le P_a$$

$$\Phi = \Phi_c \qquad P_n < P_a$$

$$P_a = -0.10 \cdot f_c{}' \cdot \frac{A_g}{\Phi_c}$$

Where P_n is the axial load (tension positive), A_g is the concrete gross area and Φ_c is the strength reduction factor for compression controlled sections:

For **ACI 349-01** (according to chapter 9.3.2 from code requirements)

- Axial tension, and axial tension with flexure $\Phi = 0.90$
- Axial compression and axial compression with flexure:
  a) Member with spiral reinforcement Φ_c=0.75
  b) Other reinforcement members Φ_c=0.70 (default value)

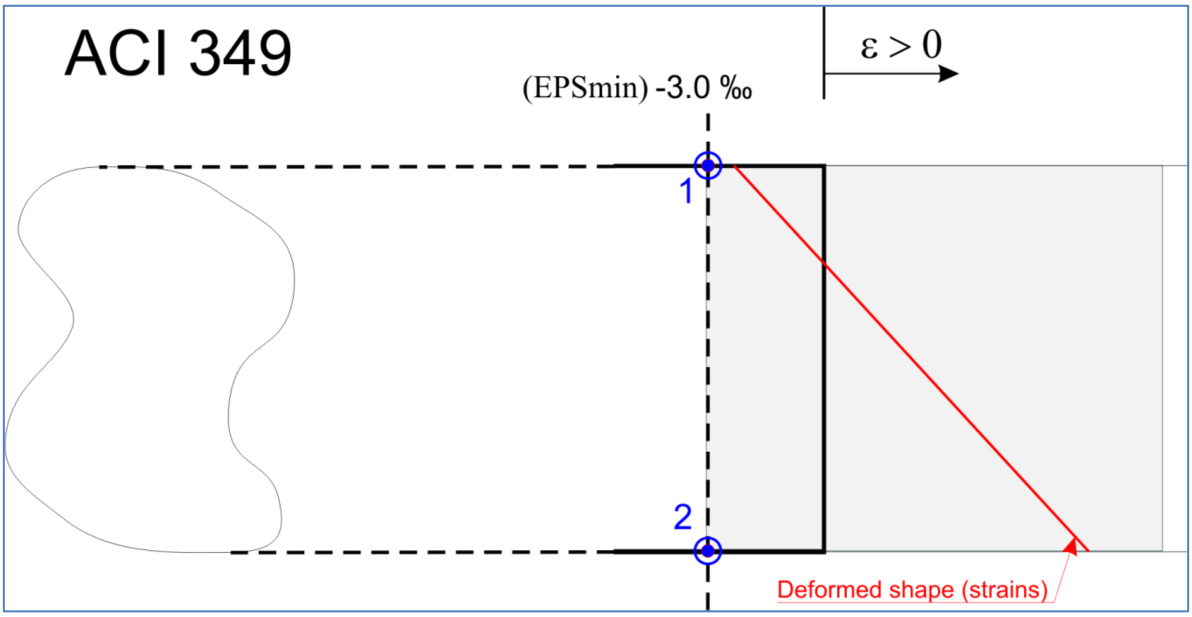
Furthermore, according to Chapter 10.3.5 of *Code Requirements for Nuclear Safety Related Concrete Structures (ACI 349-01)* document, design axial strength ϕP_n of compression members must not be greater than:

1. For member with spiral reinforcement:
$$\Phi P_{n,max} = 0.85\phi[0.85f'_c\,(A_g - A_{st}) + f_y\, A_{st}]$$
2. For other reinforcement:
$$\Phi P_{n,max} = 0.80\phi[0.85f'_c\,(A_g - A_{st}) + f_y\, A_{st}]$$

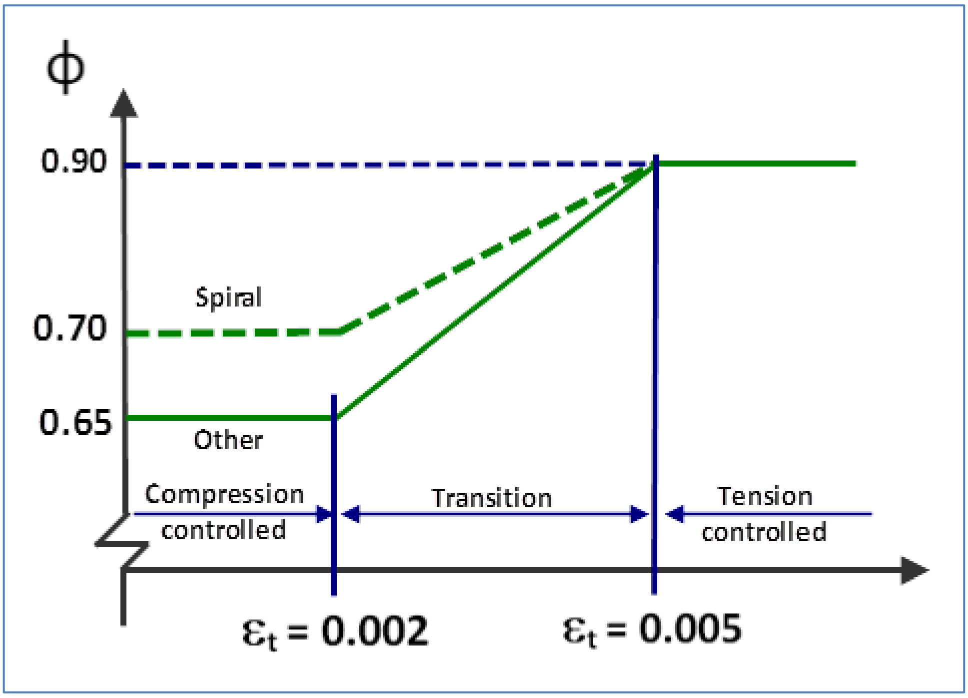
Where A_g is the gross area of concrete section and A_{st} is the total area of longitudinal reinforcement.

6.5.6. ACI 349-06

If the active code is ACI 349-06, the strain states relative to concrete and reinforcement steel are the ones defined in the following figure. It can be noted that there is no pivot relative to steel (EPSmax), considering the section does not fail due to reinforcement steel deformation.



The theoretical values of the interaction diagram are affected by the strength reduction factor ϕ according to Chapter 9.3 of *Code Requirements for Nuclear Safety-Related Concrete Structures (ACI 349-06)* document:



$$\Phi = \Phi_c \qquad \varepsilon_t \le 0.002$$

$$\Phi = \Phi_c + (\varepsilon_t - 0.002)(0.90 - \Phi_c)\frac{1000}{3} \qquad 0.002 < \varepsilon_t < 0.005$$

$$\Phi = 0.90 \qquad \varepsilon_t \ge 0.005$$

Where ε_t is the maximum strain obtained at the reinforcement and ϕ_c is the strength reduction factor for compression controlled sections:

For **ACI 349-06** (according to chapter 9.3.2 from code requirements)

- Axial tension, and axial tension with flexure $\Phi = 0.90$
- Axial compression and axial compression with flexure:
  a) Member with spiral reinforcement Φ_c=0.70
  b) Other reinforcement members Φ_c=0.65 (default value)

Furthermore, according to Chapter 10.3.6 of *Code Requirements for Nuclear Safety Related Concrete Structures (ACI 349-06)* document, design axial strength ϕP_n of compression members must not be greater than:

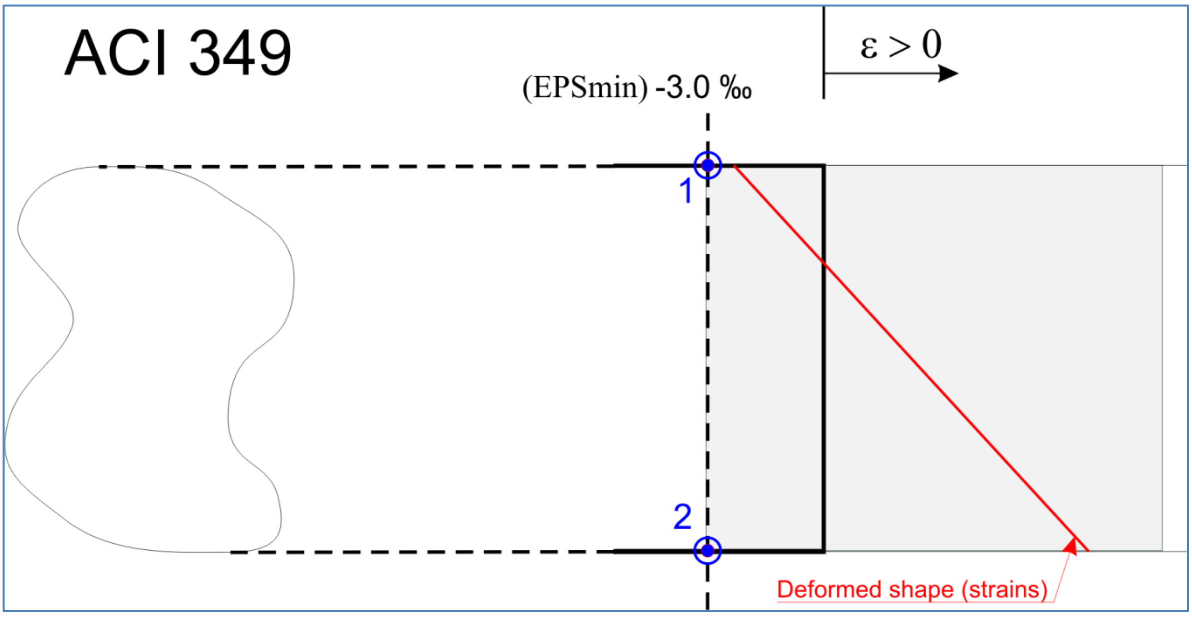
1. For member with spiral reinforcement:
$$\Phi P_{n,max} = 0.85\phi[0.85f'_c\,(A_g - A_{st}) + f_y\,A_{st}]$$
2. For other reinforcement:

$$\Phi P_{n,max} = 0.80\phi[0.85f'_c\,(A_g - A_{st}) + f_y\ A_{st}]$$

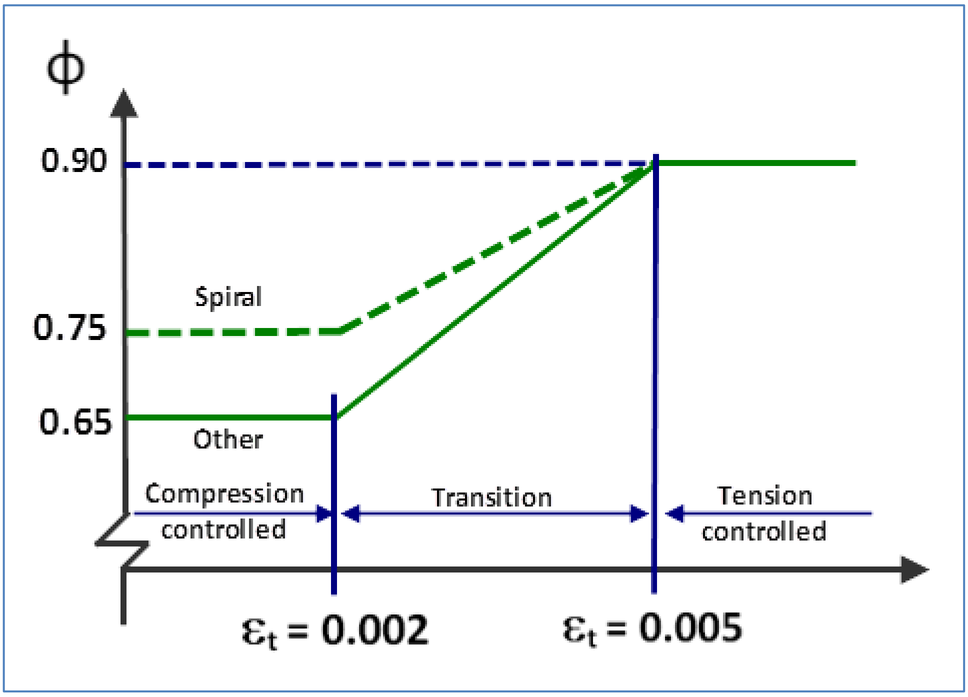
Where A_g is the gross area of concrete section and A_{st} is the total area of longitudinal reinforcement.

6.5.7. ACI 349-13

If the active code is ACI 349-13, the strain states relative to concrete and reinforcement steel are the ones defined in the following figure. It can be noted that there is no pivot relative to steel (EPSmax), considering the section does not fail due to reinforcement steel deformation.



The theoretical values of the interaction diagram are affected by the strength reduction factor ϕ according to Chapter 9.3.2 of *Code Requirements for Nuclear Safety-Related Concrete Structures (ACI 349-13)* document:



$\Phi = \Phi_c$ $\quad\quad\quad\quad \varepsilon_t \leq 0.002$

$\Phi = \Phi_c + (\varepsilon_t - 0.002)(0.90 - \Phi_c)\dfrac{1000}{3}$ $\quad\quad 0.002 < \varepsilon_t < 0.005$

$\Phi = 0.90$ $\quad\quad\quad\quad \varepsilon_t \geq 0.005$

Where ε_t is the maximum strain obtained at the reinforcement and ϕ_c is the strength reduction factor for compression controlled sections:

For **ACI 349-13** (according to chapter 9.3.2 from code requirements)

- Axial tension, and axial tension with flexure $\Phi = 0.90$
- Axial compression and axial compression with flexure:
  a) Member with spiral reinforcement Φ_c=0.75
  b) Other reinforcement members Φ_c=0.65 (default value)

Furthermore, according to Chapter 10.3.6 of *Code Requirements for Nuclear Safety Related Concrete Structures (ACI 349-13)* document, design axial strength ϕP_n of compression members must not be greater than:

3. For member with spiral reinforcement:

$$\Phi P_{n,max} = 0.85\phi[0.85f'_c(A_g - A_{st}) + f_y A_{st}]$$

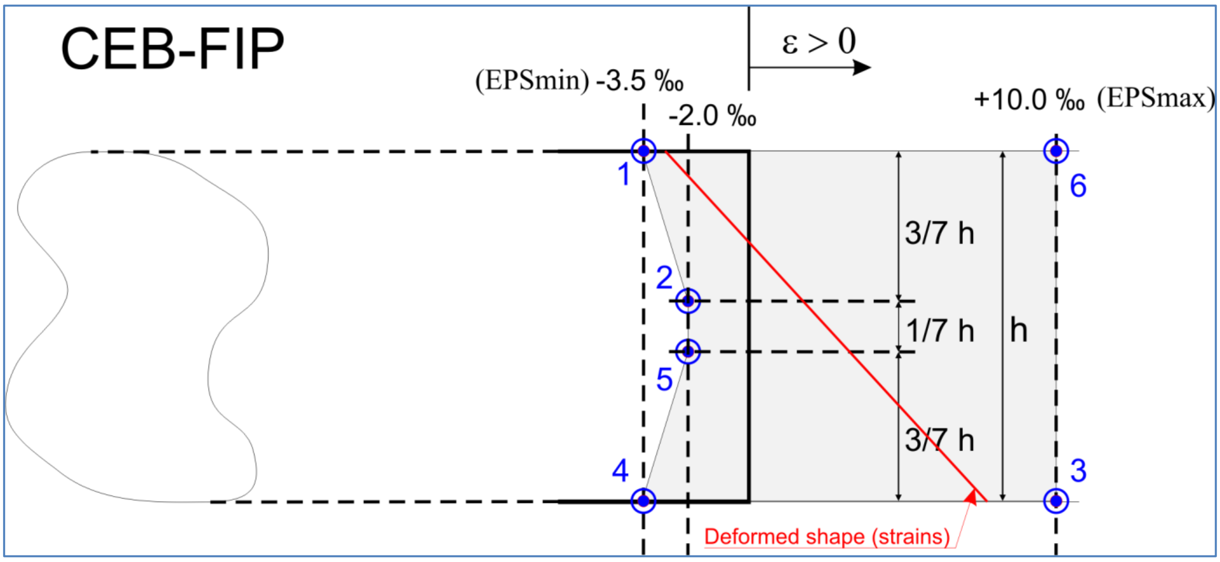
4. For other reinforcement:

$$\Phi P_{n,max} = 0.80\phi[0.85f'_c(A_g - A_{st}) + f_y A_{st}]$$

Where A_g is the gross area of concrete section and A_{st} is the total area of longitudinal reinforcement.

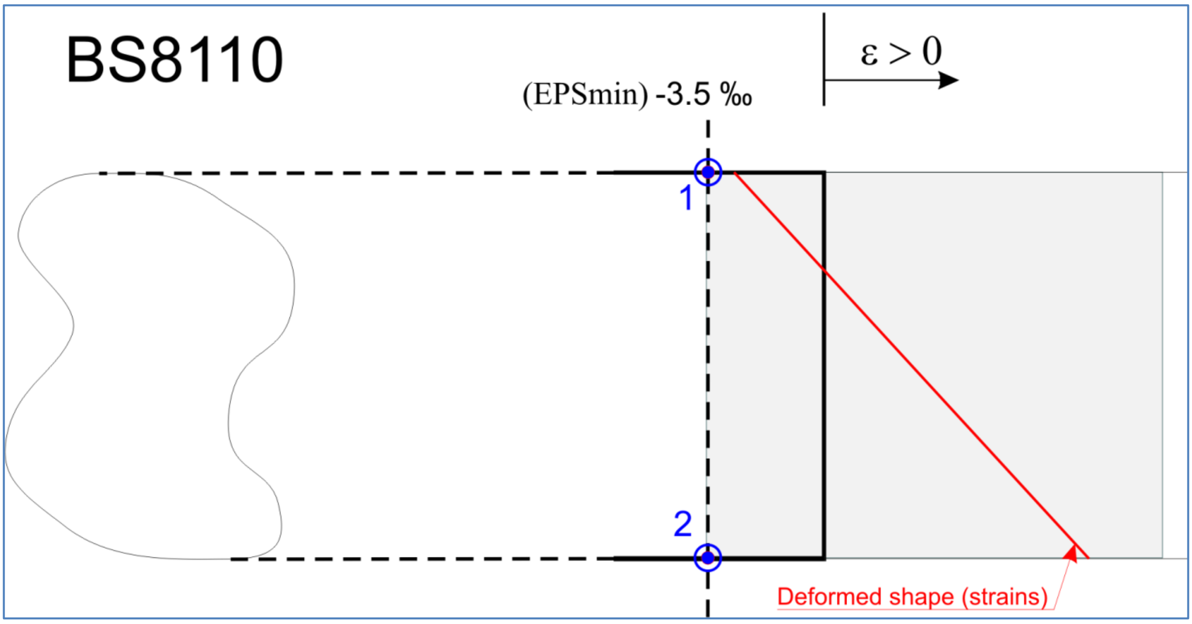
6.5.8. CEB-FIP

If the active code is CEB-FIP, the strain states relative to concrete and reinforcement steel are those defined in the following figure:



6.5.9. British Standard 8110

If the active code is BS8110, the strain states relative to concrete and reinforcement steel are those defined in the following figure. It can be noted that there is no pivot relative to steel (EPSmax), considering the section does not fail due to reinforcement steel deformation:



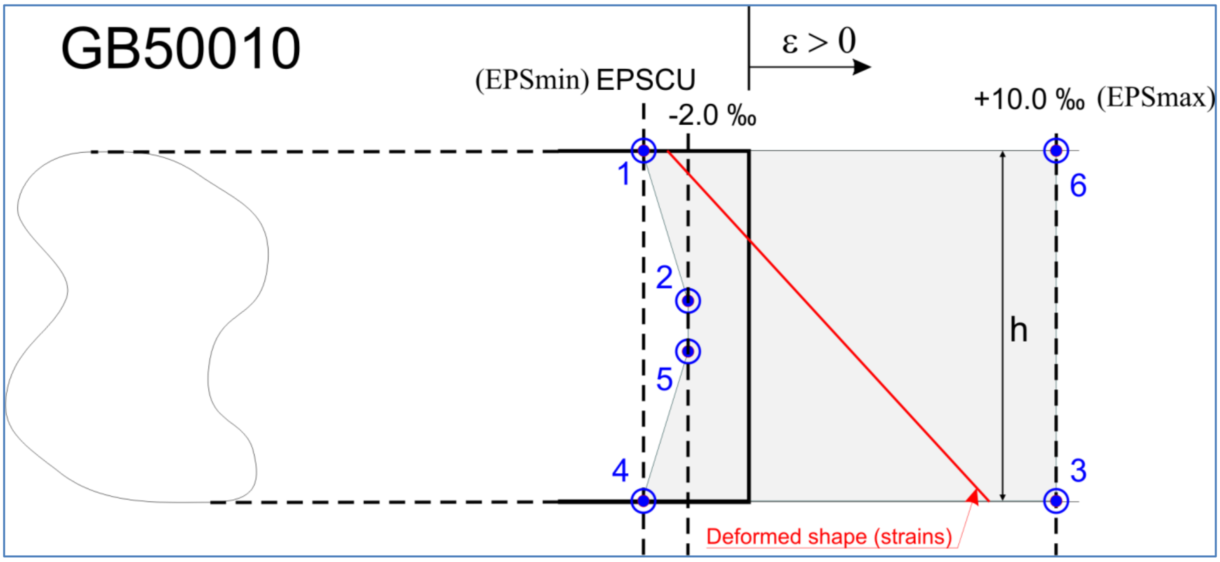
6.5.10. Australian Standard 3600

If the active code is AS3600, CivilFEM uses the same parameters as the ACI 318 code for material properties.

The theoretical values of the interaction diagram are affected by the strength reduction factor ϕ. This value is taken from the member properties.

6.5.11. Chinese Code GB50010

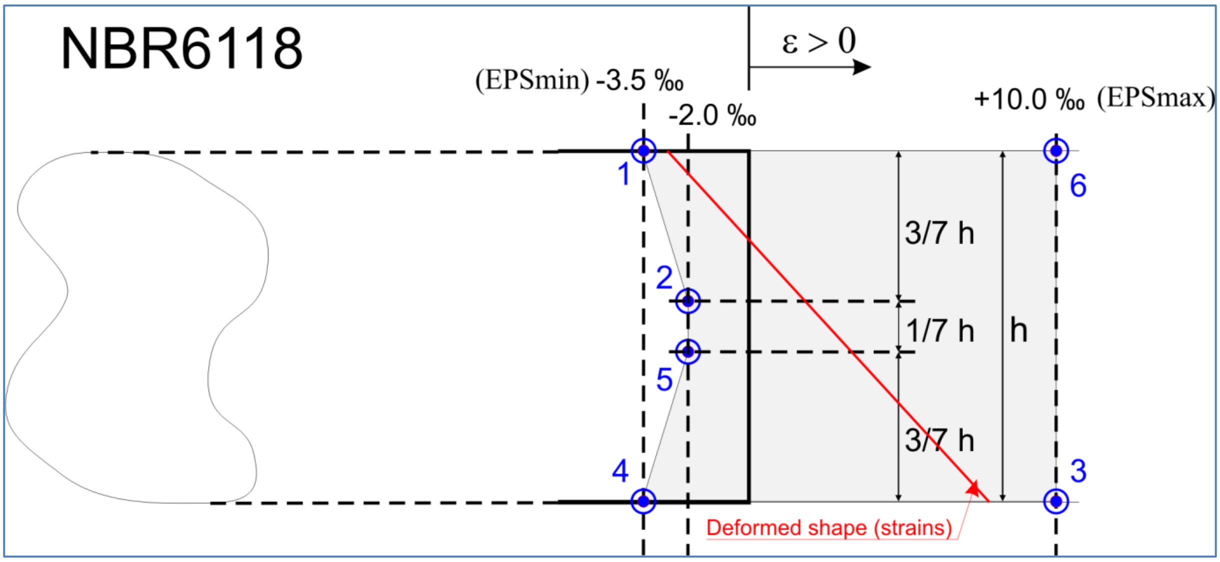
If the active code is GB50010, the strain states relative to concrete and reinforcement steel are the ones defined in the following figure:



Where EPScu = $0.0033-(f_{cuk}-50)*10^5$ [MPa]

6.5.12. Brazilian Code NBR6118

If the active code is NBR6118, the strain states relative to concrete and reinforcement steel are those defined in the following figure:



6.5.13. AASHTO Standard Specifications for Highway Bridges

If the active code is AASHTO Standard Specifications for Highway Bridges CivilFEM uses the same parameters as the ACI 318 code for material properties.

The theoretical values of the interaction diagram are affected by the strength reduction factor ϕ.

$$\phi = 0.70 \qquad \varepsilon_t \leq 0.002$$

$$\phi = 0.70 + (\varepsilon_t - 0.002)\frac{200}{3} \qquad 0.002 < \varepsilon_t < 0.005$$

$$\phi = 0.90 \qquad \varepsilon_t \leq 0.005$$

Where ε_t is the maximum strain obtained at the reinforcement.

For **AASHTO** (according to chapter 5.5.4.2 from AASHTO LRFD Bridge Design Specifications):

- Axial tension, and axial tension with flexure $\Phi = 0.90$
- Axial compression and axial compression with flexure $\Phi = 0.75$

Furthermore, according to Chapter 5.7.4.4, design axial strength ϕP_n of compression members must not be greater than:

1. For member with spiral reinforcement:

$$\Phi P_{n,max} = 0.85\phi[0.85f'_c\,(A_g - A_{st}) + f_y\,A_{st}]$$

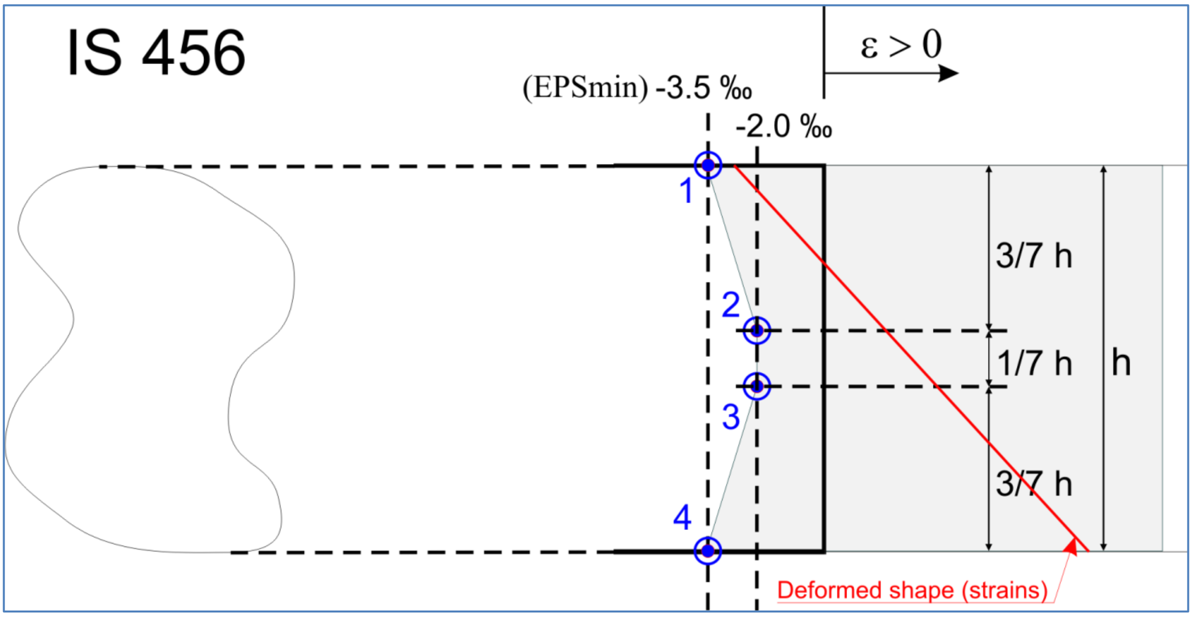
2. For other reinforcement:

$$\Phi P_{n,max} = 0.80\phi[0.85f'_c\,(A_g - A_{st}) + f_y\,A_{st}]$$

Where A_g is the gross area of concrete section and A_{st} is the total area of longitudinal reinforcement.

6.5.14. Indian Standard 456

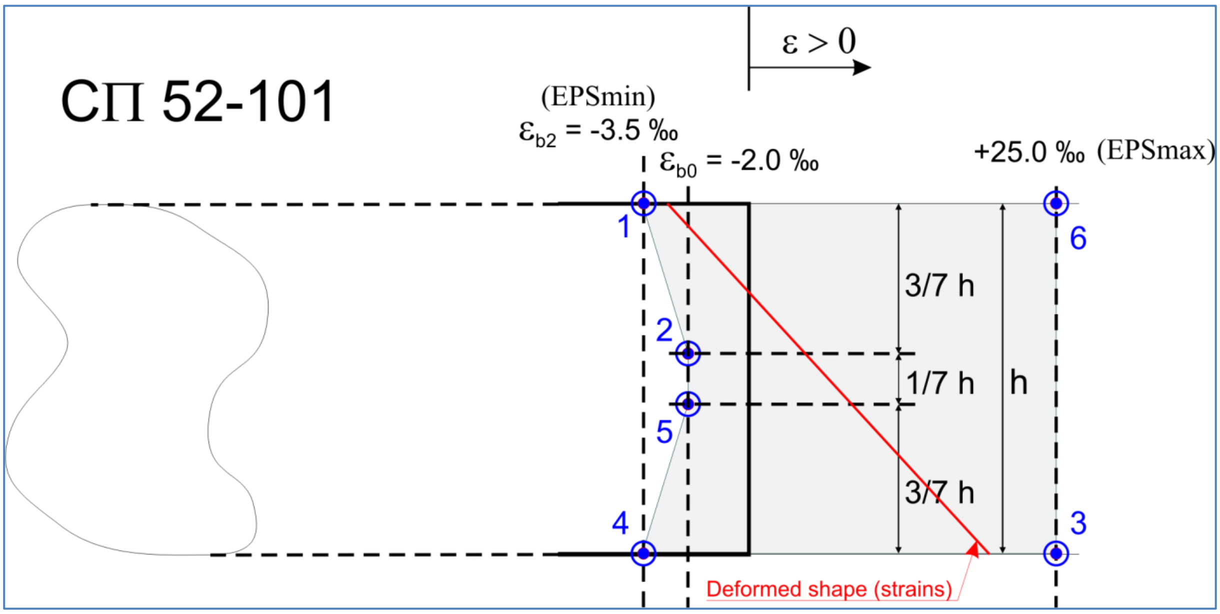
If the active code is the Indian Standard 456, the strain states relative to concrete are the ones defined in the following figure:



It can be noted that there is no pivot relative to steel (EPSmax), considering the section does not fail due to reinforcement steel deformation.

6.5.15. Russian Code SP 52-101

If the active code is SP 52-101, the strain states relative to concrete and reinforcement steel are the ones defined in the following figure:



6.6. Shear and Torsion

6.6.1. Previous considerations

Valid reinforced concrete sections for shear and torsion check and design are the following:

TABLE 1-1 VALID SECTIONS FOR SHEAR AND TORSION CHECKING

SECTION	Y SHEAR	Z SHEAR	TORSION
Rectangular	Yes	Yes	Yes
Box	Yes	Yes	Yes
Circular	Yes	Yes	Yes
Annular	Yes	Yes	Yes
Double T/I-shape	Yes	No	No
T	Yes	No	No

For each one of these sections and directions, a set of geometrical parameters in accordance with the code is automatically defined. These parameters are required for the calculating process. Later on, there is a detailed explanation on how to obtain these parameters for each valid section.

6.6.2. Shear and torsion code properties

Parameters required for the check and design processes for shear and torsion are the following:

EUROCODE 2 AND ITER

REC: Reinforcement cover.

BW_VY: Minimum width of the section over the effective depth for shear in Y.

BW_VZ: Minimum width of the section over the effective depth for shear in Z.

DY: Effective depth of the section in the Y direction.

DZ: Effective depth of the section in the Z direction.

RHO1: Reinforcement ratio:

$$\rho_1 = \frac{A_{sl}}{b_w d} < 0.02$$

Where:

A_{sl} Area of the tension reinforcement extending not less than $d + I_{b.net}$ beyond the section considered.

b_w Minimum width of the section over the effective depth.

d Effective depth.

T: Equivalent thickness of the wall:

$$T = \frac{A}{u}$$

Where:

A Total area of the cross-section within the outer circumference, including inner hollow areas.

u Outer circumference.

This equivalent thickness cannot be greater than the real thickness of the wall nor less than twice the cover.

AK: Area enclosed within the centre-line of the thin-walled cross-section.

UK: Circumference of the AK area.

KEYAST: Indicator of the position of the torsion reinforcement in the section:

= 0 if closed stirrups are placed on both faces of each wall of the equivalent hollow section or on each wall of a box section (value by default for hollow sections).

= 1 if there are only closed stirrups distributed around the periphery of the member (value by default for solid sections).

THETA: Angle of the concrete compressive struts with the longitudinal axis of member.

ACI 318-05 and ACI 349-01

REC: Reinforcement cover.

BW_VY: Web width or diameter of circular section for shear in Y (Art. 11.1).

BW_VZ: Web width, or diameter of circular section for shear in Z (Art. 11.1).

DY: Distance from extreme compression fiber to centroid of longitudinal tension reinforcement in Y, (for circular sections, this distance should not be less than the distance from extreme compression fiber to centroid of tension reinforcement in the opposite half of the member) (Art. 11.1).

DZ: Distance from extreme compression fiber to centroid of longitudinal tension reinforcement in Z, (for circular sections, this distance should not be less than the distance from extreme compression fiber to centroid of tension reinforcement in the opposite half of the member) (Art. 11.1).

ACP: Area enclosed by outside perimeter of concrete cross section (Art. 11.6.1).

PCP: Outside perimeter of the concrete cross section (Art. 11.6.1).

AOH: Area enclosed by center-line of the outermost closed transverse torsional reinforcement (Art. 11.6.3).

PH: Perimeter of centerline of outermost closed transverse torsional reinforcement (Art. 11.6.3).

AO: Gross area enclosed by shear flow path (Art. 11.6.3).

BS 8110

REC: Reinforcement cover.

BW_VY: Minimum web width, for shear in Y (Art.3.4.5.1 Part 1).

BW_VZ: Minimum web width, for shear in Z (Art.3.4.5.1 Part 1).

DY: Effective depth of section in the Y direction (Art.3.4.5.1 Part 1).

DZ: Effective depth of section in the Z direction (Art.3.4.5.1 Part 1).

AS: Longitudinal tension reinforcement (Art.3.4.5.4 Part 1).

X_W: Torsional modulus for checking and dimensioning purposes.

X1: Minimum dimension of the rectangular torsion stirrups (Art.2.4.2 Part 2).

Y1: Maximum dimension of the rectangular torsion stirrups (Art.2.4.2 Part 2).

GB 50010

REC: Reinforcement cover.

BW_VY: Minimum width of the section over the effective depth for shear in Y (Art. 7.5.1).

BW_VZ: Minimum width of the section over the effective depth for shear in Z (Art. 7.5.1).

DY: Effective depth of the section in Y (Art. 7.5.1).

DZ: Effective depth of the section in Z (Art. 7.5.1).

HW_VY: Effective depth of the web in Y (Art. 7.5.1).

HW_VZ: Effective depth of the web in Z (Art. 7.5.1).

A_{cor}: Area enclosed within the hoop reinforcements for torsion $A_{st}1$ (Art. 7.6.4).

A_{cor1}: Area enclosed within the hoop reinforcements for torsion $A_{st}1$ (Art. 7.6.4) of branch 1(e. x. Flange).

A_{cor2}: Area enclosed within the hoop reinforcements for torsion $A_{st}1$ (Art. 7.6.4) of branch 2(e. x. Flange).

U_{cor}: Perimeter of the A_{cor} area (Art. 7.6.4).

U_{cor1}: Perimeter of the A_{cor1} area of branch 1 (Art. 7.6.4).

U_{cor2}:	Perimeter of the A_{cor2} area of branch 2 (Art. 7.6.4).
W_t:	Plastic resistance of torsion moment (Art. 7.6.4).
W_{t1}:	Plastic resistance of torsion moment of branch 1 (Art. 7.6.4).
W_{t2}:	Plastic resistance of torsion moment of branch 2 (Art. 7.6.4).
ALF:	Ratio of the web depth to the web width (Art. 7.6.1).
ALF_h:	Affected factor of the thickness of web for torsion (Art. 7.6.6).
Tky	For rectangular sections: Section width in Y.
Tkz	For rectangular sections: Section width in Z.

AASHTO Standard Specifications for Highway Bridges

REC:	Reinforcement cover.
BW_VY:	Web width or diameter of circular section for shear in Y (Art. 8.15.5).
BW_VZ:	Web width or diameter of circular section for shear in Z (Art. 8.15.5).
DY:	Distance from extreme compression fiber to centroid of longitudinal tensile reinforcement in Y (for circular sections, this distance should not to be less than the distance from extreme compression fiber to centroid of tensile reinforcement in the opposite half of the member) (Art. 8.15.5).
DZ:	Distance from extreme compression fiber to centroid of longitudinal tensile reinforcement in Z, (for circular sections, this distance should not to be less than the distance from extreme compression fiber to centroid of tensile reinforcement in the opposite half of the member) (Art. 8.15.5).
ACP:	Area enclosed by outside perimeter of concrete cross section (taken from ACI 318 Art. 11.6.1).
PCP:	Outside perimeter of the concrete cross section (taken from ACI 318 Art. 11.6.1).
AOH:	Area enclosed by center-line of the outermost closed transverse torsion reinforcement (taken from ACI 318 Art. 11.6.3).
PH:	Perimeter of centerline of outermost closed transverse torsion

reinforcement (taken from ACI 318 Art. 11.6.3).

AO: Gross area enclosed by shear flow path (taken from ACI 318 Art. 11.6.3).

EHE

REC: Reinforcement cover.

BW_VY: Width of element in VY direction equal to the total width in solid sections or in case of box sections, the width equals the sum of the width of both webs.

BW_VZ: Width of element in VZ direction equal to the total width in solid sections or in case of box sections, the width equals the sum of the width of both webs.

DY: Effective depth of the section in Y (Art. 44.2.3).

DZ: Effective depth of the section in Z (Art. 44.2.3).

RHO1: Geometric ratio of the longitudinal tensile reinforcement anchored at a distance greater than or equal to d (Art. 44.2.3.2).

$$\rho_1 = \frac{A_s}{b_w d} < 0.02$$

Where:

A_s Area of the longitudinal tensile reinforcement anchored at a distance greater than or equal to d beyond the section considered.

b_w Width of the web.

d Effective depth.

HE: Equivalent thickness of the wall (Art. 45.2.1):

$$he = \frac{A}{u}$$

Where:

A Total area of the cross-section within the outer circumference, including inner hollow areas.

u Outer circumference.

This equivalent thickness cannot be greater than the real thickness of the wall nor less than twice the cover.

AE: Area inside the center-line of the design effective hollow section (Art. 45.2.2).

UE: Perimeter of the center-line of the design effective hollow section (Art. 45.2.2).

KEYAST: Indicator of the position of the torsion reinforcement in the section (Art. 45.2.2.1):

=0 if closed stirrups are placed on both faces of each wall of the equivalent hollow section or of the real hollow section (value by default for hollow sections).

= 1 if there are only closed stirrups distributed around the periphery of the member (value by default for solid sections).

THETA: Angle of the concrete compressive struts with the longitudinal axis of member (Art. 44.2.3).

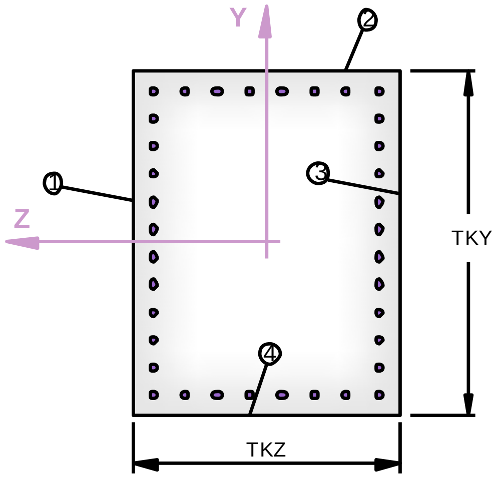
6.6.3. Code Dependent Parameters for Each Section

The following section describes how to compute the required parameters for shear and torsion according to each code. Shear and torsion calculations are performed taking for each end its section for shear considerations without accounting for reductions or enlargements due to depth variations. The mechanical cover for bending longitudinal reinforcement is required for the calculations of some parameters. The default mechanical cover for every case is equal to 5 cm.

6.6.3.1 Rectangular Section Parameters

Where Tky Section width in Y.

Tkz Section width in Z.



Eurocode 2 and **ITER**

$REC = 0.05\ m(\text{by default})$

$BW\_VY = Tkz$

$BW\_VZ = Tky$

$DY = Tky - REC$

$DZ = Tkz - REC$

$RHO1 = 0.0015$

$$T = \frac{Tky \cdot Tkz}{2(Tky + Tkz)} \geq 2 \cdot REC$$

$AK = (Tky - T) \cdot (Tkz - T)$

$UK = 2[(Tky - T) + (Tkz - T)]$

$KEYAST = 1(\text{outer reinforcement}).$

$THETA = 45°$

ACI 318 and **ACI 349**

$REC = 0.05\ m(\text{by default})$

$BW\_VY = Tkz$

$BW\_VZ = Tky$

$DY = Tky - REC$

$DZ = Tkz - REC$

$ACP = Tky \cdot Tkz$

$PCP = 2\ (Tky + Tkz)$

$AOH = (Tky - 2 \cdot REC)(Tkz - 2 \cdot REC)$

$PH = 2[(Tky - 2 \cdot REC) + (Tkz - 2 \cdot REC)]$

$AO = 0.85\ AOH$

BS 8110

$REC = 0.05\ m\ (\text{by default})$

$BW\_VY = Tkz$

$BW\_VZ = Tky$

$DY = Tky - REC$

$DZ = Tkz - REC$

$AS = 0.002 \cdot Ac$

$Ac = \text{concrete gross area}$

$Xw = \frac{1}{2}\left[h_{min}^2 \cdot (h_{max} - h_{min}/3)\right]$

$h_{min} = MIN(Tky, Tkz)$

$h_{max} = MAX(Tky, Tkz)$

$X1 = MIN(Tky, Tkz) - 2 \cdot REC$

$Y1 = MAX(Tky, Tkz) - 2 \cdot REC$

EHE

$REC = 0.04\ m(\text{by default})$

$BW\_VY = Tkz$

$BW\_VZ = Tky$

$DY = Tky - REC$

$DZ = Tkz - REC$

$RHO1 = 0.0028$

$HE = \frac{Tky \cdot Tkz}{2(Tky + Tkz)} \geq 2 \cdot REC$

$AE = (Tky - HE) \cdot (Tkz - HE)$

$UK = 2[(Tky - HE) + (Tkz - HE)]$

$KEYAST = 1(\text{outer reinforcement}).$

$THETA = 45^{O}$

GB50010

$REC = 0.05\ m(\text{default option})$

$BW\_VY = Tkz$

$BW\_VZ = Tky$

$DY = Tky - REC$

$DZ = Tkz - REC$

$HW\_VY = Tky - REC$

$HW\_VZ = Tkz - REC$

$A_{cor} = (Tkz - 2 \cdot REC)(Tky - 2 \cdot REC)$

$A_{cor1} = 0.0$

$A_{cor2} = 0.0$

$U_{cor} = 2 \cdot (Tkz + Tky - 4 \cdot REC)$

$U_{cor1} = 0.0$

$U_{cor2} = 0.0$

$W_t = \frac{{h_{min}}^2(3 \cdot h_{max} - h_{min})}{6}$

$h_{min} = min(Tkz, Tky)$

$h_{max} = max(Tkz, Tky)$

$W_{t1} = 0.0$

$W_{t2} = 0.0$

$$\text{ALF} = \max\left(\frac{\text{DZ}}{\text{Tky}}, \frac{\text{DY}}{\text{Tkz}}\right) \qquad \text{ALF}_h = 1.0$$

AASHTO Standard Specifications for Highway Bridges

$$\text{REC} = 0.05\ \text{m(by default)}$$

$$\text{BW\_VY} = \text{Tkz} \qquad \text{BW\_VZ} = \text{Tky}$$

$$\text{DY} = \text{Tky} - \text{REC} \qquad \text{DZ} = \text{Tkz} - \text{REC}$$

$$\text{ACP} = \text{Tky} \cdot \text{Tkz} \qquad \text{PCP} = 2(\text{Tky} + \text{Tkz})$$

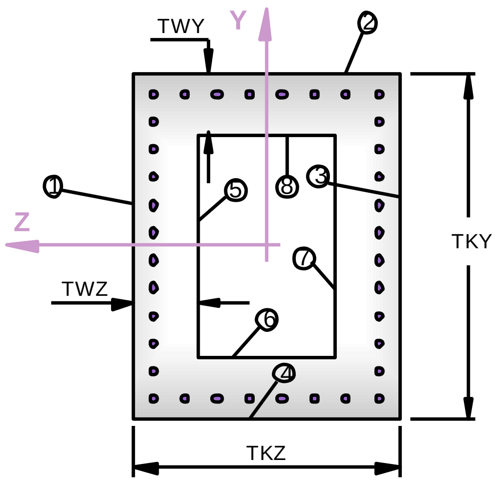
$$\text{AOH} = (\text{Tky} - 2 \cdot \text{REC})(\text{Tkz} - 2 \cdot \text{REC}) \qquad \text{PH} = 2[(\text{Tky} - 2 \cdot \text{REC}) + (\text{Tkz} - 2 \cdot \text{REC})]$$

$$\text{AO} = 0.85\ \text{AOH}$$

6.6.3.2 Box Section Parameters

Where:

Tky	Section width in Y.
Tkz	Section width in Z.
Twy	Thickness of walls in Y.
Twz	Thickness of walls in Z.



Eurocode 2 and **ITER**

$$\text{REC} = 0.05\ \text{m(by default)}$$

$BW\_VY = 2 \cdot Twz$

$BW\_VZ = 2 \cdot Twy$

$DY = Tky - REC$

$DZ = Tkz - REC$

$RH01 = 0.0015$

$T = \frac{Tky \cdot Tkz}{2(Tky + Tkz)}$

$T \geq 2 \cdot REC \,; T \leq Twy \,; T \leq Twz$

$AK = (Tky - T) \cdot (Tkz - T)$

$UK = 2[(Tky - T) + (Tkz - T)]$

$KEYAST = 0$ (outer and inner reinforcement)

$THETA = 45°$

ACI 318 and **ACI 349**

$REC = 0.05$ m(by default)

$BW\_VY = 2 \cdot Twz$

$BW\_VZ = 2 \cdot Twy$

$DY = Tky - REC$

$DZ = Tkz - REC$

$ACP = Tky \cdot Tkz$

$PCP = 2(Tky + Tkz)$

$AOH = (Tky - 2 \cdot REC)(Tkz - 2 \cdot REC)$

$PH = 2[(Tky - 2 \cdot REC) + (Tkz - 2 \cdot REC)]$

$AO = 0.85\ AOH$

BS8110

$REC = 0.05$ m(by default)

$BW\_VY = 2 \cdot Twz$

$BW\_VZ = 2 \cdot Twy$

$DY = Tky - REC$

$DZ = Tkz - REC$

$AS = 0.002 \cdot Ac$

Ac = gross concrete area

Xw is considered solid rectangular if Twy > 0.25Tky and Twz > 0.25Tkz. Otherwise: Xw = 2·MIN(Twy,Twz)·(Tkay,Twy)·(Tkz,Twz)

X1 = MIN(Tky,Tkz) – 2·REC

Y1 = MAX(Tky,Tkz) – 2·REC

EHE

$REC = 0.04$ m(by default)

$BW\_VY = 2 \cdot Twz$

$BW\_VZ = 2 \cdot Twy$

$DY = Tky - REC$

$DZ = Tkz - REC$

$RHO1 = 0.0028$

$HE = \frac{Tky \cdot Tkz}{2(Tky + Tkz)}$

$HE \geq 2 \cdot REC \,; HE \leq Twy \,; HE \leq Twz$

$AE = (Tky - HE) \cdot (Tkz - HE)$

$UE = 2[(Tky - HE) + (Tkz - HE)]$

$KEYAST = 0$ (outer and inner reinforcement)

$THETA = 45^{O}$

GB50010

REC = 0.05 m (by default)

BW_VY = 2 Twz

BW_VZ = 2 Twy

DY= Tky – REC

DZ = Tkz – REC

HW_VY = Tky– 2×TWY

HW_VZ = Tkz –2·TWZ

A_{cor} = (Tkz –2·REC) ·(Tky– 2·REC)

A_{cor1}= 0.0

A_{cor2}= 0.0

U_{cor} = 2·(Tkz +Tky– 4·REC)

U_{cor1}= 0.0

U_{cor2}= 0.0

$$W_t = \frac{{h_{min}}^2(3 \text{ x } h_{max} - h_{min})}{6} - \frac{(h_{min} - 2 \text{ x}Twy)^2}{6}[3(h_{max} - 2Twz) - (h_{min} - 2 \text{ x } Twy)]$$

$h_{min} = \min(Tkz, Tky)$

$h_{max} = \max(Tkz, Tky)$

$W_{t1} = 0.0$

$W_{t2} = 0.0$

$$ALF = \max\left(\frac{Tkz - 2 \text{ x } Twz}{Twy}, \frac{Tky - 2 \text{ x}Twy}{Twz}\right)$$

$$ALF_h = \min\left(\frac{Twy}{Tky}, \frac{Twz}{Tkz}\right)$$

AASHTO Standard Specifications for Highway Bridges

REC = 0.05 m (by default)

BW_VY = 2 Twz

BW_VZ = 2 Twy

DY= Tky – REC

DZ = Tkz – REC

$ACP = Tky \cdot Tkz$

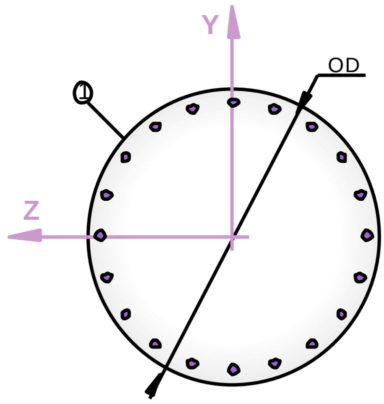
$PCP = 2(Tky + Tkz)$

$$AOH = (Tky - 2 \cdot REC)(Tkz - 2 \cdot REC)$$ $$PH = 2[(Tky - 2 \cdot REC) + (Tkz - 2 \cdot REC)]$$

AO = 0.85 AOH

6.6.3.3 Circular Section Parameters

Where: OD Diameter of the section.



Eurocode 2 and **ITER**

REC = 0.05 m (by default)

$$BW\_VY = \frac{OD}{\sqrt{2}}$$

(The width of the square within the circumference is used)

DY = OD - REC

RHO1 = 0.0015

$$AK = \frac{\pi(OD-T)^2}{4}$$

KEYAST = 1 (outer reinforcement)

$$BW\_VZ = \frac{OD}{\sqrt{2}}$$

(The width of the square within the circumference is used)

DZ = OD - REC

$$T = \frac{OD}{4} \geq 2 \cdot REC$$

$$UK = \pi(OD - T)$$

THETA = 45°

ACI 318 and **ACI 349**

REC = 0. 05 m (by default)

BW_VY = OD

BW_VZ = OD

$$DY = \frac{OD}{2} + \frac{4\left(\frac{OD}{2} - REC\right)}{3\,\pi}$$

$$DZ = \frac{OD}{2} + \frac{4\left(\frac{OD}{2} - REC\right)}{3\,\pi}$$

(In both directions, the distance from extreme compression fiber to centroid of reinforcement in the opposite half of the section is used, assuming that this reinforcement is uniformly distributed and considering the cover of REC.)

$\mathrm{ACP} = \pi \cdot \frac{\mathrm{OD}^2}{4}$

$\mathrm{PCP} = \pi \cdot \mathrm{OD}$

$\mathrm{AOH} = \pi \cdot \frac{(\mathrm{OD} - 2 \cdot \mathrm{REC})^2}{4}$

$\mathrm{PH} = \pi(\mathrm{OD} - 2 \cdot \mathrm{REC})$

AO = 0.85 AOH

BS8110

REC = 0.05 m (by default)

BW_VY = Tkz

BW_VZ = Tky

DY = Tky - REC

DZ = Tkz - REC

AS = 0.002·Ac

$\mathrm{Ww} = \frac{\pi \cdot \mathrm{OD}^3}{16}$

X1 = OD – 2·REC

Y1 = OD – 2·REC

EHE

REC = 0.04 m (by default)

$\mathrm{BW\_VY} = 2\sqrt{\left(\frac{\mathrm{OD}}{2}\right)^2 - \left(\frac{\mathrm{OD} + \mathrm{REC}}{4}\right)^2}$

$\mathrm{BW\_VZ} = 2\sqrt{\left(\frac{\mathrm{OD}}{2}\right)^2 - \left(\frac{\mathrm{OD} + \mathrm{REC}}{4}\right)^2}$

DY = OD - REC

DZ = OD - REC

RHO1 = 0.0028

$\mathrm{HE} = \frac{\mathrm{OD}}{4} \geq 2 \cdot \mathrm{REC}$

$\mathrm{AE} = \frac{\pi\,(\mathrm{OD} - \mathrm{HE})^2}{4}$

$\mathrm{UE} = \pi(\mathrm{OD} - \mathrm{HE})$

KEYAST = 1 (outer reinforcement)

THETA = 45º

GB50010

REC = 0.05 m (by default)

BW_VZ = 0.88·OD

BW_VY = 0.88·OD

DY = 0.8·OD

DZ = 0.8·OD

HW_VY = 0.8·OD

HW_VZ = 0.8·OD

$\mathrm{A_{cor}} = \pi\left(\frac{\mathrm{OD} - 2 \cdot \mathrm{REC}}{2}\right)^2$

$\mathrm{A_{cor1}} = 0.0$

$\mathrm{A_{cor2}} = 0.0$

$\mathrm{U_{cor}} = \pi \cdot (\mathrm{OD} - 2 \cdot \mathrm{REC})$

$U_{cor1} = 0.0$

$U_{cor2} = 0.0$

$W_t = \pi \frac{OD^3}{16}$

$W_{t1} = 0.0$

$W_{t2} = 0.0$

ALF = 0.91

$ALF_h = 1.0$

AASHTO Standard Specifications for Highway Bridges

REC = 0. 05 m (by default)

BW_VY = OD

BW_VZ = OD

$DY = \frac{OD}{2} + \frac{4\left(\frac{OD}{2} - REC\right)}{3\,\pi}$

$DZ = \frac{OD}{2} + \frac{4\left(\frac{OD}{2} - REC\right)}{3\,\pi}$

(In both directions the distance from extreme compression fiber to centroid of reinforcement in the opposite half of the section is used, assuming that this reinforcement is uniformly distributed and considering the cover of REC.)

$ACP = \pi \cdot \frac{OD^2}{4}$

$PCP = \pi \cdot OD$

$AOH = \pi \cdot \frac{(OD - 2 \cdot REC)^2}{4}$

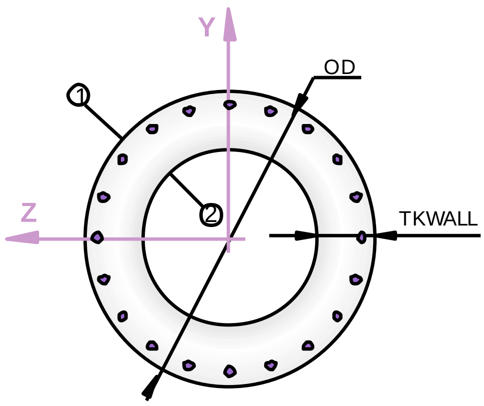
$PH = \pi(OD - 2 \cdot REC)$

AO = 0.85 AOH

6.6.3.4 Circular Hollow Section Parameters

Where: OD Diameter of the section.

TKWALL Thickness of the wall.



Eurocode 2 and **ITER**

REC = 0.05 m (by default)

BW_VY =2 · TKWALL

BW_VZ =2 · TKWALL

DY= OD - REC

DZ = OD - REC

RHO1 = 0.0015

$$T = \frac{OD}{4}$$

$$T \geq 2 \cdot REC\, ;T \leq TKWALL$$

$$AK = \frac{\pi(OD - T)^2}{4}$$

$$UK = \pi(OD - T)$$

KEYAST = 0 (inner and outer reinforcement).

THETA = 45°

ACI 318 and **ACI 349**

REC = 0. 05 m (by default)

BW_VY = 2 · TKWALL

BW_VZ = 2 · TKWALL

$$DY = \frac{OD}{2} + \frac{4\left(\frac{OD}{2} + REC\right)}{3\,\pi}$$

$$DZ = \frac{OD}{2} + \frac{4\left(\frac{OD}{2} + REC\right)}{3\,\pi}$$

(In both directions the distance from extreme compression fibre to centroid of reinforcement in the opposite half of the section is used, assuming that this reinforcement is uniformly distributed and considering the cover of REC.)

$$ACP = \pi \cdot \frac{OD^2}{4}$$

$$PCP = \pi \cdot OD$$

$$AOH = \pi \cdot \frac{(OD - 2 \cdot REC)^2}{4}$$

$$PH = \pi(OD - 2 \cdot REC)$$

AO = 0.85 AOH

BS 8110

REC = 0.04 m (by default)

BW_VY = 2 · TKWALL

BW_VZ = 2 · TKWALL

DY = OD - REC

DZ = OD - REC

AS = 0.002 · Ac

XW is considered a solid circular section if (Tkwall > 0.25·OD)

Otherwise:

$$Xw = \frac{\pi[OD^4 - (OD - TKwall)^4]}{16 \cdot OD}$$

X1 = OD - 2·REC

Y1 = OD – 2·REC

EHE

REC = 0.04 m (by default)

$$BW\_VY = 2\sqrt{\left(\frac{OD}{2}\right)^2 - \left(\frac{OD + REC}{4}\right)^2} < 2 \cdot TKWALL$$

$$BW\_VZ = 2\sqrt{\left(\frac{OD}{2}\right)^2 - \left(\frac{OD + REC}{4}\right)^2} < 2 \cdot TKWALL$$

DY = OD - REC

DZ = OD - REC

RHO1 = 0.0028

$$HE = \frac{OD}{4}$$

$$HE \geq 2 \cdot 0.04 \text{ m} \;;\; HE \leq TKWALL$$

$$AE = \frac{\pi\,(OD - HE)^2}{4}$$

$$UE = \pi(OD - HE)$$

KEYAST = 0 (outer and inner reinforcement).

THETA = 45º

GB50010

REC = 0. 05 m (by default)

BW_VY = 2 ·TKWALL

BW_VZ = 2 ·TKWALL

DY = 0.8·OD

DZ = 0.8·OD

HW_VY = not defined

HW_VZ = not defined

$$A_{cor} = \pi\left(\frac{OD - 2 \cdot REC}{2}\right)^2$$

$$A_{cor1} = 0.0$$

$$A_{cor2} = 0.0$$

$$U_{cor} = \pi x(OD - 2xREC)$$

$$U_{cor1} = 0.0$$

$$U_{cor2} = 0.0$$

$$W_t = \pi\frac{OD^4 - (OD - TKWALL)^4}{16 \cdot OD}$$

$$W_{t1} = 0.0$$

$$W_{t2} = 0.0$$

ALF = 0.91

$$ALFh = \frac{TKWALL}{OD}$$

AASHTO Standard Specifications for Highway Bridges

REC = 0. 05 m (by default)

BW_VY = 2 · TKWALL

BW_VZ = 2 · TKWALL

$$DY = \frac{OD}{2} + \frac{4\left(\frac{OD}{2} + REC\right)}{3\,\pi} \qquad DZ = \frac{OD}{2} + \frac{4\left(\frac{OD}{2} + REC\right)}{3\,\pi}$$

(In both directions the distance from extreme compression fibre to centroid of reinforcement in the opposite half of the section is used, assuming that this reinforcement is uniformly distributed and considering the cover of REC.)

$$ACP = \pi \cdot \frac{OD^2}{4} \qquad PCP = \pi \cdot OD$$

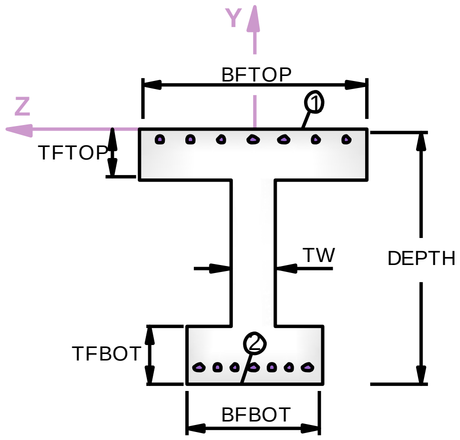
$$AOH = \pi \cdot \frac{(OD - 2 \cdot REC)^2}{4} \qquad PH = \pi(OD - 2 \cdot REC)$$

AO = 0.85 AOH

6.6.3.5 Double T / I-Section Parameters

Where: DEPTH Depth of the section (in Y).

TW Web thickness.



Eurocode 2 and **ITER**

REC = 0.05 m (by default)

BW_VY = TW

BW_VZ = undefined

DY = DEPTH - REC

DZ = undefined

RHO1 = 0.0015

T = undefined

AK = undefined

UK = undefined

KEYAST = undefined

THETA = 45°

ACI 318 and **ACI 349**

REC = 0.04 m (by default)

BW_VY = TW

BW_VZ = undefined

DY = DEPTH - REC

DZ = undefined

ACP = undefined

PCP = undefined

AOH = undefined

PH = undefined

AO = undefined

BS 8110

REC = 0.04 m (by default)

BW_VY = Tkz

BW_VZ = undefined

DY = Tky - REC

DZ = Tkz - REC

AS = 0.002·Ac

XW = undefined

Ac = concrete gross section

X1 = undefined

Y1 = undefined

EHE

REC = 0.04 m (by default)

BW_VY = TW

BW_VZ = undefined

DY = DEPTH - REC

DZ = undefined

RHO1 = 0.0028

HE = undefined

AE = undefined

U= undefined

KEYAST = undefined

THETA = 45º

GB50010

REC = 0.04 m (by default)

BW_VY = TW

BW_VZ = undefined

DY = DEPTH – REC

DZ = undefined

HW_VY = DEPTH – TFTOP – TFBOT

HW_VZ = undefined

$$A_{cor} = (DEPTH - 2 \cdot REC) \cdot (TW - 2 \cdot REC)$$

$$A_{cor1} = (TFTOP - 2 \cdot REC) \cdot (BFTOP - TW - 2 \cdot REC$$

$$A_{cor2} = (TFBOT - 2 \cdot REC) \cdot (BFBOT - TW - 2 \cdot REC)$$

$$U_{cor} = 2 \cdot (DEPTH + TW - 4 \cdot REC)$$

$$U_{cor1} = 2 \cdot (TFTOP + BFTOP - TW - 4 \cdot REC) \qquad U_{cor2} = 2 \cdot (TFBOT + BFBOP - TW - 4 \cdot REC)$$

$$W_t = \frac{TW^2(3 \cdot DEPTH - TW)}{6} + \frac{TFTOP^2 \cdot (BFTOP - TW)}{2} + \frac{TFBOT^2 \cdot (BFBOT - TW)}{2}$$

$$W_{t1} = \frac{TFTOP^2 \cdot (BFTOP - TW)}{2} \qquad W_{t2} = \frac{TFBOT^2 \cdot (BFBOT - TW)}{2}$$

$$ALF = \frac{DEPTH - TFTOP - TFBOT}{TW} \qquad ALF_h = 1.0$$

AASHTO Standard Specifications for Highway Bridges

REC = 0.04 m (by default)

BW_VY = TW

BW_VZ = undefined

DY = DEPTH - REC

DZ = undefined

ACP = undefined

PCP = undefined

AOH = undefined

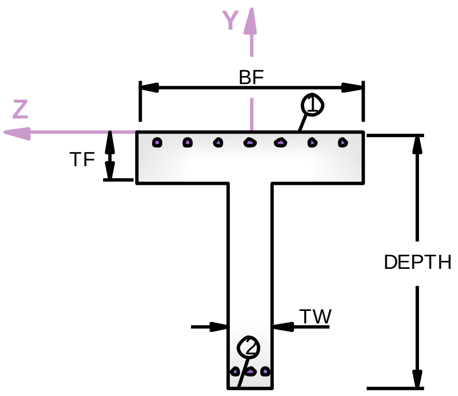
PH = undefined

AO = undefined

6.6.3.6 T-Section Parameters

Where: DEPTH Depth of the section (in Y).

TW Web thickness.



Eurocode 2 and **ITER**

REC = 0.05 m (by default)

BW_VY = TW

BW_VZ = undefined

DY = DEPTH - REC

DZ = undefined

RHO1 = 0.0015

T = undefined

AK = undefined

UK = undefined

KEYAST = undefined

THETA = 45º

ACI 318 and **ACI 349**

REC = 0.05 m (by default)

BW_VY = TW

BW_VZ = undefined

DY = DEPTH - REC

DZ = undefined

ACP = undefined

PCP = undefined

AOH = undefined

PH = undefined

AO = undefined

EHE

REC = 0.04 m (by default)

BW_VY = TW

BW_VZ = undefined

DY = DEPTH - REC

DZ = undefined

RHO1 = 0.0028

HE = undefined

AE = undefined

U = undefined

KEYAST = undefined

THETA = 45º

BS 8110

REC = 0.05 m (by default)

BW_VY = Tkz

BW_VZ = undefined

DY = Tky - REC

DZ = Tkz - REC

AS = 0.002·Ac

XW = undefined

Ac = concrete gross section

X1 = undefined

Y1 = undefined

GB50010

REC = 0.05 m (by default)

BW_VY = TW

BW_VZ = TF

DY = DEPTH – REC

DZ = BF– REC

HW_VY = DEPTH – TF – REC

HW_VZ = undefined

$A_{cor} = (DEPTH - 2 \cdot REC) \cdot (TW - 2 \cdot REC)$

$A_{cor1} = (TF - 2 \cdot REC) \cdot (BF - TW - 2 \cdot REC)$

$A_{cor2} = 0.0$

$U_{cor} = 2 \cdot (DEPTH + TW - 4 \cdot REC)$

$U_{cor1} = 2 \cdot (TF + BF - TW - 4 \cdot REC)$

$U_{cor2} = 0.0$

$$W_t = \frac{TW^2(3 \cdot DEPTH - TW)}{6} + \frac{TF^2 \cdot (BF - TW)}{2}$$

$$W_{t1} = \frac{TF^2 \cdot (BF - TW)}{2}$$

$W_{t2} = 0.0$

$$ALF = \frac{DEPTH - TF}{TW}$$

$ALF_h = 1.0$

AASHTO Standard Specifications for Highway Bridges

REC = 0.05 m (by default)

BW_VY = TW

BW_VZ = undefined

DY = DEPTH - REC

DZ = undefined

ACP = undefined

PCP = undefined

AOH = undefined

PH = undefined

AO = undefined

6.6.4. Shear and Torsion according to Eurocode 2 (EN 1992-1-1:2004/AC:2008) and ITER Design Code

6.6.4.1. Shear Check

Checking elements for shear according to Eurocode 2 (EN 1992-1-1:2004/AC:2008) and ITER Design Code follows the steps below:

1) **Obtaining strength properties of the materials.** The required material properties associated with each transverse cross section at the active time are:

f_{ck} characteristic compressive strength of concrete.

f_{cd} design strength of concrete.

f_{yk} characteristic yield strength of reinforcement.

f_{ywd} design strength of shear reinforcement.

2) **Obtaining geometrical data of the section.** Required data for shear checking are the following ones:

A_c total cross-sectional area of the concrete section.

3) **Obtaining geometrical parameters depending on code.** Required data are as follows:

b_w minimum width of the section over the effective depth.

d effective depth of the section.

ρ_1 ratio of the tension longitudinal reinforcement

$$\rho_1 = \frac{A_{sl}}{b_w \ d} < 0.02$$

where:

A_{sl} the area of the tension reinforcement extending not less than $d + I_{b,net}$ beyond the section considered.

θ angle of the compressive struts of concrete with the member's longitudinal axis, (parameter THETA):

$1.0 \leq \text{cotan} \leq 2.5$ Eurocode 2 (EN 1992-1-1:2004/AC:2008)

$1.0 \leq \text{cotan}\theta \leq \text{cotan}\theta_0$ ITER Design Code

Compressive mean stress $(\sigma_{cp} > 0)$: $\text{cotan}\ \theta_0 = 1.2 + 0.2\ \sigma_{cp}/f_{ctm}$

Tensile mean stress $(\sigma_{cp} < 0)$: $\text{cotan}\ \theta_0 = 1.2 + 0.9\ \sigma_{cp}/f_{ctm} \geq 1$

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section**. Required data are as follows:

α angle between shear reinforcement and the longitudinal axis of the member section, (parameter ALPHA).

A_{sw}/S area of reinforcement per unit length, (parameters ASSY or ASSZ).

The reinforcement ratio may also be obtained with the following data:

A_{sw} total area of the reinforcement legs, (parameters ASY or ASZ, both Y and Z directions are available).

s spacing of the stirrups.

or with the following ones:

s spacing of the stirrups.

ϕ diameter of bars, (parameter PHI).

N number of reinforcement legs, (parameters NY or NZ for Y and Z directions).

5) **Obtaining the section's internal forces and moments**. The shear force that acts on the section, as well as the concomitant axial force and bending moment, are obtained from the CivilFEM results file (.RCF).

Force	Description
V_{Ed}	Design shear force (≥ 0)
N_{Ed}	Design axial force (positive for compression)
M_{Ed}	Design bending moment (≥ 0)

6) **Checking whether the section requires shear reinforcement.** First, the design shear (V_{Ed}) is compared to the design shear resistance ($V_{Rd,c}$):

$$V_{Ed} \leq V_{Rd,c}$$

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100\rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

with the constraints:

$$V_{Ed} \leq 0.5b_w \cdot d \cdot \nu \cdot f_{cd}$$

$$V_{Rd,c} \geq \left[V_{min} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

where:

$C_{Rd,c}$ = $0.18\,\gamma_c$

f_{ck} = in MPa

k = $1 + \sqrt{\frac{200}{d}} \leq 2$ (d en mm)

k_1 = 0.15

σ_{cp} = $\frac{N_{Ed}}{A_c} < 0.2f_{cd}$ Mpa

A_c in mm^2

ν = $0.6\left(1 - \frac{f_{ck}}{250}\right)$ Eurocode 2 (EN 1992-1-1:2004/AC:2008)

$$\begin{cases} 0.6 & f_{ck} \leq 60 \text{ Mpa} \\ 0.9 - \frac{f_{ck}}{200} > 0.5 & f_{ck} > 60 \text{ Mpa} \end{cases} \text{ ITER Design Code}$$

V_{min} = $0.035(k^3 \cdot f_{ck})^{1/2}$

$V_{Rd,c}$ in N

If shear reinforcement has not been defined for the section, a check is made to ensure V_{Ed} is less than the lowest value between the shear reinforcement resistance,

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot 0.9 \cdot d \cdot f_{ywd} \cdot (\text{cotan}\theta + \text{cotan}\alpha) \cdot \sin\alpha$$

and the maximum design shear reinforcement resistance:

Eurocode 2 (EN 1992-1-1:2004/AC:2008):

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot 0.9 \cdot d \cdot \nu \cdot f_{cd} \cdot \frac{(\text{cotan}\theta + \text{cotan}\alpha)}{1 + \text{cotan}^2\theta}$$

ITER Design Code:

$$V_{Rd,max} = \alpha_{cw} \cdot \nu \cdot f_{cd} \cdot b_w \cdot 0.9 \cdot d \cdot \frac{(\text{cotan}\theta_0 + \text{cotan}\alpha)}{1 + \text{cotan}^2\theta_0}$$

Where :

$$\alpha_{cw} = \begin{cases} 1 + \frac{\sigma_{cp}}{f_{cd}}, \text{if } 0 < \sigma_{cp} \leq 0.25 f_{cd} \\ 1.25, \text{if } 0.25 f_{cd} < \sigma_{cp} \leq 0.5 f_{cd} \\ 2.5\left(1 - \frac{\sigma_{cp}}{f_{cd}}\right), \text{if } 0.5 f_{cd} < \sigma_{cp} \leq f_{cd} \end{cases}$$

The shear reinforcement must be equal to or less than (Eurocode 2 only)

$$\frac{A_{sw,max}}{S} = 0.5 \cdot \alpha_{cw} \cdot b_w \cdot \nu \cdot \frac{f_{cd}}{f_{ywd}} / \sin\alpha$$

Results are written for each end in the CivilFEM results file as the following parameters:

VRDC = $V_{Rd,c}$

VRDS = $V_{Rd,s}$

VRDMAX = $V_{Rd,max}$

TENS = $\frac{M_{Ed}}{0.9d}+\frac{V_{Ed}}{2}(\cot\theta-\cot\alpha)$

Tension resistance of the longitudinal reinforcement

CRT_1 = $\frac{V_{Ed}}{V_{Rd,c}}$

CRT_2 = 0, If there is no shear reinforcement.

$\frac{V_{Ed}}{V_{Rd,s}}$, If there is shear reinforcement.

CRT_3 = 0, If there is no shear reinforcement.

$\frac{V_{Ed}}{V_{Rd,max}}$, If there is shear reinforcement.

7) **Obtaining shear criterion.** The shear criterion indicates whether the section is valid for the design forces (if it is less than 1, the section satisfies the code provisions; whereas if it exceeds 1, the section will not be valid). Furthermore, it includes information pertaining to how close the design force is to the ultimate section strength. The shear criterion is defined as follows:

CRT_TOT = $\frac{V_{Ed}}{V_{Rd,c}}$, If there is no shear reinforcement

$\min\left\{\frac{V_{Ed}}{V_{Rd,c}},\max\left\{\frac{V_{Ed}}{V_{Rd,s}},\frac{V_{Ed}}{V_{Rd,max}}\right\}\right\}$, If there is shear reinforcement

A value of 2^{100} for this criterion indicates that $V_{Rd2,red}$ or V_{Rd3} are equal to zero.

6.6.4.2. Torsion Check

The torsion checking according to Eurocode 2 (EN 1992-1-1:2004/AC:2008) and ITER Design Code follows the steps below:

1) **Obtaining strength properties of the materials.** These properties are obtained from the material properties associated to the transverse cross section and for the active time.

The Required data are as follows:

f_{ck} characteristic strength of concrete.

f_{cd} calculation strength of concrete.

f_{yk} characteristic yield strength of reinforcement.

f_{yd} calculation torsion resistance of reinforcement. The same material is considered for transverse and longitudinal reinforcement

2) Obtaining geometrical parameters depending on specified code. The required data are as follows:

t equivalent thickness of wall.

A_k area enclosed within the centre-line of the thin-walled cross-section.

u_k circumference of area A_k.

θ Angle of the compressive struts of concrete with the member's longitudinal axis:

$1.0 \leq \cot an\ \theta \leq 2.5$ Eurocode 2 (EN 1992-1-1:2004/AC:2008)

$1.0 \leq \cot an\ \theta \leq \cot an\ \theta_0$ ITER Design Code

Compressive mean stress $(\sigma_{cp} > 0)$: $\cot an\theta_0 = 1.2 + 0.2\sigma_{cp}/f_{ctm}$

Tensile mean stress $(\sigma_{cp} < 0)$: $\cot an\theta_0 = 1.2 + 0.9\sigma_{cp}/f_{ctm} \geq 1$

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

3) Obtaining reinforcement data of the section. Required data are as follows:

Transverse reinforcement

A_{sw}/S area of transverse reinforcement per unit length.

The reinforcement ratio can alternatively be defined using the following data:

A_{sw} closed stirrups area for torsion.

s spacing of closed stirrups.

Or with the following data:

s spacing of closed stirrups.

Φ_t diameter of the closed stirrups.

Longitudinal Reinforcement

A_{sl} total area of the longitudinal reinforcement.

The reinforcement ratio can alternatively be defined using the following data:

Φ_I diameter of longitudinal bars.

N number of longitudinal bars.

4) **Obtaining section internal forces and moments**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	Description
T_{sd}	Design torsional moment

5) **Calculating the maximum torsional moment that can be resisted by the concrete compressive struts.** The design torsional moment (T_{sd}) must be less than or equal to the maximum torsional moment that can be resisted by the concrete compressive struts ($T_{Rd,max}$); therefore, the following condition must be fulfilled:

$$T_{Ed} \leq T_{Rd,max}$$

$$T_{Rd,max} = 2 \cdot \nu \cdot \alpha_{cw} \cdot f_{cd} \cdot t \cdot A_k \cdot \sin\theta \cdot \cos\theta$$

Where the values de of ν and $\alpha_{cw}$are the same as those used in shear checking,

Results are written in the CivilFEM results file for both element ends as the parameters:

$$\text{TRDMAX} = T_{Rd,max}$$

$$\text{CRT\_1} = \frac{T_{Ed}}{T_{Rd,max}}$$

6) **Calculating the maximum torsional moment that can be resisted by the reinforcement.** The design torsional moment (T_{Ed}) must be less than or equal to the maximum design torsional moment that can be resisted by the reinforcement (T_{Rd}); consequently, the following condition must be fulfilled:

$$T_{Ed} \leq T_{Rd}$$

$$T_{Rd} = 2A_k\left(f_{yd} \cdot \frac{A_{sw}}{S}\right)\text{cotan}\theta$$

Calculation results are written in the CivilFEM results file for both element ends as the parameters:

TRD = T_{Rd}

CRT_2 = $\frac{T_{Ed}}{T_{Rd}}$

If transverse reinforcement is not defined, $T_{Rd2} = 0$ and the criterion will take the value of 2^{100}.

7) **Calculating the required longitudinal reinforcement.** The required longitudinal reinforcement is calculated from T_{Rd} as follows:

$$A_{sl,nec} = T_{Rd}\frac{U_k}{2A_k f_{yd}}\text{cotan}\theta$$

If longitudinal reinforcement is not defined, $A_{sl} = 0$ and the criterion will be 2^{100}.

ALT = $A_{sl,nec}$

CRTALT = $\frac{A_{sl,nec}}{A_{sl}}$

8) **Obtaining torsion criterion.** The torsion criterion is defined as the ratio of the design moment to the section ultimate resistance: if it is less than 1, the section is valid; whereas if it exceeds 1, the section is not valid. The criterion pertaining to the validity for torsion is defined as follows:

$$CRT\_TOT = \max\left(\frac{T_{Ed}}{T_{Rd,max}}, \frac{T_{Ed}}{T_{Rd}}, \frac{A_{sl,nec}}{A_{sl}}\right) \leq 1$$

This value is stored in the CivilFEM results file for each end.

A value 2^{100} for this criterion indicates that any one of the torsion reinforcement groups are undefined.

6.6.4.3. Combined Shear and Torsion Check

For checking sections subjected to shear force and concomitant torsional moment, we follow the steps below:

1) **Torsion checking considering a null shear force.** This check follows the same procedure as for the check of elements subjected to pure torsion according to Eurocode 2 (EN 1992-1-1:2004/AC:2008) and ITER Design Code.
2) **Shear checking assuming a null torsional moment. .** This check follows the same steps as for the check of elements subjected to pure shear according to Eurocode 2 (EN 1992-1-1:2004/AC:2008) and ITER Design Code.

3) **Checking the concrete ultimate strength condition.** The design torsional moment (T_{Sd}) and the design shear force (V_{Sd}) must satisfy the following condition:

$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \leq 1$$

4) **Obtaining the combined shear and torsion criterion.** This criterion comprehends pure shear, pure torsion and ultimate strength condition criteria of concrete. The criterion determines whether the section is valid and is defined as follows:

$$\mathrm{CRT\_TOT} = \max\left\{\frac{V_{Ed}}{V_{Rd,s}}, \frac{V_{Ed}}{V_{Rd,max}}, \frac{T_{Ed}}{T_{Rd,max}}, \frac{T_{Ed}}{T_{Rd}}, \frac{A_{sl,nec}}{A_{sl}}, \frac{T_{Ed}}{T_{Rd}} + \frac{V_{Ed}}{V_{Rd}}\right\} \leq 1$$

A value 2^{100} for this criterion indicates that $V_{Rd1,red}$ or V_{Rd3} are equal to zero or that one of the torsion reinforcement groups has not been defined.

6.6.4.4. Shear Design

Shear reinforcement design according to Eurocode 2 (EN 1992-1-1:2004/AC:2008) and ITER Design Code follows the steps below:

1) **Obtaining strength properties of the materials.** The required material properties associated with each transverse cross section at the active time are:

f_{ck} characteristic strength of concrete.

f_{cd} characteristic design strength of concrete.

f_{yk} characteristic yield strength of reinforcement.

f_{ywd} design strength of shear reinforcement.

2) **Obtaining geometrical data of the section.** Required data for shear design are the following:

A_c total cross-sectional area of the concrete section.

3) **Obtaining geometrical parameters depending on specified code.** Required data are as follows:

b_w minimum width of the section over the effective depth.

d effective depth of the section.

ρ_1 ratio of the longitudinal tensile :

$$\rho_1 = \frac{A_{sl}}{b_w d} < 0.02$$

where:

A_{sl} the area of the tensile reinforcement extending not less than $d + I_{b,net}$beyond the section considered.

θ angle of the compressive struts of concrete with the member's longitudinal axis:

$1.0 \leq \cot\theta \leq 2.5$ Eurocode 2 (EN 1992-1-1:2004/AC:2008)

$1.0 \leq \cot\theta \leq \cot\theta_0$ ITER Design Code

Compressive mean stress $(\sigma_{cp} > 0)$: $\cot\theta_0 = 1.2 + 0.2\sigma_{cp}/f_{ctm}$

Tensile mean stress $(\sigma_{cp} < 0)$: $\cot\theta_0 = 1.2 + 0.9\,\sigma_{cp}/f_{ctm} \geq 1$

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** For shear reinforcement design, it is possible to define the angle between the reinforcement and the longitudinal axis of member can be indicated. This angle should be included in the reinforcement definition of each element. If this angle is null or is not defined, =90° is used. Other reinforcement data will be ignored.

5) **Obtaining forces and moments acting on the section**. The shear force that acts on the section as well as the concomitant axial force and bending moment, are obtained from the CivilFEM results file (.RCF).

Force	**Description**
V_{Ed}	Design shear force
N_{Ed}	Design axial force (positive for compression)
M_{Ed}	Design bending moment (≥ 0)

6) **Checking whether the section requires shear reinforcement.** First, the design shear (V_{Ed}) is compared to the design shear resistance ($V_{Rd,c}$):

$$V_{Ed} \leq V_{Rd,c}$$

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100\ \rho_1 \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

with the constraints:

$$V_{Ed} \leq 0.5 b_w \cdot d \cdot \nu \cdot f_{cd}$$
$$V_{Rd,c} \geq \left[V_{min} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

where:

$C_{Rd,c}$ = $0.18\ \gamma_c$

f_{ck} in MPa

k = $1 + \sqrt{\frac{200}{d}}$ (d in mm)

ρ_1 = $\frac{A_{sl}}{b_w\ d} \leq 0.02$

k_1 = 0.15

σ_{cp} = $\frac{N_{Ed}}{A_c} < 0.2 f_{cd}$ MPa

A_c in mm^2

$0.6\left(1 - \frac{f_{ck}}{250}\right)$ Eurocode 2 (EN 1992-1-1:2004/AC:2008)

ν = $\begin{cases} 0.6 & f_{ck} \leq 60\ \text{Mpa} \\ 0.9 - \frac{f_{ck}}{200} > 0.5 & f_{ck} > 60\ Mpa \end{cases}$ ITER Design Code

V_{min} = $0.035(k^3 \cdot f_{ck})^{1/2}$

$V_{Rd,c}$ in N

Results are written for each element end in the CivilFEM results file as the parameters:

$$VRDC = V_{Rd,c}$$

$$CRT\_1 = \frac{V_{Ed}}{V_{Rd,c}}$$

7) Calculating the maximum shear force that can be resisted by the concrete compressive struts.

A check is made to ensure that V_{Ed} is less than $V_{Rd,max}$:

Eurocode 2 (EN 1992-1-1:2004/AC:2008):

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot 0.9 \cdot d \cdot \nu \cdot f_{cd} \cdot \frac{(\cot an\theta + \cot an\alpha)}{(1 + \cot an^2\theta)}$$

ITER Design Code:

$$V_{Rd\ max} = \alpha_{cw} \cdot \nu \cdot f_{cd} \cdot b_w \cdot d \cdot \frac{(\cot an\theta_0 + \cot an\alpha)}{(1 + \cot an^2\theta_0)}$$

where:

$$\alpha_{cw} = \begin{cases} 1, \text{if the section is not prestessing} \\ 1 - \dfrac{\sigma_{cp}}{f_{cd}}, \text{if } 0 < \sigma_{cp} \leq 0.25 f_{cd} \\ 1.25, \text{if } 0.25 f_{cd} < \sigma_{cp} \leq 0.5 f_{cd} \\ 2.5\left(1 - \dfrac{\sigma_{cp}}{f_{cd}}\right), \text{if } 0.5 f_{cd} < \sigma_{cp} \leq f_{cd} \end{cases}$$

α = 90º if shear reinforcement was determined as not necessary in the previous step. If reinforcement is necessary, the angle α will be read from in the reinforcement definition data.

Results are written for each element end in the CivilFEM results file as the parameters:

$$VRDMAX = V_{Rd,max}$$

$$CRT\_3 = \frac{V_{Ed}}{V_{Rd,max}}$$

If design shear force is greater than the force required to crush the concrete compressive struts, the reinforcement design will not be feasible, so the parameter containing this datum will be marked with 2^{100}.

If the struts are not crushed by oblique compression, the calculating process continues.

8) Calculating required amount of transverse reinforcement. The section validity condition pertaining to shear force is:

$$V_{Rd,s} = \begin{cases} V_{Ed} & \text{if } V_{Ed} < V_{Rd,max} \\ V_{Rd,max} & \text{if } V_{Ed} \geq V_{Rd,max} \end{cases}$$

Therefore, the reinforcement amount per length unit should be:

$$\frac{A_{sw}}{S} = \frac{V_{Rd,s}}{0.9 \cdot d \cdot f_{ywd} \cdot (\cot\theta + \cot\alpha) \cdot \sin\alpha}$$

While also satisfying the following condition (Eurocode 2 only):

$$\frac{A_{sw}}{S} \leq 0.5 \cdot \alpha_{cw} \cdot b_w \cdot \nu \cdot \frac{f_{cd}}{f_{ywd}} / \sin\alpha$$

If the design is not possible, the reinforcement will be defined as 2^{100} and labeled as not designed.

The design criterion will be 1 (Ok) if the element was designed or 0 (Not Ok) if not.

For each element end, the results are included in the CivilFEM results file as the following parameters:

$$VRDS = V_{Rd,s}$$

$$ASSH = \frac{A_{sw}}{S}$$

DSG_CRT = Design criterion

6.6.4.5. Torsion Design

Torsion reinforcement design according to Eurocode 2 (EN 1992-1-1:2004/AC:2008) and ITER Design Code follows the steps below:

1) **Obtaining strength properties of the materials.** These properties are obtained from the material properties associated to each transverse cross section and for the active time. Those material properties should be previously defined. The Required data are as follows:

f_{ck} characteristic strength of concrete.

f_{cd} characteristic design strength of concrete.

f_{yk} characteristic yield strength of reinforcement.

f_{yd} design strength of shear reinforcement. The same material will be considered for transverse and longitudinal reinforcement.

2) **Obtaining geometrical parameters depending on specified code.** The required data are as follows:

t equivalent thickness of wall.

A_k area enclosed within the centre-line of the thin-walled cross-section.

U_k circumference of area A_k .

θ angle between the concrete compressive struts and the longitudinal axis of the member:

$1.0 \leq \cot\theta \leq 2.5$ Eurocode 2 (EN 1992-1-1:2004/AC:2008)

$1.0 \leq \cot\theta \leq \cot\theta_0$ ITER Design Code

Compressive mean stress $(\sigma_{cp} > 0)$: $\cot\theta_0 = 1.2 + 0.2\ \sigma_{cp}/f_{ctm}$

Tensile mean stress $(\sigma_{cp} < 0)$: $\cot\theta_0 = 1.2 + 0.9\ \sigma_{cp}/f_{ctm}$

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining forces and moments acting on the section**. The torsional moment that acts on the section is obtained from the CivilFEM results file.

Moment	Description
T_{Ed}	Design torsional moment in I-section.

4) **Checking crushing of concrete compressive struts.** First, it is necessary to check that the design torsional moment (T_{Ed}) is less than or equal to the maximum torsional moment that can be resisted by the concrete compressive struts ($T_{Rd,max}$):

$$T_{Ed} \leq T_{Rd,max}$$

$$T_{Rd,max} = 2 \cdot \nu \cdot \alpha_{cw} \cdot f_{cd} \cdot t \cdot A_{cd} \cdot \sin\theta \cdot \cos\theta$$

Where the values ν and α_{cw} are the same as the used previously.

Calculation results are written in the CivilFEM results file for both element ends as the parameters:

$$\text{TRDMAX} = \text{T}_{\text{Rd,max}}$$

$$\text{CRT\_1} = \frac{\text{T}_{\text{Ed}}}{\text{T}_{\text{Rd,max}}}$$

If the design torsional moment is greater than the moment required to crush the concrete compressive struts, the reinforcement design will not be feasible. As a result, the parameter for the reinforcement will contain a value of 2^{100}.

$$\text{ASTT} = \frac{\text{Ast}}{\text{S}} = 2^{100} \text{ for transverse reinforcement}$$

$$\text{ASLT} = \text{Asl} = 2^{100} \text{ for longitudinal reinforcement}$$

In this case, the element will be labeled as not designed.

If there is no crushing due to compression, the calculation process continues.

5) **Determining the required transverse reinforcement ratio.** The required transverse reinforcement is defined by this expression:

$$\frac{\text{A}_{\text{sw}}}{\text{S}} = \frac{\text{T}_{\text{Ed}}}{2\ \text{A}_{\text{k}}\text{f}_{\text{yd}}} \tan\theta$$

The area of the designed transverse reinforcement per unit length is stored in the CivilFEM results file as the parameter:

$$\text{ASTT} = \frac{\text{A}_{\text{sw}}}{\text{S}}$$

6) **Determining the required longitudinal reinforcement ratio.** The longitudinal reinforcement is calculated as:

$$\text{A}_{\text{sl}} = \frac{\text{T}_{\text{Ed}}\ \text{U}_{\text{k}}}{2\ \text{A}_{\text{k}}\ \text{f}_{\text{yd}}}$$

The area of the designed longitudinal reinforcement is stored in the CivilFEM results file as the parameter:

$$\text{ASLT} = \text{A}_{\text{sl}}$$

If both transverse and longitudinal reinforcements are designed for both element sections, this element will be labeled as designed.

Design criterion (DSG_CRT) is 1 (Ok) if the element was designed, 0 (Not OK) if not.

6.6.4.6. Combined Shear and Torsion Design

The design of sections subjected to shear force and concomitant torsional moment, follows the steps below:

1) **Torsion design considering a null shear force.** This design follows the same steps as for the design of elements subjected to pure torsion according to Eurocode 2 (EN 1992-1-1:2004/AC:2008) and ITER Design Code.
2) **Shear design considering a null torsion force.** This design is accomplished with the same steps as for the design of elements subjected to pure shear according to Eurocode 2 (EN 1992-1-1:2004/AC:2008) and ITER Design Code.
3) **Checking concrete ultimate strength condition.** The design torsional moment (T_{Ed}) and the design shear force (V_{Ed}) must satisfy the following condition:

$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \leq 1$$

4) **Obtaining required shear and torsion reinforcement ratios.** If the concrete ultimate strength condition is fulfilled (i.e. the concrete can resist the combined shear and torsion action) the reinforcements calculated in steps 1 and 2 are taken as the designed reinforcements. The element is then labeled as designed.

If the concrete ultimate strength condition is not fulfilled, the parameters corresponding to each type of reinforcement will take the value of 2^{100}.

The design criterion is 1 (Ok) if the element has been designed, and 0 if not.

6.6.5. Shear and Torsion according Structural Code (Spanish Code)

6.6.4.7. Shear Check

Checking elements for shear according to Structuaral Code (Annex 19) follows the steps below:

1) **Obtaining strength properties of the materials.** The required material properties associated with each transverse cross section at the active time are:

f_{ck} characteristic compressive strength of concrete.

f_{cd} design strength of concrete.

f_{yk} characteristic yield strength of reinforcement.

f_{ywd} design strength of shear reinforcement.

2) **Obtaining geometrical data of the section.** Required data for shear checking are the following ones:

A_c total cross-sectional area of the concrete section.

3) **Obtaining geometrical parameters depending on code.** Required data are as follows:

b_w minimum width of the section over the effective depth.

d effective depth of the section.

ρ_1 ratio of the tension longitudinal reinforcement

$$\rho_1 = \frac{A_{sl}}{b_w\ d} < 0.02$$

where:

A_{sl} the area of the tension reinforcement extending not less than $d + I_{b,net}$ beyond the section considered.

θ angle of the compressive struts of concrete with the member's longitudinal axis, (parameter THETA):

$0.5 \leq \text{cotan} \leq 2$

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section**. Required data are as follows:

α angle between shear reinforcement and the longitudinal axis of the member section, (parameter ALPHA).

A_{sw}/S area of reinforcement per unit length, (parameters ASSY or ASSZ).

The reinforcement ratio may also be obtained with the following data:

A_{sw} total area of the reinforcement legs, (parameters ASY or ASZ, both Y and Z directions are available).

s spacing of the stirrups.

or with the following ones:

s spacing of the stirrups.

ϕ diameter of bars, (parameter PHI).

N number of reinforcement legs, (parameters NY or NZ for Y and Z directions).

5) **Obtaining the section's internal forces and moments**. The shear force that acts on the section, as well as the concomitant axial force and bending moment, are obtained from the CivilFEM results file (.RCF).

Force	Description
V_{Ed}	Design shear force (≥ 0)
N_{Ed}	Design axial force (positive for compression)
M_{Ed}	Design bending moment (≥ 0)

6) **Checking whether the section requires shear reinforcement.** First, the design shear (V_{Ed}) is compared to the design shear resistance ($V_{Rd,c}$):

$$V_{Ed} \leq V_{Rd,c}$$

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100\rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

with the constraints:

$$V_{Ed} \leq 0.5 b_w \cdot d \cdot \nu \cdot f_{cd}$$

$$V_{Rd,c} \geq \left[V_{min} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

where:

$C_{Rd,c}$ = $0.18\,\gamma_c$

f_{ck} = in MPa

k = $1 + \sqrt{\frac{200}{d}} \leq 2$ (d en mm)

k_1 = 0.15

σ_{cp} = $\frac{N_{Ed}}{A_c} < 0.2 f_{cd}$ Mpa

A_c in mm^2

ν = $0.6\left(1 - \frac{f_{ck}}{250}\right)$

V_{min} = $0.035(k^3 \cdot f_{ck})^{1/2}$

$V_{Rd,c}$ in N

If shear reinforcement has not been defined for the section, a check is made to ensure V_{Ed} is less than the lowest value between the shear reinforcement resistance,

$$V_{Rd,s} = \frac{A_{sw}}{s} \cdot 0.9 \cdot d \cdot f_{ywd} \cdot (\text{cotan}\theta + \text{cotan}\alpha) \cdot \sin\alpha$$

and the maximum design shear reinforcement resistance:

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot 0.9 \cdot d \cdot \nu \cdot f_{cd} \cdot \frac{(\text{cotan}\theta + \text{cotan}\alpha)}{1 + \text{cotan}^2\theta}$$

Where :

$$\alpha_{cw} = \begin{cases} 1 + \dfrac{\sigma_{cp}}{f_{cd}}, \text{if } 0 < \sigma_{cp} \le 0.25 f_{cd} \\ 1.25, \text{if } 0.25 f_{cd} < \sigma_{cp} \le 0.5 f_{cd} \\ 2.5\left(1 - \dfrac{\sigma_{cp}}{f_{cd}}\right), \text{if } 0.5 f_{cd} < \sigma_{cp} \le f_{cd} \end{cases}$$

The shear reinforcement must be equal to or less than

$$\frac{A_{sw,max}}{S} = 0.5 \cdot \alpha_{cw} \cdot b_w \cdot \nu \cdot \frac{f_{cd}}{f_{ywd}} / \sin\alpha$$

Results are written for each end in the CivilFEM results file as the following parameters:

VRDC = $V_{Rd,c}$

VRDS = $V_{Rd,s}$

VRDMAX = $V_{Rd,max}$

TENS = $\frac{M_{Ed}}{0.9d} + \frac{V_{Ed}}{2}(\cot\theta - \cot\alpha)$

Tension resistance of the longitudinal reinforcement

CRT_1 = $\frac{V_{Ed}}{V_{Rd,c}}$

CRT_2 = 0, If there is no shear reinforcement.

$\frac{V_{Ed}}{V_{Rd,s}}$, If there is shear reinforcement.

CRT_3 = 0, If there is no shear reinforcement.

$\frac{V_{Ed}}{V_{Rd,max}}$, If there is shear reinforcement.

8) **Obtaining shear criterion.** The shear criterion indicates whether the section is valid for the design forces (if it is less than 1, the section satisfies the code provisions; whereas if it exceeds 1, the section will not be valid). Furthermore, it includes information pertaining to how close the design force is to the ultimate section strength. The shear criterion is defined as follows:

CRT_TOT = $\frac{V_{Ed}}{V_{Rd,c}}$, If there is no shear reinforcement

$\min\left\{\frac{V_{Ed}}{V_{Rd,c}}, \max\left\{\frac{V_{Ed}}{V_{Rd,s}}\right.\right.$ If there is shear reinforcement

$\left.\left.\frac{V_{Ed}}{V_{Rd,max}}\right\}\right\}$,

A value of 2^{100} for this criterion indicates that $V_{Rd2,red}$ or V_{Rd3} are equal to zero.

6.6.4.8. *Torsion Check*

The torsion checking according to Structuaral Code (Annex 19) follows the steps below:

1) **Obtaining strength properties of the materials.** These properties are obtained from the material properties associated to the transverse cross section and for the active time.

The Required data are as follows:

f_{ck} characteristic strength of concrete.

f_{cd} calculation strength of concrete.

f_{yk} characteristic yield strength of reinforcement.

f_{yd} calculation torsion resistance of reinforcement. The same material is considered for transverse and longitudinal reinforcement

2) **Obtaining geometrical parameters depending on specified code. The required data are as follows:**

t	equivalent thickness of wall.
A_k	area enclosed within the centre-line of the thin-walled cross-section.
u_k	circumference of area A_k.
θ	Angle of the compressive struts of concrete with the member's longitudinal axis:

$$0.5 \leq \cot\theta \leq 2$$

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining reinforcement data of the section.** Required data are as follows:

Transverse reinforcement

A_{sw}/S area of transverse reinforcement per unit length.

The reinforcement ratio can alternatively be defined using the following data:

A_{sw} closed stirrups area for torsion.

s spacing of closed stirrups.

Or with the following data:

s spacing of closed stirrups.

Φ_t diameter of the closed stirrups.

Longitudinal Reinforcement

A_{sl} total area of the longitudinal reinforcement.

The reinforcement ratio can alternatively be defined using the following data:

Φ_I diameter of longitudinal bars.

N number of longitudinal bars.

4) **Obtaining section internal forces and moments**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	Description
T_{sd}	Design torsional moment

5) **Calculating the maximum torsional moment that can be resisted by the concrete compressive struts.** The design torsional moment (T_{sd}) must be less than or equal to the maximum torsional moment that can be resisted by the concrete compressive struts ($T_{Rd,max}$); therefore, the following condition must be fulfilled:

$$T_{Ed} \leq T_{Rd,max}$$

$$T_{Rd,max} = 2 \cdot \nu \cdot \alpha_{cw} \cdot f_{cd} \cdot t \cdot A_k \cdot \sin\theta \cdot \cos\theta$$

Where the values de of ν and $\alpha_{cw}$are the same as those used in shear checking,

Results are written in the CivilFEM results file for both element ends as the parameters:

$$\text{TRDMAX} = T_{Rd,max}$$

$$\text{CRT\_1} = \frac{T_{Ed}}{T_{Rd,max}}$$

6) **Calculating the maximum torsional moment that can be resisted by the reinforcement.** The design torsional moment (T_{Ed}) must be less than or equal to the maximum design torsional moment that can be resisted by the reinforcement (T_{Rd}); consequently, the following condition must be fulfilled:

$$T_{Ed} \leq T_{Rd}$$

$$T_{Rd} = 2A_k \left(f_{yd} \cdot \frac{A_{sw}}{S} \right) \cot\text{an}\theta$$

Calculation results are written in the CivilFEM results file for both element ends as the parameters:

$$\text{TRD} = T_{Rd}$$

$$\text{CRT\_2} = \frac{T_{Ed}}{T_{Rd}}$$

If transverse reinforcement is not defined, $T_{Rd2} = 0$ and the criterion will take the value of 2^{100}.

7) **Calculating the required longitudinal reinforcement.** The required longitudinal reinforcement is calculated from T_{Rd} as follows:

$$A_{sl,nec} = T_{Rd}\frac{U_k}{2A_k f_{yd}}\cot an\theta$$

If longitudinal reinforcement is not defined, $A_{sl} = 0$ and the criterion will be 2^{100}.

$$ALT = A_{sl,nec}$$

$$CRTALT = \frac{A_{sl,nec}}{A_{sl}}$$

8) **Obtaining torsion criterion.** The torsion criterion is defined as the ratio of the design moment to the section ultimate resistance: if it is less than 1, the section is valid; whereas if it exceeds 1, the section is not valid. The criterion pertaining to the validity for torsion is defined as follows:

$$CRT\_TOT = \max\left(\frac{T_{Ed}}{T_{Rd,max}}, \frac{T_{Ed}}{T_{Rd}}, \frac{A_{sl,nec}}{A_{sl}}\right) \leq 1$$

This value is stored in the CivilFEM results file for each end.

A value 2^{100} for this criterion indicates that any one of the torsion reinforcement groups are undefined.

6.6.4.9. Combined Shear and Torsion Check

For checking sections subjected to shear force and concomitant torsional moment, we follow the steps below:

5) **Torsion checking considering a null shear force.** This check follows the same procedure as for the check of elements subjected to pure torsion according to Structuaral Code (Annex 19).
6) **Shear checking assuming a null torsional moment. .** This check follows the same steps as for the check of elements subjected to pure shear according to Structuaral Code (Annex 19).

7) **Checking the concrete ultimate strength condition.** The design torsional moment (T_{Sd}) and the design shear force (V_{Sd}) must satisfy the following condition:

$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \leq 1$$

8) **Obtaining the combined shear and torsion criterion.** This criterion comprehends pure shear, pure torsion and ultimate strength condition criteria of concrete. The criterion determines whether the section is valid and is defined as follows:

$$CRT\_TOT = \max\left\{\frac{V_{Ed}}{V_{Rd,s}}, \frac{V_{Ed}}{V_{Rd,max}}, \frac{T_{Ed}}{T_{Rd,max}}, \frac{T_{Ed}}{T_{Rd}}, \frac{A_{sl,nec}}{A_{sl}}, \frac{T_{Ed}}{T_{Rd}} + \frac{V_{Ed}}{V_{Rd}}\right\} \leq 1$$

A value 2^{100} for this criterion indicates that $V_{Rd1,red}$ or V_{Rd3} are equal to zero or that one of the torsion reinforcement groups has not been defined.

6.6.4.10. Shear Design

Shear reinforcement design according to Structuaral Code (Annex 19) follows the steps below:

1) **Obtaining strength properties of the materials.** The required material properties associated with each transverse cross section at the active time are:

f_{ck} characteristic strength of concrete.

f_{cd} characteristic design strength of concrete.

f_{yk} characteristic yield strength of reinforcement.

f_{ywd} design strength of shear reinforcement.

2) **Obtaining geometrical data of the section.** Required data for shear design are the following:

A_c total cross-sectional area of the concrete section.

3) **Obtaining geometrical parameters depending on specified code.** Required data are as follows:

b_w minimum width of the section over the effective depth.

d effective depth of the section.

ρ_1 ratio of the longitudinal tensile :

$$\rho_1 = \frac{A_{sl}}{b_w d} < 0.02$$

where:

A_{sl} the area of the tensile reinforcement extending not less than $d + I_{b,net}$ beyond the section considered.

θ angle of the compressive struts of concrete with the member's longitudinal axis:

$0.5 \leq \cot\theta \leq 2$

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** For shear reinforcement design, it is possible to define the angle ▯▯between the reinforcement and the longitudinal axis of member can be indicated. This angle should be included in the reinforcement definition of each element. If this angle is null or is not defined, ▯=90° is used. Other reinforcement data will be ignored.

5) **Obtaining forces and moments acting on the section**. The shear force that acts on the section as well as the concomitant axial force and bending moment, are obtained from the CivilFEM results file (.RCF).

Force	**Description**
V_{Ed}	Design shear force
N_{Ed}	Design axial force (positive for compression)
M_{Ed}	Design bending moment (≥ 0)

6) **Checking whether the section requires shear reinforcement.** First, the design shear (V_{Ed}) is compared to the design shear resistance ($V_{Rd,c}$):

$$V_{Ed} \leq V_{Rd,c}$$

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100\, \rho_1 \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

with the constraints:

$$V_{Ed} \leq 0.5 b_w \cdot d \cdot \nu \cdot f_{cd}$$

$$V_{Rd,c} \geq \left[V_{min} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

where:

$C_{Rd,c}$ = $0.18\,\gamma_c$

f_{ck} in MPa

k = $1 + \sqrt{\frac{200}{d}}$ (d in mm)

ρ_1 = $\frac{A_{sl}}{b_w\, d} \leq 0.02$

k_1 = 0.15

σ_{cp} =

$$\frac{N_{Ed}}{A_c} < 0.2f_{cd} \text{ MPa}$$

A_c in mm^2

ν = $0.6\left(1 - \frac{f_{ck}}{250}\right)$

V_{min} = $0.035(k^3 \cdot f_{ck})^{1/2}$

$V_{Rd,c}$ in N

Results are written for each element end in the CivilFEM results file as the parameters:

$$VRDC = V_{Rd,c}$$

$$CRT\_1 = \frac{V_{Ed}}{V_{Rd,c}}$$

7) **Calculating the maximum shear force that can be resisted by the concrete compressive struts.**

A check is made to ensure that V_{Ed} is less than $V_{Rd,max}$:

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot 0.9 \cdot d \cdot \nu \cdot f_{cd} \cdot \frac{(\cot\theta + \cot\alpha)}{(1 + \cot^2\theta)}$$

where:

$$\alpha_{cw} = \begin{cases} 1, \text{if the section is not prestessing} \\ 1 - \frac{\sigma_{cp}}{f_{cd}}, \text{if } 0 < \sigma_{cp} \leq 0.25f_{cd} \\ 1.25, \text{if } 0.25f_{cd} < \sigma_{cp} \leq 0.5f_{cd} \\ 2.5\left(1 - \frac{\sigma_{cp}}{f_{cd}}\right), \text{if } 0.5f_{cd} < \sigma_{cp} \leq f_{cd} \end{cases}$$

α = 90º if shear reinforcement was determined as not necessary in the previous step. If reinforcement is necessary, the angle α will be read from in the reinforcement definition data.

Results are written for each element end in the CivilFEM results file as the parameters:

$$\mathrm{VRDMAX} = V_{Rd,max}$$

$$\mathrm{CRT\_3} = \frac{V_{Ed}}{V_{Rd,max}}$$

If design shear force is greater than the force required to crush the concrete compressive struts, the reinforcement design will not be feasible, so the parameter containing this datum will be marked with 2^{100}.

If the struts are not crushed by oblique compression, the calculating process continues.

8) **Calculating required amount of transverse reinforcement.** The section validity condition pertaining to shear force is:

$$V_{Rd,s} = \begin{cases} V_{Ed} & \text{if } V_{Ed} < V_{Rd,max} \\ V_{Rd,max} & \text{if } V_{Ed} \geq V_{Rd,max} \end{cases}$$

Therefore, the reinforcement amount per length unit should be:

$$\frac{A_{sw}}{S} = \frac{V_{Rd,s}}{0.9 \cdot d \cdot f_{ywd} \cdot (\cot\theta + \cot\alpha) \cdot \sin\alpha}$$

While also satisfying the following condition (Eurocode 2 only):

$$\frac{A_{sw}}{S} \leq 0.5 \cdot \alpha_{cw} \cdot b_w \cdot \nu \cdot \frac{f_{cd}}{f_{ywd}} / \sin\alpha$$

If the design is not possible, the reinforcement will be defined as 2^{100} and labeled as not designed.

The design criterion will be 1 (Ok) if the element was designed or 0 (Not Ok) if not.

For each element end, the results are included in the CivilFEM results file as the following parameters:

$$\mathrm{VRDS} = V_{Rd,s}$$

$$\mathrm{ASSH} = \frac{A_{sw}}{S}$$

DSG_CRT = Design criterion

6.6.4.11. Torsion Design

Torsion reinforcement design according to Structuaral Code (Annex 19) follows the steps below:

1) **Obtaining strength properties of the materials.** These properties are obtained from the material properties associated to each transverse cross section and for the active time. Those material properties should be previously defined. The Required data are as follows:

f_{ck}	characteristic strength of concrete.
f_{cd}	characteristic design strength of concrete.
f_{yk}	characteristic yield strength of reinforcement.
f_{yd}	design strength of shear reinforcement. The same material will be considered for transverse and longitudinal reinforcement.

2) **Obtaining geometrical parameters depending on specified code.** The required data are as follows:

t	equivalent thickness of wall.
A_k	area enclosed within the centre-line of the thin-walled cross-section.
U_k	circumference of area A_k .
θ	angle between the concrete compressive struts and the longitudinal axis of the member:

$$0.5 \leq \text{cotan}\ \theta \leq 2.0$$

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining forces and moments acting on the section**. The torsional moment that acts on the section is obtained from the CivilFEM results file.

Moment	**Description**
T_{Ed}	Design torsional moment in I-section.

4) **Checking crushing of concrete compressive struts.** First, it is necessary to check that the design torsional moment (T_{Ed}) is less than or equal to the maximum torsional moment that can be resisted by the concrete compressive struts ($T_{Rd,max}$):

$$T_{Ed} \leq T_{Rd,max}$$
$$T_{Rd,max} = 2 \cdot \nu \cdot \alpha_{cw} \cdot f_{cd} \cdot t \cdot A_{cd} \cdot \sin\theta \cdot \cos\theta$$

Where the values ν and α_{cw} are the same as the used previously.

Calculation results are written in the CivilFEM results file for both element ends as the parameters:

$$TRDMAX = T_{Rd,max}$$
$$CRT\_1 = \frac{T_{Ed}}{T_{Rd,max}}$$

If the design torsional moment is greater than the moment required to crush the concrete compressive struts, the reinforcement design will not be feasible. As a result, the parameter for the reinforcement will contain a value of 2^{100}.

$$ASTT = \frac{Ast}{s} = 2^{100} \text{ for transverse reinforcement}$$
$$ASLT = Asl = 2^{100} \text{ for longitudinal reinforcement}$$

In this case, the element will be labeled as not designed.

If there is no crushing due to compression, the calculation process continues.

5) **Determining the required transverse reinforcement ratio.** The required transverse reinforcement is defined by this expression:

$$\frac{A_{sw}}{S} = \frac{T_{Ed}}{2\ A_k f_{yd}} \tan\theta$$

The area of the designed transverse reinforcement per unit length is stored in the CivilFEM results file as the parameter:

$$ASTT = \frac{A_{sw}}{S}$$

6) **Determining the required longitudinal reinforcement ratio.** The longitudinal reinforcement is calculated as:

$$A_{sl} = \frac{T_{Ed}\ U_k}{2\ A_k\ f_{yd}}$$

The area of the designed longitudinal reinforcement is stored in the CivilFEM results file as the parameter:

$$ASLT = A_{sl}$$

If both transverse and longitudinal reinforcements are designed for both element sections, this element will be labeled as designed.

Design criterion (DSG_CRT) is 1 (Ok) if the element was designed, 0 (Not OK) if not.

6.6.4.12. Combined Shear and Torsion Design

The design of sections subjected to shear force and concomitant torsional moment, follows the steps below:

1) **Torsion design considering a null shear force.** This design follows the same steps as for the design of elements subjected to pure torsion according to Structuaral Code (Annex 19).
2) **Shear design considering a null torsion force.** This design is accomplished with the same steps as for the design of elements subjected to pure shear according to Structuaral Code (Annex 19).
3) **Checking concrete ultimate strength condition.** The design torsional moment (T_{Ed}) and the design shear force (V_{Ed}) must satisfy the following condition:

$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \leq 1$$

4) **Obtaining required shear and torsion reinforcement ratios.** If the concrete ultimate strength condition is fulfilled (i.e. the concrete can resist the combined shear and torsion action) the reinforcements calculated in steps 1 and 2 are taken as the designed reinforcements. The element is then labeled as designed.

If the concrete ultimate strength condition is not fulfilled, the parameters corresponding to each type of reinforcement will take the value of 2^{100}.

The design criterion is 1 (Ok) if the element has been designed, and 0 if not.

6.6.6. Shear and Torsion according to ACI 318-05

Strength reduction factor ϕ is taken as ϕ = 0.75 for shear and torsion according to Chapter 9.3.2 of *Building Code Requirements for Structural Concrete Structures (ACI 318-05)* document.

6.6.5.1. Shear Check

Shear checking according to ACI 318-05 is described in this section. These equations refer to US (British) units which include force, length, and time units of lb, in, and sec.

1) **Obtaining strength properties of the materials.** The required material properties associated with each transverse cross section at the active time are:

   f'_c specified compressive strength of concrete.

   f_{yk} specified yield strength of reinforcement.

2) **Obtaining geometrical data of the section.** Required data for shear checking:

   A_g area of concrete section.

3) **Obtaining geometrical parameters depending on specified code.** Required data:

   b_w web width or diameter of circular section.

   d distance from the extreme compressed fiber to the centroid of the longitudinal tensile reinforcement in the Y direction, (for circular sections this should not be less than the distance from the extreme compressed fiber to the centroid of the tensile reinforcement in the opposite half of the member).

   Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** Required data are as follows:

   α angle between shear reinforcement and the longitudinal axis of the member section.

   A_s/S area of the reinforcement per unit length (reinforcement ratio) in both the Y and Z directions.

   The reinforcement ratio may also be obtained with the following data:

   A_s total area of the reinforcement legs.

   s spacing of the stirrups.

   or with the following input:

   s spacing of the stirrups.

ϕ	diameter of bars.
N	number of reinforcement legs.

5) **Obtaining forces and moments acting on the section**. The forces that act on the section are obtained from the CivilFEM results file (.RCF).

Force	Description
V_u	Factored design shear force
N_u	Factored axial force occurring simultaneously to the shear force (positive for compression).

6) **Calculating the shear strength provided by concrete** for nonprestressed members. First, the shear strength provided by concrete (V_c) is calculated with the following expression:

$$V_c = 2\sqrt{f'_c} b_w\, d$$

where:

$\sqrt{f'_c}$	square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force,

$$V_c = 2\left(1 + \frac{N_u}{2000\, A_g}\right)\sqrt{f'_c} b_w\ d$$

If section is subjected to a tensile force so that the tensile stress is less than 500 psi,

$$V_c = 2\left(1 + \frac{N_u}{2000\, A_g}\right)\sqrt{f'_c} b_w\ d$$

If the section is subjected to a tensile force so that the tensile stress exceeds 500 psi, it is assumed $V_c = 0$.

The calculation result for both element ends is stored in the CivilFEM results file as the parameter VC:

VC	Shear strength provided by concrete.

$$VC = V_c$$

7) **Calculating the shear strength provided by shear reinforcement.** The strength provided by shear reinforcement (V_s) is calculated with the following expression:

$$V_s = \frac{A_s}{S} f_y\ (\sin\alpha + \cos\alpha) d \leq 8\sqrt{f'_c} b_w\ d$$

where:

f_y yield strength of the shear reinforcement (not greater than 60000 psi).

The calculation result for both element ends is stored in the CivilFEM results file as the parameter VS:

V_S Shear strength provided by transverse reinforcement.

8) **Calculating the nominal shear strength of section.** The nominal shear strength (V_n) is the summation of the provided by concrete and by the shear reinforcement:

$$V_n = V_c + V_s$$

This nominal strength as well as its ratio to the design shear are stored in the CivilFEM results file as the parameters:

VN Nominal shear strength.

$$VN = V_n$$

CRTVN Ratio of the design shear force (V_u) to the resistance V_n.

$$CRTVN \frac{V_u}{V_n}$$

If the strength provided by concrete is null, and the shear reinforcement is not defined in the section, then $V_n = 0$, and the criterion is equal to –1.

9) **Obtaining shear criterion.** The section will be valid for shear if the following condition is fulfilled:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

ϕ strength reduction factor of the section. ϕ = 0.75 for shear and torsion according to Chapter 9.3 of *Building Code Requirements for Structural Concrete Structures (ACI 318-05)* document.

Therefore, the validity shear criterion is defined as follows:

$$CRT\_TOT = \frac{V_u}{\phi V_n} \leq 1$$

For each element, this value is stored in the CivilFEM results file as the parameter CRT_TOT.

If the strength provided by concrete is null and the shear reinforcement is not defined in the section, then $V_n = 0$, and the criterion is equal to 2^{100}.

The $\phi \cdot V_n$ value is stored in CivilFEM results file as the parameter VFI.

6.6.5.2. Torsion Check

The torsion checking according to ACI 318-05 is described in this section. These equations refer to US (British) units which include force, length, and time units of lb, in, and sec.

1) **Obtaining strength properties of the materials.** These properties are obtained from the material properties associated with each transverse cross section at the active time:

   f'_c specified compressive strength of concrete.

   f_{yk} specified yield strength of reinforcement.

2) **Obtaining geometrical parameters depending on specified code.** The required data are as follows:

   b_w web width or diameter of circular section.

   d distance from the extreme compression fiber to the centroid of the longitudinal tensile reinforcement in Y, (for circular sections this should not be less than the distance from the extreme compression fiber to the centroid of the tensile reinforcement in the opposite half of the member).

   A_{cp} area enclosed by outside perimeter of concrete cross section.

   P_{cp} outside perimeter of the concrete cross section.

   A_{oh} area enclosed by centerline of the outermost closed transverse torsional reinforcement.

   P_h perimeter of centerline of outermost closed transverse torsional reinforcement.

   A_o gross area enclosed by shear flow path.

   Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining reinforcement data of the section.** Required data are as follows:

   Transverse Reinforcement

   $A_{st/S}$ area of transverse reinforcement per unit of length.

   The reinforcement ratio can alternatively be defined using the following data:

   A_{st} closed stirrups area for torsion.

   s spacing of closed stirrups.

Or with the following data:

s spacing of closed stirrups.

Φ_t diameter of the closed stirrups.

Longitudinal Reinforcement

A_{sl} total area of the longitudinal reinforcement.

The reinforcement ratio can alternatively be defined using the following data:

ϕ_l diameter of longitudinal bars.

N number of longitudinal bars.

4) **Obtaining section's internal forces and moments**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	**Description**
T_u	Factored design torsional moment.

5) **Checking if torsion effects will be considered.** Torsion effects are only considered if the design torsional moment (T_u) satisfies the condition below:

$$T_u > \phi\left(\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\right)$$

If the design torsional moment is less than this value, its effects can be neglected and it will be considered as null for checking.

Checking section dimensions. Section dimensions must satisfy the following requirements:

$$\frac{T_u P_h}{1.7\ A_{oh}^2} \leq \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right)$$

In hollow sections, if the section wall's thickness is less than A_{oh}/P_h, this value will be replaced by the minimum thickness of the section in the previous formula.

The ratio of the two coefficients is stored in the CivilFEM results file for both element ends as the parameter:

$$CRTTC = \frac{\frac{T_u P_h}{1.7\ A_{oh}^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

6) Calculating the nominal torsional moment strength of the section. The nominal torsional moment strength (T_n) is evaluated with the following expression:

$$T_n = 2\ A_c \frac{A_{st}}{S} f_y$$

where:

f_y specified yield strength of torsional reinforcement (not greater than 60,000 psi).

This nominal torsional moment strength and its ratio to the design shear force are stored in the CivilFEM results file for both element ends as the parameters:

TN Nominal torsional moment strength.

$$TN = T_n$$

CRTTN Ratio of the design torsional moment (T_u) to the torsional moment strength T_n .

$$CRTTN = \frac{T_u}{T_n}$$

The required longitudinal reinforcement area is given by:

$$(A_l)_{nec} = \frac{A_{st}}{S} P_h$$

Calculation results are stored in the CivilFEM results file for both element ends as the parameters:

ALT Area of longitudinal torsion reinforcement required in accordance with the transverse torsion reinforcement defined.

$$ALT = (A_l)_{nec}$$

CRTALT Ratio of the area of longitudinal torsion reinforcement required to the area of longitudinal torsion reinforcement defined.

$$CRTALT = \frac{(A_l)_{nec}}{A_{sl}}$$

If longitudinal reinforcement is not defined, then $A_{sl} = 0$ and the criterion is equal to 2^{100}.

7) Obtaining torsion criterion. The section will be valid for torsion if the following condition is fulfilled:

$$T_u \leq \Phi T_n$$

$$A_{sl} \geq (A_l)_{nec}$$

$$\frac{T_u P_h}{1.7\ A_{ch}^2} \leq \phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)$$

ϕ strength reduction factor of the section, (0.75 for shear and torsion).

Therefore, the validity torsion criterion is defined as follows:

$$CRT\_TOT = Max\left(\frac{T_u}{\phi\ T_n}\ ;\ \frac{(A_l)_{nec}}{A_{sl}}\ ;\ \frac{\frac{T_u P_h}{1.7\ A_{oh}^2}}{\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}\right) \leq 1$$

For each element end, this value is stored in the CivilFEM results file as CRT_TOT.

If the strength provided by concrete is null and the torsion reinforcement is not defined in the section, the criterion will be 2^{100}.

The $\phi \cdot T_n$ value is stored in the CivilFEM results file for both element ends as the parameter TFI.

6.6.5.3. Combined Shear and Torsion Check

For checking sections subjected to shear force and concomitant torsional moment, the following steps are taken:

1) Checking if torsion effects will be considered. Torsion effects are only considered if the design torsional moment (T_u) satisfies the condition below:

$$T_u > \phi\left(\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\right)$$

If the design torsional moment is less than this value, its effects can be neglected and it is considered as null for checking.

2) Checking section dimensions. For shear force and the associated torsional moment, section dimensions must satisfy the following requirements:

a) Solid sections:

$$\sqrt{\left(\frac{V_u}{b_w\ d}\right)^2+\left(\frac{T_u P_h}{1.7\ A_{oh}^2}\right)^2}\le \phi\left(\frac{V_c}{b_w\ d}+8\sqrt{f'_c}\right)$$

b) Hollow sections:

$$\left(\frac{V_u}{b_w\ d}\right)+\left(\frac{T_u P_h}{1.7A_{oh}^2}\right)\le \phi\left(\frac{V_c}{b_w\ d}+8\sqrt{f'_c}\right)$$

In hollow sections, if the section wall's thickness is less than A_{oh}/P_h , this value is replaced in the expression above by the section's minimum thickness.

The ratio between these two factors is stored in the CivilFEM results file for both element ends.

a) Solid sections:

$$CRTTC=\frac{\sqrt{\left(\frac{V_u}{b_w\ d}\right)^2+\left(\frac{T_u P_n}{A_{oh}^2}\right)^2}}{\phi\left(\frac{V_c}{b_w\ \ d}+8\sqrt{f'_c}\right)}$$

b) Hollow sections:

$$CRTTC=\frac{\frac{V_u}{b_w\ d}+\frac{T_u P_n}{1.7\ A_{oh}^2}}{\phi\left(\frac{V_c}{b_w\ \ d}+8\sqrt{f'_c}\right)}$$

3) **Checking for shear force with concomitant torsional moment.** This check is accomplished with the same steps as the check of elements subjected to pure shear force according to ACI 318-05. The same results as for shear checking will be calculated.

   Except for this check, the CRT_TOT criterion is stored in the CivilFEM results file as CRTSHR for each element end.

4) **Checking for torsion with shear force.** This check follows the same steps considered for the check of elements subjected to pure torsion according to ACI 318-05. The same results as in torsion checking will be calculated.

Except for this check, the CRT_TOT criterion is stored in the CivilFEM results file as CRTTRS for each element end.

5) **Obtaining the combined shear and torsion criterion.** This criterion determines whether the section is valid or not. It is defined as follows:

$$\mathrm{CRT\_TOT} = \mathrm{Max}\left[\frac{V_u}{\phi\, V_n};\frac{T_u}{\phi\, T_n};\frac{(A_l)_{nec}}{A_{sl}};\mathrm{CRTTC}\right] \leq 1$$

For each end, this value is stored in the CivilFEM results file.

A value equal to 2^{100} for this criterion indicates:

- the shear strength provided by concrete is equal to zero and the shear reinforcement has not been defined.
- the shear strength provided by concrete is equal to zero and the transverse torsion reinforcement has not been defined.
- the longitudinal torsion reinforcement has not been defined.

6.6.5.4. Shear Design

The shear designing according to ACI 318-05 is described in this section. These equations refer to US (British) units which include force, length, and time units of lb, in, and sec.

1) **Obtaining strength properties of the materials.** The required material properties associated with each transverse cross section at the active time are:

f'_c specified compressive strength of concrete.

f_{yk} specified yield strength of reinforcement.

2) **Obtaining geometrical data of the section.** Required data for shear designing are the following ones:

A_g area of concrete section.

3) **Obtaining geometrical parameters depending on specified code.** The Required data are as follows:

b_w web width or diameter of the circular section.

d distance from the extreme compressed fiber to the centroid of the longitudinal tension reinforcement in Y, (for circular sections this should not be less than the distance from the extreme compressed fiber to the centroid of the tension reinforcement in the opposite half of the member).

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** For shear reinforcement design, it is possible to define the angle α between the reinforcement and the longitudinal axis of the member. This angle must be stored in the shear reinforcement data of each element. If this angle is equal to zero or it is not defined, □□= 90º. Other data pertaining to reinforcements will be ignored.

5) **Obtaining forces and moments acting on the section.** The shear force that acts on the section as well as the concomitant axial force are obtained from the CivilFEM results file (.RCF).

Force	Description
V_u	Design shear force
Nu	Axial force (positive for compression)

6) **Calculating the shear strength provided by concrete.** The shear strength provided by concrete (V_c) is calculated with the following expression:

$$V_c = 2\sqrt{f'_c} b_w\, d$$

where:

$\sqrt{f'_c}$	square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force,

$$V_c = 2\left(1 + \frac{N_u}{2000\, A_g}\right)\sqrt{f'_c} b_w\ \ d$$

If the section is subjected to a tensile force so that the tensile stress is less than 500 psi,

$$V_c = 2\left(1 + \frac{N_u}{500\, A_g}\right)\sqrt{f'_c} b_w\ \ d$$

If the section is subjected to a tensile force so that the tensile stress exceeds 500 psi, it is assumed $V_c = 0$.

The calculation result is stored in the CivilFEM results file for both element ends as the parameter:

VC	Shear strength provided by concrete: $VC = V_c$

7) **Calculating the required reinforcement contribution to the shear strength.** The section must satisfy the following condition to resist the shear force:

$$V_u \leq \phi\, V_n = \phi(V_c + V_s)$$

Therefore, the reinforcement shear resistance must satisfy:

$$V_s = \frac{V_u}{\phi} - V_c \leq 8\sqrt{f'_c} b_w\, d$$

If the shear resistance of the reinforcement does not satisfy the expression above, the section cannot be designed. As a result, the parameters for the reinforcement ratio will be equal to 2^{100}.

$$ASSH = \frac{A_s}{S} = 2^{100}$$

For this case, the element will be labeled as not designed.

Calculation results are stored in the CivilFEM results file for both element ends as the parameter:

VS Shear resistance provided by the transverse reinforcement.

$$VS = V_s \leq 8\sqrt{f'_c}\, b_w\, d$$

8) **Calculating the required reinforcement ratio.** Once the shear resistance of the reinforcement has been obtained, the reinforcement can be calculated with the following expression:

$$\frac{A_s}{S} = \frac{V_s}{f_y(\sin\alpha + \cos\alpha)d}$$

Where:

A_s area of the cross-section of the shear reinforcement.

s spacing of the stirrups measured along the longitudinal axis.

f_y yield strength of the shear reinforcement (not greater than 60000 psi).

The area of the designed reinforcement per unit length is stored in the CivilFEM results file for both element ends:

$$ASSH = \frac{A_s}{S}$$

In this case, the element will be labeled as designed (providing the design process is correct for both element sections).

6.6.5.5. Torsion Design

The torsion designing according to ACI 318-05 is described in this section. These equations refer to US (British) units which include force, length, and time units of lb, in, and sec.

1) **Obtaining strength properties of the materials.** These properties are obtained from the material properties associated with each transverse cross section at the active time.

f'_c specified compressive strength of concrete.

f_{yk} specified yield strength of reinforcement.

2) **Obtaining geometrical parameters depending on specified code.** The required data are as follows:

b_w web width or diameter of the circular section.

d distance from the extreme compressed fiber to the centroid of the longitudinal tensile reinforcement in Y, (for circular sections this should not be less than the distance from the extreme compressed fiber to the centroid of the tensile reinforcement in the opposite half of the member).

A_{cp} Area enclosed by outside perimeter of concrete cross section.

P_{cp} Outside perimeter of the concrete cross section.

A_{oh} Area enclosed by centerline of the outermost closed transverse torsional reinforcement.

P_h Perimeter of centerline of outermost closed transverse torsional reinforcement.

A_o Gross area enclosed by shear flow path.

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining forces and moments acting on the section**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	**Description**
T_u	Design torsional moment in I section.

4) **Checking if torsion effects will be considered.** Torsion effects are only considered if the design torsional moment (T_u) satisfies the condition below:

$$T_u > \phi\left(\sqrt{f'_c}\,\frac{A_{cp}^2}{P_{cp}}\right)$$

If the design torsional moment is less than this value, its effects can be neglected and it is consider as null for the design.

5) **Checking section dimensions.** Section dimensions must satisfy the following requirements:

$$\frac{T_u P_h}{1.7A_{oh}^2} \leq \phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)$$

For hollow sections, if the thickness of the section walls is less than A_{oh}/P_h, this value will be replaced by the minimum thickness of the section in the equation above.

The torsion reinforcement will not be designed if the previous expression is not fulfilled; consequently, the parameters for the reinforcement will be equal to 2^{100}.

$$ASTT = \frac{A_{st}}{s} = 2^{100} \quad \text{for transverse reinforcement}$$

$$ASLT = A_{sl} = 2^{100} \quad \text{for longitudinal reinforcement}$$

In this case the element will be marked as not designed and it will be stored in the TRS_NOT OK component.

The ratio of the two coefficients is stored in the CivilFEM results file for both element ends:

$$CRTTC = \frac{\dfrac{T_u P_h}{1.7\ A_{oh}^2}}{\phi\left(\dfrac{V_c}{b_w\ b} + 8\sqrt{f'_c}\right)}$$

6) **Calculating the required transverse reinforcement.** In order to resist the torsional moment, the section must satisfy the condition below:

$$T_u \leq \phi\ T_n = \phi\left(2A_0\,\frac{A_t}{S}\,f_y\right)$$

A_t cross-sectional area of one leg of a closed stirrup resisting torsion.

s spacing of the stirrups.

Therefore, the required transverse torsion reinforcement is:

$$\frac{A_t}{S} = \frac{T_u}{\phi\ 2\ A_0\ f_y}$$

The area of designed transverse reinforcement per unit length is stored in the CivilFEM results file for both element ends:

$$ASTT = \frac{A_t}{S}$$

7) Calculating the required longitudinal reinforcement. The longitudinal reinforcement area is given by the following expression:

$$A_l = \frac{A_t}{S} P_h$$

The area of the designed longitudinal reinforcement is stored in the CivilFEM results file for both element ends:

$$ASLT = A_l$$

If transverse and longitudinal reinforcements are designed for both element ends, this element will be labeled as designed.

6.6.5.6. Combined Shear and Torsion Design

The designing of sections subjected to shear force and concomitant torsional moment, follows the steps below:

1) Checking if torsion effects will be considered. Torsion effects are only considered if the design torsional moment (T_u) satisfies the condition below:

$$T_u > \phi \left(\sqrt{f'_c} \frac{A_{cp}^2}{P_{cp}} \right)$$

If the design torsional moment is less than this value, its effects can be neglected and it is considered as null for the design.

2) Checking section dimensions. For shear force and concomitant torsional moment, section dimensions must satisfy the following requirements:

a) Solid sections:

$$\sqrt{\left(\frac{V_u}{b_w\, d}\right)^2 + \left(\frac{T_u P_h}{1.7\, A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_w\, d} + 8\sqrt{f'_c}\right)$$

b) Hollow sections:

$$\left(\frac{V_u}{b_w\ d}\right) + \left(\frac{T_u P_h}{1.7\ A_{oh}^2}\right) \leq \phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)$$

For hollow sections, if the section wall's thickness is less than A_{oh}/P_h, this value will be replaced by the minimum thickness of the section in the expression above.

The torsion reinforcement will not be designed if the expression above is not fulfilled; consequently, the parameters for the reinforcement will be equal to 2^{100}.

$$\text{ASTT} = \frac{A_{st}}{s} = 2^{100} \text{for transverse reinforcement}$$

$$\text{ASLT} = A_{sl} = 2^{100} \text{for longitudinal reinforcement}$$

In this case, the element will be marked as not designed.

The ratio of the two coefficients is stored in the CivilFEM results file for both element ends.

a) Solid sections:

$$\text{CRTTC} = \frac{\sqrt{\left(\frac{V_u}{b_w\ d}\right)^2 + \left(\frac{T_u P_h}{1.7\ A_{oh}^2}\right)^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

b) Hollow sections:

$$\text{CRTTC} = \frac{\left(\frac{V_u}{b_w\ d}\right)^2 + \left(\frac{T_u P_h}{1.7\ A_{oh}^2}\right)^2}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

3) **Shear design assuming a null torsional moment.** This design follows the same procedure as for the design of elements subjected to pure shear force according to ACI 318-05.

4) **Torsion design considering a null shear force.** This design is accomplished with the same steps as for the design of elements subjected to pure torsion according to ACI 318-05.

6.6.7. Shear and Torsion according to ACI 318-14

Strength reduction factor ϕ is taken as ϕ = 0.75 for shear and torsion according to Chapter 21.2.1 of *Building Code Requirements for Structural Concrete Structures (ACI 318-14)* document.

6.6.6.1. Shear Check

Shear checking according to ACI 318-14 is described in this section. These equations refer to US (British) units which include force, length, and time units of lb, in, and sec.

1) **Obtaining strength properties of the materials.** The required material properties associated with each transverse cross section at the active time are:

   f'_c specified compressive strength of concrete.

   f_{yk} specified yield strength of reinforcement.

   λ modification factor for lightweight concrete ($\lambda = 1.0$ default value).

2) **Obtaining geometrical data of the section.** Required data for shear checking:

   A_g area of concrete section.

3) **Obtaining geometrical parameters depending on specified code.** Required data:

   b_w web width or diameter of circular section.

   d distance from the extreme compressed fiber to the centroid of the longitudinal tensile reinforcement in the Y direction, (for circular sections this should not be less than the distance from the extreme compressed fiber to the centroid of the tensile reinforcement in the opposite half of the member).

   Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** Required data are as follows:

   α angle between shear reinforcement and the longitudinal axis of the member section.

   A_s/S area of the reinforcement per unit length (reinforcement ratio) in both the Y and Z directions.

   The reinforcement ratio may also be obtained with the following data:

   A_s total area of the reinforcement legs.

s spacing of the stirrups.

or with the following input:

s spacing of the stirrups.

ϕ diameter of bars.

N number of reinforcement legs.

5) **Obtaining forces and moments acting on the section**. The forces that act on the section are obtained from the CivilFEM results file (.RCF).

Force	**Description**
V_u	Factored design shear force
N_u	Factored axial force occurring simultaneously to the shear force (positive for compression).

6) **Calculating the shear strength provided by concrete** for nonprestressed members. First, the shear strength provided by concrete (V_c) is calculated with the following expression for sections without axial force:

$$V_c = 2\lambda\sqrt{f'_c} b_w\, d$$

where:

$\sqrt{f'_c}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force,

$$V_c = 2\left(1 + \frac{N_u}{2000\, A_g}\right)\lambda\sqrt{f'_c} b_w\ d$$

If section is subjected to significant tensile force,

$$V_c = 2\left(1 + \frac{N_u}{500\, A_g}\right)\lambda\sqrt{f'_c} b_w\ d$$

The calculation result for both element ends is stored in the CivilFEM results file as the parameter VC:

VC Shear strength provided by concrete.

$$VC = V_c$$

7) **Calculating the shear strength provided by shear reinforcement.** The strength provided by shear reinforcement (V_s) is calculated with the following expression:

$$V_s = \frac{A_s}{S} f_y \,(\sin\alpha + \cos\alpha)d \leq 8\sqrt{f'_c}b_w \;\text{ d and} \geq \; V_s = \frac{V_u}{\phi} - V_c$$

where:

f_y yield strength of the shear reinforcement (not greater than 60000 psi).

The calculation result for both element ends is stored in the CivilFEM results file as the parameter VS:

V_S Shear strength provided by transverse reinforcement.

8) **Calculating the nominal shear strength of section.** The nominal shear strength (V_n) is the summation of the provided by concrete and by the shear reinforcement:

$$V_n = V_c + V_s$$

This nominal strength as well as its ratio to the design shear are stored in the CivilFEM results file as the parameters:

VN Nominal shear strength.

$$VN = V_n$$

CRTVN Ratio of the design shear force (V_u) to the resistance V_n.

$$CRTVN \frac{V_u}{V_n}$$

If the strength provided by concrete is null, and the shear reinforcement is not defined in the section, then $V_n = 0$, and the criterion is equal to –1.

9) **Obtaining shear criterion.** The section will be valid for shear if the following condition is fulfilled:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

ϕ strength reduction factor of the section (0.75 for shear and torsion).

Therefore, the validity shear criterion is defined as follows:

$$CRT\_TOT = \frac{V_u}{\phi V_n} \leq 1$$

For each element, this value is stored in the CivilFEM results file as the parameter CRT_TOT.

If the strength provided by concrete is null and the shear reinforcement is not defined in the section, then $V_n = 0$, and the criterion is equal to 2^{100}.

The $\phi \cdot V_n$ value is stored in CivilFEM results file as the parameter VFI.

6.6.6.2. *Torsion Check*

The torsion checking according to ACI 318-14 is described in this section. These equations refer to US (British) units which include force, length, and time units of lb, in, and sec.

1) **Obtaining strength properties of the materials.** These properties are obtained from the material properties associated with each transverse cross section at the active time:

   f'_c specified compressive strength of concrete.

   f_{yk} specified yield strength of reinforcement.

   λ modification factor for lightweight concrete.

2) **Obtaining geometrical parameters depending on specified code.** The required data are as follows:

   b_w web width or diameter of circular section.

   d distance from the extreme compression fiber to the centroid of the longitudinal tensile reinforcement in Y, (for circular sections this should not be less than the distance from the extreme compression fiber to the centroid of the tensile reinforcement in the opposite half of the member).

   A_{cp} area enclosed by outside perimeter of concrete cross section.

   P_{cp} outside perimeter of the concrete cross section.

   A_{oh} area enclosed by centerline of the outermost closed transverse torsional reinforcement.

   P_h perimeter of centerline of outermost closed transverse torsional reinforcement.

   A_o gross area enclosed by shear flow path.

   Section 6.6.1.. provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining reinforcement data of the section.** Required data are as follows:

   Transverse Reinforcement

   $A_{st/S}$ area of transverse reinforcement per unit of length.

   The reinforcement ratio can alternatively be defined using the following data:

A_{st} closed stirrups area for torsion.

s spacing of closed stirrups.

Or with the following data:

s spacing of closed stirrups.

Φ_t diameter of the closed stirrups.

Longitudinal Reinforcement

A_{sl} total area of the longitudinal reinforcement.

The reinforcement ratio can alternatively be defined using the following data:

ϕ_l diameter of longitudinal bars.

N number of longitudinal bars.

4) **Obtaining section's internal forces and moments**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	Description
T_u	Factored design torsional moment.

5) **Checking if torsion effects will be considered.** Torsion effects are only considered if the design torsional moment (T_u) satisfies the condition below:

$$T_u > \phi\left(\lambda\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\right)$$

If the design torsional moment is less than this value, its effects can be neglected and it will be considered as null for checking.

Checking section dimensions. Section dimensions must satisfy the following requirements for solid sections:

$$\sqrt{\left(\frac{V_u}{b_w\ d}\right)^2 + \left(\frac{T_u P_h}{1.7\ A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right)$$

Hollow sections:

$$\left(\frac{V_u}{b_w\ d}\right) + \left(\frac{T_u P_h}{1.7 A_{oh}^2}\right) \le \phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)$$

The ratio of the two coefficients is stored in the CivilFEM results file for both element ends as the parameter (solid sections):

$$CRTTC = \frac{\sqrt{\left(\frac{V_u}{b_w\ d}\right)^2 + \left(\frac{T_u P_h}{1.7\ A_{oh}^2}\right)^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

Hollow sections:

$$CRTTC = \frac{\frac{V_u}{b_w\ d} + \frac{T_u P_n}{1.7\ A_{oh}^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

6) **Calculating the nominal torsional moment strength of the section.** The nominal torsional moment strength (T_n) is evaluated with the following expression:

$$T_n = 2\ A_c \frac{A_{st}}{S} f_y$$

where:

f_y specified yield strength of torsional reinforcement (not greater than 60,000 psi).

This nominal torsional moment strength and its ratio to the design shear force are stored in the CivilFEM results file for both element ends as the parameters:

TN Nominal torsional moment strength.

$$TN = T_n$$

CRTTN Ratio of the design torsional moment (T_u) to the torsional moment strength T_n .

$$CRTTN = \frac{T_u}{T_n}$$

The required longitudinal reinforcement area is given by:

$$(A_l)_{nec} = \frac{A_{st}}{S} P_h$$

Calculation results are stored in the CivilFEM results file for both element ends as the parameters:

ALT Area of longitudinal torsion reinforcement required in accordance with the transverse torsion reinforcement defined.

$$ALT = (A_l)_{nec}$$

CRTALT Ratio of the area of longitudinal torsion reinforcement required to the area of longitudinal torsion reinforcement defined.

$$CRTALT = \frac{(A_l)_{nec}}{A_{sl}}$$

If longitudinal reinforcement is not defined, then $A_{sl} = 0$ and the criterion is equal to 2^{100}.

7) Obtaining torsion criterion. The section will be valid for torsion if the following condition is fulfilled:

$$T_u \leq \Phi T_n$$

$$A_{sl} \geq (A_l)_{nec}$$

$$\sqrt{\left(\frac{V_u}{b_w\,d}\right)^2 + \left(\frac{T_u P_h}{1.7\,A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)$$

ϕ strength reduction factor of the section, (0.75 for shear and torsion).

Therefore, the validity torsion criterion is defined as follows:

$$CRT\_TOT = Max\left(\frac{T_u}{\phi\ T_n}\ ;\ \frac{(A_l)_{nec}}{A_{sl}}\ ;\ \frac{\sqrt{\left(\frac{V_u}{b_w\,d}\right)^2 + \left(\frac{T_u P_h}{1.7\,A_{oh}^2}\right)^2}}{\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}\right) \leq 1$$

For each element end, this value is stored in the CivilFEM results file as CRT_TOT.

If the strength provided by concrete is null and the torsion reinforcement is not defined in the section, the criterion will be 2^{100}.

The $\phi \cdot T_n$ value is stored in the CivilFEM results file for both element ends as the parameter TFI.

6.6.6.3. Combined Shear and Torsion Check

For checking sections subjected to shear force and concomitant torsional moment, the following steps are taken:

6) Checking if torsion effects will be considered. Torsion effects are only considered if the design torsional moment (T_u) satisfies the condition below:

$$T_u > \phi\left(\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\right)$$

If the design torsional moment is less than this value, its effects can be neglected and it is considered as null for checking.

7) Checking section dimensions. For shear force and the associated torsional moment, section dimensions must satisfy the following requirements:

a) Solid sections:

$$\sqrt{\left(\frac{V_u}{b_w\, d}\right)^2 + \left(\frac{T_u P_h}{1.7\, A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w\, d} + 8\sqrt{f'_c}\right)$$

b) Hollow sections:

$$\left(\frac{V_u}{b_w\, d}\right) + \left(\frac{T_u P_h}{1.7 A_{oh}^2}\right) \leq \phi\left(\frac{V_c}{b_w\, d} + 8\sqrt{f'_c}\right)$$

In hollow sections, if the section wall's thickness is less than A_{oh}/P_h, this value is replaced in the expression above by the section's minimum thickness.

The ratio between these two factors is stored in the CivilFEM results file for both element ends.

a) Solid sections:

$$CRTTC = \frac{\sqrt{\left(\frac{V_u}{b_w\, d}\right)^2 + \left(\frac{T_u P_n}{A_{oh}^2}\right)^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

b) Hollow sections:

$$CRTTC = \frac{\frac{V_u}{b_w\ d} + \frac{T_u P_n}{1.7\ A_{oh}^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

8) Checking for shear force with concomitant torsional moment. This check is accomplished with the same steps as the check of elements subjected to pure shear force according to ACI 318-14. The same results as for shear checking will be calculated.

Except for this check, the CRT_TOT criterion is stored in the CivilFEM results file as CRTSHR for each element end.

9) Checking for torsion with shear force. This check follows the same steps considered for the check of elements subjected to pure torsion according to ACI 318-05. The same results as in torsion checking will be calculated.

Except for this check, the CRT_TOT criterion is stored in the CivilFEM results file as CRTTRS for each element end.

10) Obtaining the combined shear and torsion criterion. This criterion determines whether the section is valid or not. It is defined as follows:

$$CRT\_TOT = \text{Max}\left[\frac{V_u}{\phi\ V_n}; \frac{T_u}{\phi\ T_n}; \frac{(A_l)_{nec}}{A_{sl}}; CRTTC\right] \leq 1$$

For each end, this value is stored in the CivilFEM results file.

A value equal to 2^{100} for this criterion indicates:

- the shear strength provided by concrete is equal to zero and the shear reinforcement has not been defined.
- the shear strength provided by concrete is equal to zero and the transverse torsion reinforcement has not been defined.
- the longitudinal torsion reinforcement has not been defined.

6.6.6.4. Shear Design

The shear designing according to ACI 318-14 is described in this section. These equations refer to US (British) units which include force, length, and time units of lb, in, and sec.

1) Obtaining strength properties of the materials. The required material properties associated with each transverse cross section at the active time are:

f'_c specified compressive strength of concrete.

f_{yk} specified yield strength of reinforcement.

λ modification factor for lightweight concrete.

2) **Obtaining geometrical data of the section.** Required data for shear designing are the following ones:

A_g area of concrete section.

3) **Obtaining geometrical parameters depending on specified code.** The Required data are as follows:

b_w web width or diameter of the circular section.

d distance from the extreme compressed fiber to the centroid of the longitudinal tension reinforcement in Y, (for circular sections this should not be less than the distance from the extreme compressed fiber to the centroid of the tension reinforcement in the opposite half of the member).

Section 6.6.1.. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** For shear reinforcement design, it is possible to define the angle α between the reinforcement and the longitudinal axis of the member. This angle must be stored in the shear reinforcement data of each element. If this angle is equal to zero or it is not defined, alpha= 90º. Other data pertaining to reinforcements will be ignored.

5) **Obtaining forces and moments acting on the section.** The shear force that acts on the section as well as the concomitant axial force are obtained from the CivilFEM results file (.RCF).

Force	**Description**
V_u	Design shear force
Nu	Axial force (positive for compression)

6) **Calculating the shear strength provided by concrete.** The shear strength provided by concrete (V_c) is calculated with the following expression:

$$V_c = 2\lambda\sqrt{f'_c}b_w\, d$$

where:

$\sqrt{f'_c}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force,

$$V_c = 2\left(1 + \frac{N_u}{2000\ A_g}\right)\lambda\sqrt{f'_c}b_w\ d$$

If the section is subjected to a significant tensile force,

$$V_c = 2\left(1 + \frac{N_u}{500\ A_g}\right)\lambda\sqrt{f'_c}b_w\ d$$

The calculation result is stored in the CivilFEM results file for both element ends as the parameter:

VC Shear strength provided by concrete: $VC = V_c$

7) Calculating the required reinforcement contribution to the shear strength. The section must satisfy the following condition to resist the shear force:

$$V_u \leq \phi\ V_n = \phi(V_c + V_s)$$

Therefore, the reinforcement shear resistance must satisfy:

$$V_s = \frac{V_u}{\phi} - V_c \leq 8\sqrt{f'_c}b_w\ d$$

If the shear resistance of the reinforcement does not satisfy the expression above, the section cannot be designed. As a result, the parameters for the reinforcement ratio will be equal to 2^{100}.

$$ASSH = \frac{A_s}{S} = 2^{100}$$

For this case, the element will be labeled as not designed.

Calculation results are stored in the CivilFEM results file for both element ends as the parameter:

VS Shear resistance provided by the transverse reinforcement.

$$VS = V_s \leq 8\sqrt{f'_c}\ b_w\ d$$

8) Calculating the required reinforcement ratio. Once the shear resistance of the reinforcement has been obtained, the reinforcement can be calculated with the following expression:

$$\frac{A_s}{S} = \frac{V_s}{f_y(\sin\alpha + \cos\alpha)d}$$

Where:

A_s area of the cross-section of the shear reinforcement.

s spacing of the stirrups measured along the longitudinal axis.

f_y yield strength of the shear reinforcement (not greater than 60000 psi).

The area of the designed reinforcement per unit length is stored in the CivilFEM results file for both element ends:

$$ASSH = \frac{A_s}{S}$$

In this case, the element will be labeled as designed (providing the design process is correct for both element sections).

6.6.6.5. *Torsion Design*

The torsion designing according to ACI 318-14 is described in this section. These equations refer to US (British) units which include force, length, and time units of lb, in, and sec.

1) **Obtaining strength properties of the materials.** These properties are obtained from the material properties associated with each transverse cross section at the active time.

f'_c specified compressive strength of concrete.

f_{yk} specified yield strength of reinforcement.

λ modification factor for lightweight concrete.

2) **Obtaining geometrical parameters depending on specified code.** The required data are as follows:

b_w web width or diameter of the circular section.

d distance from the extreme compressed fiber to the centroid of the longitudinal tensile reinforcement in Y, (for circular sections this should not be less than the distance from the extreme compressed fiber to the centroid of the tensile reinforcement in the opposite half of the member).

A_{cp} Area enclosed by outside perimeter of concrete cross section.

P_{cp} Outside perimeter of the concrete cross section.

A_{oh} Area enclosed by centerline of the outermost closed transverse torsional reinforcement.

P_h Perimeter of centerline of outermost closed transverse torsional reinforcement.

A_o Gross area enclosed by shear flow path.

Section 6.6.1.. provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining forces and moments acting on the section**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	Description
T_u	Design torsional moment in I section.

4) **Checking if torsion effects will be considered.** Torsion effects are only considered if the design torsional moment (T_u) satisfies the condition below:

$$T_u > \phi\left(\lambda\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\right)$$

If the design torsional moment is less than this value, its effects can be neglected and it is consider as null for the design.

Checking section dimensions. Section dimensions must satisfy the following requirements for solid sections:

$$\sqrt{\left(\frac{V_u}{b_w\ d}\right)^2+\left(\frac{T_u P_h}{1.7\ A_{oh}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w d}+8\sqrt{f'_c}\right)$$

Hollow sections:

$$\left(\frac{V_u}{b_w\ d}\right)+\left(\frac{T_u P_h}{1.7 A_{oh}^2}\right) \leq \phi\left(\frac{V_c}{b_w\ d}+8\sqrt{f'_c}\right)$$

For hollow sections, if the thickness of the section walls is less than A_{oh}/P_h, this value will be replaced by the minimum thickness of the section in the equation above.

The torsion reinforcement will not be designed if the previous expression is not fulfilled; consequently, the parameters for the reinforcement will be equal to 2^{100}.

$$ASTT = \frac{A_{st}}{s} = 2^{100} \quad \text{for transverse reinforcement}$$

$$ASLT = A_{sl} = 2^{100} \quad \text{for longitudinal reinforcement}$$

In this case the element will be marked as not designed and it will be stored in the TRS_NOT OK component.

The ratio of the two coefficients is stored in the CivilFEM results file for both element ends:

a) Solid sections:

$$\mathrm{CRTTC} = \frac{\sqrt{\left(\frac{V_u}{b_w\, d}\right)^2 + \left(\frac{T_u P_n}{A_{oh}^2}\right)^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

b) Hollow sections:

$$\mathrm{CRTTC} = \frac{\frac{V_u}{b_w\, d} + \frac{T_u P_n}{1.7\, A_{oh}^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

5) **Calculating the required transverse reinforcement.** In order to resist the torsional moment, the section must satisfy the condition below:

$$T_u \leq \phi\, T_n = \phi\left(2 A_0 \frac{A_t}{S} f_y\right)$$

A_t cross-sectional area of one leg of a closed stirrup resisting torsion.

s spacing of the stirrups.

Therefore, the required transverse torsion reinforcement is:

$$\frac{A_t}{S} = \frac{T_u}{\phi\ 2\ A_0\ f_y}$$

The area of designed transverse reinforcement per unit length is stored in the CivilFEM results file for both element ends:

$$\mathrm{ASTT} = \frac{A_t}{S}$$

6) **Calculating the required longitudinal reinforcement.** The longitudinal reinforcement area is given by the following expression:

$$A_l = \frac{A_t}{S} P_h$$

The area of the designed longitudinal reinforcement is stored in the CivilFEM results file for both element ends:

$$\mathrm{ASLT} = A_l$$

If transverse and longitudinal reinforcements are designed for both element ends, this element will be labeled as designed.

6.6.6.6. Combined Shear and Torsion Design

The designing of sections subjected to shear force and concomitant torsional moment, follows the steps below:

5) **Checking if torsion effects will be considered.** Torsion effects are only considered if the design torsional moment (T_u) satisfies the condition below:

$$T_u > \phi\left(\sqrt{f'_c}\,\frac{A_{cp}^2}{P_{cp}}\right)$$

If the design torsional moment is less than this value, its effects can be neglected and it is considered as null for the design.

6) **Checking section dimensions.** For shear force and concomitant torsional moment, section dimensions must satisfy the following requirements:

a) Solid sections:

$$\sqrt{\left(\frac{V_u}{b_w\,d}\right)^2+\left(\frac{T_u P_h}{1.7\,A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w\,d}+8\sqrt{f'_c}\right)$$

b) Hollow sections:

$$\left(\frac{V_u}{b_w\,d}\right)+\left(\frac{T_u P_h}{1.7\,A_{oh}^2}\right) \le \phi\left(\frac{V_c}{b_w\,d}+8\sqrt{f'_c}\right)$$

For hollow sections, if the section wall's thickness is less than A_{oh}/P_h, this value will be replaced by the minimum thickness of the section in the expression above.

The torsion reinforcement will not be designed if the expression above is not fulfilled; consequently, the parameters for the reinforcement will be equal to 2^{100}.

$$\mathrm{ASTT} = \frac{A_{st}}{s} = 2^{100} \text{for transverse reinforcement}$$

$$\mathrm{ASLT} = A_{sl} = 2^{100} \text{for longitudinal reinforcement}$$

In this case, the element will be marked as not designed.

The ratio of the two coefficients is stored in the CivilFEM results file for both element ends.

a) Solid sections:

$$CRTTC = \frac{\sqrt{\left(\frac{V_u}{b_w\ d}\right)^2 + \left(\frac{T_u P_h}{1.7\ A_{oh}^2}\right)^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

b) Hollow sections:

$$CRTTC = \frac{\left(\frac{V_u}{b_w\ d}\right)^2 + \left(\frac{T_u P_h}{1.7\ A_{oh}^2}\right)^2}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

7) **Shear design assuming a null torsional moment.** This design follows the same procedure as for the design of elements subjected to pure shear force according to ACI 318-14.

8) **Torsion design considering a null shear force.** This design is accomplished with the same steps as for the design of elements subjected to pure torsion according to ACI 318-14.

6.6.8. Shear and Torsion according to ACI 349-01 and ACI349-06

6.6.7.1. Shear Check

Shear checking according to ACI 349-01 and 349-06 is described in this section. Units in these equations refer to the US (British) force, length, time units measured in pounds, inches, and seconds.

1) **Obtaining material strength properties.** The required material properties associated with each transverse cross section at the active time are:

f'_c specified compressive strength of concrete.

f_{yk} specified yield strength of reinforcement.

2) **Obtaining geometrical data of the section.** Required data for shear checking:

A_g area of concrete section.

3) **Obtaining geometrical parameters depending on specified code.** Geometrical parameters used for shear calculations must be defined. The required data:

b_w web width or diameter of circular section.

d distance from the extreme compressed fiber to the centroid of the longitudinal tensile reinforcement in the Y direction, (for circular sections, this must not be less than the distance from the extreme compressed fiber to the centroid of the tensile reinforcement in the opposite half of the member).

Section 6.6.1. "Previous Considerations to Shear and Torsion Calculation" provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining section reinforcement data.** Required data includes:

α angle between shear reinforcement and the longitudinal axis of the member section.

A_s/S area of reinforcement per unit length (reinforcement ratio) in both the Y and Z directions.

The reinforcement ratio may also be obtained with the following data:

A_s total area of the reinforcement legs.

s spacing of the stirrups.

or with the data below:

s spacing of the stirrups.

diameter of bars.

N number of reinforcement legs.

5) **Obtaining forces acting on the section**. The forces that act on the section are obtained from the CivilFEM results file (.RCF).

Force	Description
V_u	Factored design shear force in the section
N_u	Factored axial force occurring simultaneously to the shear force (positive for compression).

6) **Calculating the shear strength provided by concrete for nonprestressed members.** First, the shear strength provided by concrete (V_c) is calculated with the following expression:

$$V_c = 2\sqrt{f_c'}b_w\, d$$

where:

$\sqrt{f_c'}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force:

$$V_c = 2\left(1 + \frac{N_u}{2000A_g}\right)\sqrt{f_c'}b_w\, d$$

If the section is subjected to a tensile force such that the tensile stress is less than 500 psi:

$$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\sqrt{f_c'}b_w\, d$$

If the section is subjected to a tensile force such that the tensile stress exceeds 500 psi, it is assumed that $V_c = 0$.

The calculated result at both element ends is stored in the CivilFEM results file as the parameter VC:

VC Shear strength provided by concrete.

$$VC = V_c$$

7) **Calculating the shear strength provided by the shear reinforcement.** The strength provided by the shear reinforcement (V_s) is calculated with the following expression:

$$V_s = \frac{A_s}{S} f_y(\sin\alpha + \cos\alpha)d \leq 8\sqrt{f_c'}b_w\, d$$

where:

f_y yield strength of the shear reinforcement (not greater than 60,000 psi).

The calculated result at both element ends is stored in the CivilFEM results file as the parameter VS:

VS Shear strength provided by transverse reinforcement.

$$VS = V_s$$

8) **Calculating the nominal shear strength of section.** The nominal shear strength (V_n) is the sum of the shear strength provided by the concrete and the shear reinforcement as described in the previous sections:

$$V_n = V_c + V_s$$

This nominal shear strength as well as its ratio to the design shear are stored in the CivilFEM results file as the parameters:

VN Nominal shear strength.

$$VN = V_n$$

CRTVN Ratio of the design shear force (V_u) to the resistance V_n.

$$CRTVN = \frac{V_u}{V_n}$$

If the shear strength provided by the concrete is null and shear reinforcement is not defined in the section, then $V_n = 0$, and the criterion is set equal to –1.

9) **Obtaining shear criterion.** The section will be valid for shear if the following condition is satisfied

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

ϕ strength reduction factor of the section, (0.85 for shear and torsion).

Therefore, the validity of the shear criterion is defined as follows:

$$CRT\_TOT = \frac{V_u}{\phi\ V_n} \leq 1$$

For each element, this shear utilization value is stored in the CivilFEM results file as the parameter CRT_TOT.

In cases where the strength provided by the concrete is null and the shear reinforcement is not defined in the section, the shear strength $V_n = 0$, and the criterion is set equal to 2^{100}.

The $\phi \cdot V_n$ value is stored in the CivilFEM results file as the parameter VFI.

6.6.7.2. Torsion Check

Torsion checking according to ACI 349-01 and 349-06 is described in this section. Units in these equations refer to the US (British) force, length, time units measured in pounds, inches, and seconds.

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated with each transverse cross section at the active time are:

f'_c specified compressive strength of concrete.

f_{yk} specified yield strength of reinforcement.

2) **Obtaining geometrical parameters depending on specified code.** Geometrical parameters used for torsion calculations must be defined. The required data are as follows:

b_w web width or diameter of circular section.

d distance from the extreme compression fiber to the centroid of the longitudinal tensile reinforcement in Y, (for circular sections, this must not be less than the distance from the extreme compression fiber to the centroid of the tensile reinforcement in the opposite half of the member).

A_{cp} Area enclosed by outside perimeter of the concrete cross section.

P_{cp} Outside perimeter of the concrete cross section.

A_{0h} Area enclosed by the centerline of the outermost closed transverse torsional reinforcement.

P_h Perimeter of the centerline of the outermost closed transverse torsional reinforcement.

A_0 Gross area enclosed by the shear flow path.

Section 6.6.1. "Previous Considerations to Shear and Torsion Calculation" provides detailed information on how to calculate the required data for each valid section.

3) **Obtaining reinforcement data of the section.** Required data are as follows:

Transverse Reinforcement

A_{st}/s area of transverse reinforcement per unit length.

The reinforcement ratio can alternatively be defined using the following data:

A_{st} closed stirrups area for torsion.

s spacing of closed stirrups.

Or with the data below:

s spacing of closed stirrups.

ϕ_t diameter of the closed stirrups.

Longitudinal Reinforcement

A_{sl} total area of the longitudinal reinforcement.

The reinforcement ratio can also be defined using the following data:

ϕ_l diameter of longitudinal bars.

N number of longitudinal bars.

4) **Obtaining section internal forces and moments**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	Description
T_u	Factored design torsional moment.

5) **Checking whether torsion effects will be considered.** Torsion effects are only considered if the design torsional moment (T_u) satisfies the following equation:

$$T_u > \phi\left(\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\right)$$

If the design torsional moment is less than this value, its effects can be neglected and it is considered as null for checking.

6) **Checking section dimensions.** Section dimensions must satisfy the following requirements:

$$\frac{T_u P_h}{1.7\ A_{0h}^2} \leq \phi\left(\frac{V_c}{b_w} + 8\sqrt{f'_c}\right)$$

In hollow sections, if the section wall thickness is less than A_{oh}/P_h, this value must be substituted with the minimum thickness of the section in the expression above.

The ratio of the two coefficients is stored in the CivilFEM results file at both element ends as the parameter:

$$CRTTC = \frac{\frac{T_u P_u}{1.7\ A_{0h}^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

7) **Calculating the nominal torsional moment strength of the section.** The nominal torsional moment strength (T_n) is evaluated with the following expression:

$$T_n = 2A_0\frac{A_{st}}{S}f_y$$

where:

f_y specified yield strength of torsional reinforcement (not greater than 60000 psi).

This nominal torsional moment strength and its ratio to the design shear force are stored in the CivilFEM results file at both element ends as the parameters:

TN Nominal torsional moment strength.

$$TN = T_n$$

CRTTN Ratio of the design torsional moment (T_u) to the torsional moment strength T_n .

$$CRTTN = \frac{T_u}{T_n}$$

The needed longitudinal reinforcement area is given by:

$$(A_l)_{nec} = \frac{A_{st}}{S} P_h$$

The calculated results are stored in the CivilFEM results file at both element ends as the parameters:

ALT Area of torsion longitudinal reinforcement required in accordance to the torsion transverse reinforcement defined.

$$ALT = (A_l)_{nec}$$

CRTALT Ratio of the area of torsion longitudinal reinforcement required to the area of torsion longitudinal reinforcement defined.

$$CRTALT = \frac{(A_l)_{nec}}{A_{sl}}$$

If longitudinal reinforcement is not defined, then A_{sl}, and the criterion is set equal to 2^{100}.

8) Obtaining torsion criterion. The section will be valid for torsion if the following condition is satisfied:

$$T_u \leq \phi T_n$$

$$A_{sl} \geq (A_l)_{nec}$$

$$\frac{T_u P_h}{1.7 A_{0h}^2} \leq \phi \left(\frac{V_c}{b_w\, d} + 8\sqrt{f_c'} \right)$$

ϕ strength reduction factor of the section, (=0.85 for shear and torsion).

Therefore, the torsion design utilization is defined as follows:

$$CRT\_TOT = Max \left(\frac{T_u}{\phi\, T_n}; \frac{(A_l)_{nec}}{A_{sl}}; \frac{\frac{T_u P_h}{1.7 A_{0h}^2}}{\frac{V_c}{b_w\, d} + 8\sqrt{f_c'}} \right) \leq 1$$

For each element end, this value is stored in the CivilFEM results file.

In cases where the strength provided by concrete is null and the torsion reinforcement is not defined in the section, the criterion will be set to 2^{100}.

The $\phi \cdot T_n$ value is stored in the CivilFEM results file at both element ends as the parameter TFI.

6.6.7.3. Combined Shear and Torsion Checking

For checking sections subjected to combined shear force and torsional moment, the following steps are taken:

1) Checking if torsion effects must be considered. Torsion effects are only considered if the design torsional moment (T_u) satisfies the condition below:

$$T_u > \phi\left(\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\right)$$

If the design torsional moment is less than this value its effects can be neglected and it is considered as null for checking.

2) Checking section dimensions. For shear force and associated torsional moment, section dimensions must satisfy the following requirements:

a) Solid sections:

$$\sqrt{\left(\frac{V_u}{b_w\,d}\right)^2+\left(\frac{T_u P_h}{1.7\,A_{0h}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w\,d}+8\sqrt{f'_c}\right)$$

b) Hollow sections:

$$\left(\frac{V_u}{b_w\,d}\right)+\left(\frac{T_u P_h}{1.7A_{0h}^2}\right) \leq \phi\left(\frac{V_c}{b_w\,d}+8\sqrt{f'_c}\right)$$

In hollow sections if the section wall thickness is lower than A_{0h}/P_h, this value is changed in the previous expression by the section minimum thickness.

The ratio between these two factors is stored in the CivilFEM results file at both element ends.

a) Solid sections:

$$CRTTC = \frac{\sqrt{\left(\frac{V_u}{b_w\ d}\right)^2 + \left(\frac{T_u P_h}{1.7\ A_{0h}^2}\right)^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

b) Hollow sections:

$$CRTTC = \frac{\frac{V_u}{b_w\ d} + \frac{T_u P_h}{1.7\ A_{0h}^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)}$$

3) **Checking for shear force with associated torsional moment.** This checking is accomplished following the same steps considered for the checking of elements subjected only to shear force according to ACI 349. The same results as defined in the shear check are calculated.

4) **Checking for torsion with shear force.** This checking is accomplished following the same steps considered for the checking of elements subjected only to torsion according to ACI 349. The same results as defined in the torsion check are calculated.

5) **Obtaining the combined shear and torsion criterion.** This criterion determines whether the section is valid or not. The utilization is defined as follows:

$$CRT\_TOT = Max\left[\frac{V_u}{\phi\ V_n}; \frac{T_u}{\phi\ T_n}; \frac{(A_l)_{nec}}{A_{sl}}; CRTTC\right] \leq 1$$

For each end, this value is stored in the CivilFEM results file.

A value equals to 2^{100} for this criterion would indicate one of the following:

- the shear strength provided by concrete is equal to zero and shear reinforcement has not been defined
- the shear strength provided by concrete is equal to zero and transverse torsion reinforcement has not been defined
- the longitudinal torsion reinforcement has not been defined

6.6.7.4. Shear Design

Shear design according to ACI 349-01 and 349-06 is described in this section. Units in these equations refer to the US (British) force, length, time units measured in pounds, inches, and seconds.

1) **Obtaining material strength properties.** The required material properties associated with each transverse cross section at the active time are:

f'_c	specified compressive strength of concrete.
f_{yk}	specified yield strength of reinforcement.

2) **Obtaining section geometrical data.** Required data for shear design:

A_g	area of concrete section.

3) **Obtaining geometrical parameters depending on specified code.** Geometrical parameters used for shear designing must be defined. The required data:

b_w	web width or diameter of the circular section
d	distance from the extreme compressed fiber to the centroid of the longitudinal tensile reinforcement in Y, (for circular sections, this must not be less than the distance from the extreme compressed fiber to the centroid of the tensile reinforcement in the opposite half of the member).

Section "6.6.1. Previous Considerations to Shear and Torsion Calculation" provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** In shear reinforcement designing, it is possible to define the angle α between the reinforcement and the longitudinal axis of the member. This angle must be stored in the shear reinforcement data of each element. If this angle is equal to zero or is not defined, α=90º. Other data concerning to reinforcements are ignored.

5) **Obtaining forces and moments acting on the section.** The shear force that acts on the section as well as the associated axial force are obtained from the CivilFEM results file (.RCF).

Force	**Description**
V_u	Factored design shear force.
N_u	Factored axial force occurring simultaneously with the shear force (positive for compression).

6) **Calculating the shear strength provided by concrete for nonprestressed members.** First, the shear strength provided by the concrete (V_c) is calculated with the following expression:

$$V_c = 2\sqrt{f'_c} b_w \, d$$

where:

$\sqrt{f'_c}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force:

$$V_c = 2\left(1 + \frac{N_u}{2000 \, A_g}\right)\sqrt{f'_c} b_w \, d$$

If section is subjected to a tensile force so that the tensile stress is less than 500 psi:

$$V_c = 2\left(1 + \frac{N_u}{500 \, A_g}\right)\sqrt{f'_c} b_w \, d$$

If the section is subjected to a tensile force such that the tensile stress exceeds 500 psi, it is assumed that V_c=0.

The calculated result is stored in the CivilFEM results file at both element ends as the parameter:

VC Shear strength provided by concrete.

$$VC = V_c$$

7) **Calculating the required reinforcement contribution to the shear strength.** The section must satisfy the following condition to resist the shear force:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

Therefore, the required shear strength of the reinforcement must be:

$$V_s = \frac{V_u}{\phi} - V_c \leq 8\sqrt{f'_c} b_w \, d$$

If the required shear strength of the reinforcement does not satisfy the expression above, the section cannot be designed; consequently, the reinforcement parameter will be defined as 2^{100}. Then:

$$ASSH = \frac{A_s}{S} = 2^{100}$$

In this case, the element will be labeled as not designed, the program then advances to the following element.

The calculated result at both element ends is stored in the CivilFEM results file as the parameter VS:

VS Shear resistance provided by the transverse reinforcement.

$$VS = V_s \leq 8\sqrt{f_c'} b_w\ d$$

8) Calculating the required reinforcement ratio. Once the shear force that the shear reinforcement must support has been obtained, the reinforcement is obtained from the following expression:

$$\frac{A_s}{S} = \frac{V_s}{f_y(\sin\alpha + \cos\alpha)d}$$

Where:

A_s area of the cross-section of the shear reinforcement.

s spacing of the stirrups measured along the longitudinal axis.

f_y yield strength of the shear reinforcement (not greater than 60000 psi).

The area of the designed reinforcement per unit length is stored in the CivilFEM results file at both element ends:

$$ASSH = \frac{A_s}{S}$$

In this case, the element will be labeled as designed (providing the design process is correct at both element ends).

6.6.7.5. Torsion Design

The design of torsion reinforcements according to ACI 349-01 and 349-06 follows these steps:

1) Obtaining material resistant properties. These properties are obtained from the material properties associated with each transverse cross section and for the active time:

f_c' specified compressive strength of concrete.

f_{yk} specified yield strength of reinforcement.

2) Obtaining geometrical parameters depending on specified code. The required data is as follows:

b_w	web width or diameter of the circular section
d	distance from the extreme compressed fiber to the centroid of the longitudinal tensile reinforcement in Y, (for circular sections, this should not be less than the distance from the extreme compressed fiber to the centroid of the tensile reinforcement in the opposite half of the member).
A_{cp}	Area enclosed by outside perimeter of the concrete cross section.
P_{cp}	Outside perimeter of the concrete cross section.
A_{0h}	Area enclosed by the centerline of the outermost closed transverse torsional reinforcement.
P_h	Perimeter of the centerline of the outermost closed transverse torsional reinforcement.
A_0	Gross area enclosed by the shear flow path.

Section 11-A.7 “Previous Considerations to Shear and Torsion Calculation” provides detailed information on how to calculate the required data for each valid section.

3) **Obtaining forces and moments acting on the section**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	**Description**
T_u	Factored design torsional moment.

4) **Checking whether torsion effects will be considered.** Torsion effects are only considered if the design torsional moment (T_u) satisfies the condition below:

$$T_u > \phi\left(\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\right)$$

If the design torsional moment is less than this value, its effects can be neglected and it is consider as null for the design.

5) **Checking section dimensions.** Section dimensions must satisfy the following requirements:

$$\frac{T_u P_h}{1.7\ A_{0h}^2} \leq \phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f'_c}\right)$$

In hollow sections, if the section's wall thickness is less than A_{oh}/P_h, this value will be equal to the minimum thickness of the section in the formula above.

The torsion reinforcement will not be designed if the previous expression is not satisfied, so the parameters where the reinforcement is stored would be marked with 2^{100}. Then:

$$\mathrm{ASTT} = \frac{A_{st}}{S} = 2^{100} \quad \text{for transverse reinforcement}$$

$$\mathrm{ASLT} = A_l = 2^{100} \quad \text{for longitudinal reinforcement}$$

In this case, the element will be marked as not designed.

The ratio of the two coefficients is stored in the CivilFEM results file at both element ends:

$$\mathrm{CRTTC} = \frac{\dfrac{T_u P_h}{1.7\, A_{0h}^2}}{\phi\left(\dfrac{V_c}{b_w\, d} + 8\sqrt{f_c'}\right)}$$

6) **Calculating the required transverse reinforcement.** In order to resist the torsional moment, the section must satisfy the following condition:

$$T_u \le \phi\, T_n = \phi\left(2\, A_O \frac{A_{st}}{S} f_y\right)$$

A_{st} cross-sectional area of one leg of a closed stirrup of the transverse reinforcement.

s spacing of the stirrups.

Therefore, the required transverse torsion reinforcement is:

$$\frac{A_{st}}{S} = \frac{T_u}{\phi\, 2\, A_O\, f_y}$$

The area of the designed transverse reinforcement per unit length is stored in the CivilFEM results file at both element ends:

$$\mathrm{ASTT} = \frac{A_{st}}{S}$$

7) **Determining the longitudinal reinforcement requirement.** The longitudinal reinforcement area is given by the following expression:

$$A_l = \frac{A_{st}}{S} P_h$$

The area of the designed longitudinal reinforcement is stored in the CivilFEM results file at both element ends:

$$\mathrm{ASLT} = \mathrm{A_l}$$

If both transverse and longitudinal reinforcements are designed at both element ends, this element will be labeled as designed.

6.6.7.6. Combined Shear and Torsion Design

The design of sections subjected to combined shear force and torsional moment, follows the steps below:

1) Checking whether torsion effects will be considered. Torsion effects are only considered if the design torsional moment (T_u) satisfies the condition below:

$$T_u > \phi\left(\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\right)$$

If the design torsional moment is less than this value, its effects can be neglected and it will be considered as null for designing.

2) Checking section dimensions. For shear force and associated torsional moment, section dimensions must satisfy the following requirements:

a) Solid sections:

$$\sqrt{\left(\frac{V_u}{b_w\,d}\right)^2 + \left(\frac{T_u P_h}{1.7\,A_{0h}^2}\right)^2} \leq \phi\left(\frac{V_c}{b_w\,d} + 8\sqrt{f'_c}\right)$$

b) Hollow sections:

$$\left(\frac{V_u}{b_w\,d}\right) + \left(\frac{T_u P_h}{1.7\,A_{0h}^2}\right) \leq \phi\left(\frac{V_c}{b_w\,d} + 8\sqrt{f'_c}\right)$$

In hollow sections, if the section wall thickness is less than A_{oh}/P_h, this last value will be equal to the minimum thickness of the section in the equation above.

If the expression above is not satisfied, the torsion reinforcement will not be designed; as a result, the reinforcement parameters will be defined as:

$$\mathrm{ASTT} = \frac{A_{st}}{s} = 2^{100} \text{for transverse reinforcement}$$

$$ASLT = A_l = 2^{100}\text{for longitudinal reinforcement}$$

In this case, the element will be labeled as not designed, and the program will then advance to the next element.

The ratio of the two coefficients is stored in the CivilFEM results file at both element ends.

a) Solid sections:

$$CRTTC = \frac{\sqrt{\left(\frac{V_u}{b_w\ d}\right)^2 + \left(\frac{T_u P_h}{1.7\ A_{0h}^2}\right)^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f_c'}\right)}$$

b) Hollow sections:

$$CRTTC = \frac{\frac{V_u}{b_w\ d} + \frac{T_u P_h}{1.7\ A_{0h}^2}}{\phi\left(\frac{V_c}{b_w\ d} + 8\sqrt{f_c'}\right)}$$

3) **Shear design assuming a null torsional moment.** This design is accomplished with the same procedure as for the design of elements subjected to pure shear force according to ACI 349-01 and 349-06.

4) **Torsion design considering a null shear force.** This design is follows the same procedure as for the design of elements subjected to pure torsion according to ACI 349-01 and 349-06.

6.6.9. Shear and Torsion according to BS8110

6.6.8.1. Shear Check

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated to each transverse cross section and for the active time.

The required data are the following:

f_{cu} characteristic compressive strength of concrete.

f_y characteristic yield strength of reinforcement.

γ_{cs} concrete partial safety factor.

2) **Obtaining geometrical data of the section.** Required data for shear checking are the following:

A_c total area of the concrete transverse section.

h total depth in the shear direction considered.

3) **Obtaining geometrical parameters depending on specified code.** Required data are the following ones:

b_w minimum width of the section.

d effective depth of the section.

A_{sb} Area of the longitudinal tension reinforcement that extends at least a distance d beyond the considered section.

Section 6.6.1.. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** Required data are the following:

α angle between shear reinforcement and the longitudinal axis of the member. For this code, α = 90º.

A_s/S area of reinforcement per unit length.

The reinforcement ratio may also be obtained with the following data:

A_s total area of the reinforcement legs.

s spacing of the stirrups.

Or with the data below:

s spacing of the stirrups.

ϕ diameter of bars.

N number of reinforcement legs.

5) **Obtaining forces and moments acting on the section**. The shear force that acts on the section as well as the concomitant axial force and bending moment are obtained from the CivilFEM results file (.RCF).

Force	**Description**
V_d	Design shear force

N_d	Concomitant axial force
M	Concomitant bending moment

6) **Checking compression failure in the web.** First, a check is made to ensure the design shear force (V_d) is less than or equal to the oblique compression resistance of concrete section (V_{u1}):

$$V_d \leq V_{u1}$$

$$V_{u1} = MIN\left\{0.8\sqrt{f_{cu}}, 5\ N/mm^2\right\} \cdot \quad b_w \cdot d$$

For each element end, calculated results are written in the CivilFEM results file in the following parameters:

VU1 Ultimate shear strength due to oblique compression of the concrete in web.

$$VU1 = V_{u1}$$

CRTVU1 Ratio of the design shear (V_d) to the resistance V_{u1}.

$$CRTVU1 = \frac{V_d}{V_{u1}}$$

7) **Calculating the shear resistance of concrete.** Shear resistance of concrete (V_c) is checked using the following expression:

$$V_c = \frac{0.79}{\gamma_{cs}}\left(100\frac{A_{sb}}{b_w\ d}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4} \cdot b_w \cdot d$$

Where:

A_{sb} Area of the longitudinal tension reinforcement that extends at least a distance d beyond the considered section.

Considering the following restrictions:

$$100\frac{A_{sb}}{b_w\ d} = 3$$

$$\frac{400}{d} = 1$$

If $f_{cu} > 25\ N/mm^2$, the results are multiplied by $\left(\frac{f_{cu}}{25}\right)^{1/3}$

If the section is subjected to an axial force, then the following expression will be used:

$$V_c{}' = V_c + 0.6\frac{NVh}{A_c M} \cdot b_w \cdot d$$

Where:

h total depth in the shear direction considered.

V_c concrete shear resistance without axial forces.

Taking into account that V_dh / M always has to be = 1

For each element end, calculated results are written in the CivilFEM results file as the following parameters:

VC concrete shear resistance:

$$VC = V_c$$

8) **Calculaing the steel reinforcement shear resistance.** Shear resistance provided by the steel reinforcement (V_s) is checked using the following expression:

$$V_s = \frac{A_s}{S} \cdot 0.95 \cdot f_{yv} \cdot d$$

Where:

A_s/S area per unit length of shear reinforcement.

f_{yv} characteristic yield strength of shear reinforcement.

$f_{yv} = f_y$, always less than 460 N/mm^2

For each element end, calculated results are written in the CivilFEM results file in the following parameters:

VS shear resistance provided by the transverse reinforcement

$$VS = V_s$$

9) **Calculating the total shear resistance of section.** The total shear resistance (V_{U2}) is the sum of the shear resistance provided by the concrete and the shear resistance provided by the reinforcement:

$$V_{u2} = V_c + V_s$$

For each element end, calculated results are written in the CivilFEM results file in the following parameters:

VU2 Total shear resistance of section.

$$VU2 = V_{u2} = V_c + V_s$$

CRTVU2 ratio of shear design force (V) and the resistance force V_n

$$CRTVU2 = \frac{V_d}{V_{u2}}$$

If V_{u2} = 0, a value of 2^{100} is assigned to criterion CRTVU2.

10) **Calculating the shear criterion.** The shear criterion indicates the validity of the section (if less than 1, the section conforms to code specifications, and if greater than 1, the section is not valid). Moreover, it provides information with regards to how much more additional load the section can resist. The shear criterion is defined as follows:

$$CRT\_TOT = Max\left(\frac{V_d}{V_{u1}};\frac{V_d}{V_{u2}}\right) \leq 1$$

For each element end, calculated results are written in the CivilFEM results file in the parameter CRT_TOT.

A value of 2^{100} for this criterion indicates that the shear resistance (V_{u2}) has a value of zero, as indicated in the previous step.

6.6.8.2. Torsion Check

The torsion checking according to BS8110 follows the steps below:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated to each transverse cross section and for the active time.

The required data are the following ones:

f_{cu} characteristic compressive strength of concrete.

f_y characteristic yield strength of reinforcement.

2) **Obtaining geometrical parameters depending on specified code.** Geometrical parameters used for torsion calculations must be defined within CivilFEM database. The required data are the following ones:

X_w torsion modulus for torsion checking and design.

X_1 minimum distance of the rectangular stirrups.

y_1 maximum distance of the rectangular stirrups.

Section 6.6.1.. provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining section reinforcement data.** Required data are the following:

Transverse Reinforcement

A_{st}/S area of transverse reinforcement per unit length.

The reinforcement ratio can also be obtained with the following data:

A_{st} closed stirrups area for torsion.

s spacing of closed stirrups.

Or with the data below:

s spacing of closed stirrups.

ϕ_t diameter of the closed stirrups bars.

Longitudinal Reinforcement

A_{sl} total area of the longitudinal reinforcement.

The reinforcement ratio can also be obtained with the following data:

ϕ diameter of longitudinal bars.

N number of longitudinal bars.

4) **Obtaining section internal forces and moments**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	Description
T_d	Design torsional moment

5) **Checking if torsion effects will be considered.** Torsion effects are only considered if design torsional moment (T_d) satisfies the condition below:

$$T_d \geq T_{min}$$

$$\text{with } T_{min} = V_{t,min} \cdot X_w$$

$$V_{t,min} = 0.0067 \cdot \sqrt{f_{cu}} \leq 0.4\ N/mm^2$$

Where $V_{t,min}$ is the minimum torsional stress.

If the design torsional moment is less than this value, its effects can be neglected and its default value taken as 0 for checking purposes.

6) **Checking concrete failure.** The design torsional moment T_d must be less than or equal to the maximum torsional moment resisted by the concrete (T_{u1}):

$$T_d \leq T_{u1}$$

$$T_{u1} = V_{tu} \cdot X_w$$

$$\text{If y1< 550 mm } T_{u1} = V_{tu} \cdot X_w \cdot \frac{y1}{550}$$

where:

$V_{tu} = 0.8\sqrt{f_{cu}} \leq 5.0$ N/mm^2 is the maximum allowable stress.

X_w torsion modulus for torsion check and design.

For each element end, calculated results are written in the CivilFEM results file in the following parameters:

TU1 Maximum torsional moment resisted by the section.

$$TU1 = T_{u1}$$

CRTTU1 Ratio of the design torsional moment (T_d) to the resistance T_{u1}.

$$CRTTR = \frac{T_d}{TU1}$$

If the torsion transverse reinforcement is not defined, the criterion is taken as 2^{100}.

7) **Checking the maximum torsional moment resisted by the reinforcement.** The design torsional moment T_d must be less than or equal to the maximum torsional moment that the reinforcement can resist (T_{u2}), therefore:

$$T_d \leq T_{u2}$$

$$T_{u2} = \frac{A_{st}}{S} \cdot 0.8 \cdot x_1 y_1 \cdot 0.87 \cdot f_{yv}$$

where:

A_{st}/S area of transverse reinforcement per unit length

For each element end, calculated results are written in the CivilFEM results file in the following parameters:

TU2 Maximum torsional moment that can be resisted by the reinforcement.

$$TU2 = T_{u2}$$

CRTTU2 Ratio of the design torsional moment (T_d) to the resistance T_{u2}.

$$CRTTU2 = \frac{T_d}{TU2}$$

In case the longitudinal reinforcement is not defined, the criterion is taken as 2^{100}.

8) **Obtaining the necessary torsion reinforcement.** The necessary longitudinal reinforcement is calculated as a function of the transverse reinforcement, using the following expression:

$$(A_{sl})_{nec} = \frac{A_{st}}{S} \cdot (x_1 + y_1)$$

Where:

A_{sl} defined longitudinal reinforcement

$(A_{sl})_{nec}$ necessary longitudinal reinforcement

A_s area of transverse reinforcement

For each element end, calculated results are written in the CivilFEM results file in the following parameters:

ALT Area of necessary longitudinal torsion reinforcement in compliance with the defined transverse reinforcement.

$$ALT = (A_{sl})_{nec}$$

CRTALT Ratio between the area of the required longitudinal torsion reinforcement and the area of the defined longitudinal torsion reinforcement.

$$CRTALT = \frac{(A_{sl})_{nec}}{A_{sl}}$$

9) **Obtaining torsion criterion.** The torsion criterion identifies the ratio of the design moment to the section's ultimate strength (if it is less than 1, the section is valid; whereas if it exceeds 1, the section is not valid). The criterion concerning the validity for torsion is defined as follows:

$$CRT\_TOT = Max\left(\frac{T_d}{T_{u1}}; \frac{T_d}{T_{u2}}; \frac{(A_{sl})_{nec}}{A_{sl}}\right) \leq 1$$

For each element end, this value is stored in the CivilFEM results file as CRT_TOT.

A value 2^{100} for this criterion indicates that any one of the torsion reinforcements are not defined.

6.6.8.3. Combined Shear and Torsion Check

Checking sections subjected to shear force and concomitant torsional moment follows the steps below:

1) **Shear checking disregarding the torsional moment.** This check follows the same procedure as the check of elements subjected to shear.
In this case, the total shear criterion CRT_TOT is named as CRTSHR.

2) **Torsion checking disregarding the shear force.** This check will be accomplished with the same procedure as the check of elements subjected to torsion, considering the torsional force due to shear in the calculation of concrete failure.

$$T_d \leq T_{u1}$$

$$T_{u1} = V_{tu} \cdot \chi_w - \frac{V_d}{b_w\, d}$$

In this case, the total torsion criterion CRT_TOT is named as CRTTRS.

3) **Obtaining the criterion of combined shear and torsion.** This criterion contains both shear and torsion criteria:

$$\mathrm{CRT\_TOT} = \mathrm{Max}\left[\frac{V_d}{V_{u1}}; \frac{V_d}{V_{u2}}; \frac{T_d}{T_{u1}}; \frac{T_d}{T_{u2}}; \frac{(A_{sl})_{nec}}{A_{sl}}\right]$$

For each element end, calculated results are written in the CivilFEM results file in the parameter CRT_TOT.

6.6.8.4. Shear Design

The shear design according to BS8110 follows these steps:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated to each transverse cross section and for the active time.

The required data are the following:

f_{cu} characteristic compressive strength of concrete.

f_y characteristic yield strength of reinforcement.

γ_{cs} concrete partial safety factor.

2) **Obtaining geometrical data of the section.** Required data for shear checking are the following:

A_c total area of the concrete transverse section.

h total depth in the shear direction considered.

3) **Obtaining geometrical parameters depending on specified code.** Required data are the following:

b_w minimum width of the section.

d effective depth of the section,.

A_{sb} Area of the longitudinal tension reinforcement that extends at least a distance d beyond the considered section.

Chapter 6.6.1. "Previous Considerations to Shear and Torsion Calculation" provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** Required data are the following ones:

α angle between shear reinforcement and the longitudinal axis of the member. For this code, α = 90º.

5) **Obtaining forces and moments acting on the section**. The shear force that acts on the section as well as the concomitant axial force and bending moment are obtained from the CivilFEM results file (.RCF).

Force	**Description**
V_d	Design shear force
N_d	Concomitant axial force
M	Concomitant bending moment

6) **Checking the crushing of the web in compression.** First, a check is made to ensure the design shear force (V_d) is less than or equal to the oblique compression resistance of concrete section (V_{u1}):

$$V_d \leq V_{u1}$$

$$V_{u1} = \text{MIN}\left\{0.8\sqrt{\text{fcu}},\ \ 5\ \text{N/mm}^2\right\} * b_w * d$$

For each element end, calculated results are written in the CivilFEM results file in the following parameters:

VU1 Ultimate shear strength due to oblique compression of the concrete in web.

$$\text{VU1} = V_{u1}$$

CRTVU1 Ratio of the design shear (V_d) to the resistance V_{u1}.

$$\text{CRTVU1} = \frac{V_d}{V_{u1}}$$

If the design shear force is greater than the shear force that causes failure in the web, the section will not be designed. Therefore, the parameter for the reinforcement data will be defined as 2^{100}.

$$\text{ASSH} = \frac{\text{As}}{\text{S}} = 2^{100}$$

For this case, the element will be labeled as not designed.

7) **Calculating the concrete shear resistance.** The shear resistance of concrete (Vc) is checked using the following expression:

$$V_c = \frac{0.79}{\gamma_{cs}} \left(100 \frac{A_{sb}}{b_w\, d}\right)^{1/3} \left(\frac{400}{d}\right)^{1/4} \cdot b_w \cdot d$$

Where:

A_{sb} Area of the longitudinal tension reinforcement that extends at least a distance d beyond the considered section.

Taking into account the following restrictions:

$$100 \frac{A_{sb}}{b_w\, d} \leq 3$$

$$\frac{400}{d} \geq 1$$

If $f_{cu} > 25\ N/\text{mm}^2$, the results are multiplied by $\left(\frac{f_{cu}}{25}\right)^{1/3}$

If the section is subjected to an axial force, then the following expression will be used:

$$V_c{}' = V_c + 0.6 \frac{NVh}{A_c M} \cdot b_w \cdot d$$

Where:

h total depth in the shear direction considered.

V_c concrete shear resistance without axial forces.

Taking into account that ${V_d \cdot h}/{M} \leq 1$

For each element end, calculated results are written in the CivilFEM results file in the following parameters:

VC concrete shear resistance:

$$VC = V_c$$

8) **Determining the contribution of the required transverse reinforcement to the shear force.** If the section requires shear reinforcement, the condition for the validity of sections subjected to shear force is the following:

$$V_d \leq V_{u2}$$

$$V_{u2} = V_c + V_s$$

V_s is the reinforcement contribution.

V_c is the concrete contribution.

$$V_s = V_{u2} - V_c = V_d - V_c$$

For each element end, calculated results are written in the CivilFEM results file in the following parameter:

VS Transverse reinforcement shear resistance.

$$VS = V_s$$

9) **Calculating the required reinforcement ratio.** Once the shear force that must be carried by the shear reinforcement has been obtained, this can be calculated from the equation below:

$$\frac{A_s}{S} = \frac{V_s}{0.95 \cdot f_{yv} \cdot d}$$

where:

A_s/S area per unit length of shear reinforcement.

f_{yv} characteristic yield strength of shear reinforcement.

The area of designed reinforcement per unit length is stored in the CivilFEM results file for both ends:

$$ASSH = \frac{A_s}{S}$$

In this case the element is marked as designed (provided that the design process is correct for both element sections).

6.6.8.5. Torsion Design

Torsion reinforcement design according to BS8110 follows the following steps:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated to each transverse cross section and for the active time.

The required data are the following ones:

f_{cu} characteristic compressive strength of concrete.

f_y characteristic yield strength of reinforcement.

2) **Obtaining geometrical parameters depending on specified code.** Geometrical parameters used for torsion calculations must be defined within CivilFEM database. The required data are the following ones:

X_w torsion modulus for torsion checking and dimensioning.

x_1 minimum distance of the rectangular stirrups.

y_1 maximum distance of the rectangular stirrups.

Section 6.6.1.. provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining section internal forces and moments**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	**Description**
T_d	Design torsional moment

4) **Checking if torsion effects must be considered.** Torsion effects are only considered if design torsional moment (T_d) satisfies the condition below:

$$T_d \geq T_{min}$$

$$\text{with } T_{min} = v_{t,min} \cdot X_w$$

$$v_{t,min} = 0.0067 \cdot \sqrt{f_{cu}} \leq 0.4 N/mm^2$$

Where:

$V_{t,min}$ minimum torsional stress

If the design torsional moment is less than this value, its effects can be neglected and its default value will be defined as 0 for checking purposes.

5) **Checking concrete failure.** The design torsional moment T_d must be less than or equal to the maximum torsional moment that concrete can resist (T_{u1}); therefore:

$$T_d \leq T_{u1}$$

$$T_{u1} = V_{tu} \cdot X_w$$

$$\text{If y1< 550mm } T_{u1} = V_{tu} \cdot X_w \cdot \frac{y1}{550}$$

Where:

$V_{tu} = 0.8\sqrt{f_{cu}} \leq 5.0$ N/mm^2 is the maximum allowable stress.

X_w torsion modulus for torsion check and design.

For each element end, calculated results are written in the CivilFEM results file in the following parameters:

TU1 Maximum torsional moment that can be resisted by the section.

$$TU1 = T_{u1}$$

CRTTU1 Ratio of the design torsional moment (T_d) to the resistance T_{u1}.

$$CRTTR = \frac{T_d}{TU1}$$

In case the torsion transverse reinforcement is not defined, the criterion is taken as 2^{100}.

If the design torsional moment is greater than the torsional moment that causes the compression failure of concrete, the reinforcement design will not be feasible. Therefore, the parameters for reinforcement data will be assigned a value of 2^{100}.

$$AST/ST = \frac{Ast}{S} = 2^{100} \text{ for transverse reinforcement}$$

$$AST/ST = Asl = 2^{100} \text{ for longitudinal reinforcement}$$

In this case, the element is marked as not designed, and the program then advances to the next element.

If there is no failure due to oblique compression, the calculation process continues.

6) **Calculating the transverse reinforcement required.** The design torsional reinforcement must be less than or equal to the resistance torsional reinforcement:

$$T_d \leq T_{u2}$$

$$T_{u2} = \frac{A_{st}}{S} \cdot 0.8 \cdot x_1 y_1 \cdot 0.87 \cdot f_{yv}$$

Where:

$A_{st/S}$ area of transverse reinforcement per unit length

f_y characteristic yield strength of reinforcement $f_{yv} \leq 460\ N/mm^2$

Therefore, the required transverse reinforcement is:

$$\frac{A_{st}}{S} = \frac{T_d}{0.8 \cdot x_1 y_1 \cdot 0.87 \cdot f_{yv}}$$

The area per unit length of the designed transverse reinforcement is stored in the CivilFEM results file for both element ends as:

$$ASTT = \frac{A_t}{S}$$

7) **Calculating the longitudinal reinforcement required.** The longitudinal reinforcement is calculated as a function of the transverse reinforcement using the expression:

$$(A_{sl})_{nec} = \frac{A_{st}}{S} \cdot (x_1 + y_1)$$

Where:

$(A_{sl})_{nec}$ required longitudinal reinforcement.

A_{st}/s area per unit length of transverse reinforcement.

For each element end, calculated results are written in the CivilFEM results file in the following parameter:

ASLT Area of longitudinal torsion reinforcement.

$$\text{ASLT} = (A_{sl})_{nec}$$

6.6.8.6. Shear and Torsion Design

The design of sections subjected to shear force and concomitant torsional moment follows the steps below:

1) **Shear design assuming a null torsional moment.** This design follows the same steps as for the design of elements subjected to pure shear according to BS8110.

2) **Torsion design assuming a null shear force.** This design is accomplished with the same procedure as for the designing of elements subjected to torsion force according to BS8110. However, this design considers the stress due to shear in the calculation of concrete failure.

$$T_d \leq T_{u1}$$

$$T_{u1} = V_{tu} \cdot X_w - \frac{V_d}{b_w\, d}$$

Where:

V_{tu} maximum combined shear stress (shear plus torsion).

X_w torsion modulus for torsion check and design.

6.6.10. Shear and Torsion according to GB50010

6.6.9.1. Shear Check

Shear checking for elements according to GB50010-2010 follows the steps below:

1) **Obtaining materials strength properties.** The required data are the following:

f_c	design compressive strength of concrete.
f_t	design tensile strength of concrete.
f_{yv}	steel design tensile strength for of shear reinforcement.

2) **Obtaining geometrical data of the section.** Required data for shear checking are the following:

A_c	total cross-sectional area of the concrete section.

3) **Obtaining geometrical parameters depending on specified code.** Required data are the following:

b	minimum width of the section over the effective depth.
h_0	effective height of the section.
h_w	the web height.

4) **Obtaining the reinforcement data of the section.** The necessary data are:

α	angle between shear reinforcement and the longitudinal axis of the member.
A_s/S	Reinforcement area per length unit.

Alternatively, the amount of reinforcement can be determined from:

A_s	total area in the reinforcement legs.
s	spacing among stirrups.

Or from the data below:

s	spacing among stirrups.
ϕ	diameter among bars.
N	reinforcement leg number.

5) **Obtaining the section internal forces and moments**. The shear force that acts on the section as well as the concomitant axial force and bending moment are obtained from the CivilFEM results file (.RCF).

Force	**Description**
V	Design shear force
N	Axial force

6.6.9.2. Shear Check without Seismic Action

1) **Checking whether the section dimensions meet the requirement.** First, a check is made to ensure the design shear (V) is less than or equal to maximum shear resistance of the section (V_{Rd1}):

$$V \le V_{Rd1}$$

$$\text{If } h_w/b \le 4\,, \quad V_{Rd1} = 0.25\beta_c f_c b h_0$$

$$\text{If } h_w/b \ge 6\,, \quad V_{Rd1} = 0.2\beta_c f_c b h_0$$

where β_c is a coefficient depending on the concrete strength:

- For concrete C50 (fc= 23.1 N/mm^2) or under, $\beta_c = 1.0$;
- For concrete C80 (fc= 35.9 N/mm^2), $\beta_c = 0.8$,
- For concrete C55-75, a linear interpolation is made for β_c according to the values of fc.

Results are written for each end in the CivilFEM results file as the following parameters:

VRD1 Maximum shear resistance.

$$VRD1 = V_{Rd1}$$

CRVRD1 Ratio of the design shear force V to the resistance V_{Rd1}.

$$CRVRD1 = \frac{V}{V_{Rd1}}$$

2) **Checking if shear reinforcement will be required.**

If shear reinforcement has not been defined for the section, a check is made to ensure the design shear force V is less than the maximum design shear force that can be resisted by the concrete without reinforcements (V_{Rd2}):

$$V \le V_{Rd2}$$

Where

$$V_{Rd2} = 0.7\beta_h f_t b h_0$$

$\beta_h = (\frac{800}{h_0})^{\frac{1}{4}}$ is the section height factor,

if $h_0 < 800mm$, assume $h_0 = 800\text{mm}$;

if $h_0 > 2000mm$, assume $h_0 = 2000\text{mm}$;

If reinforcement has been defined, axial forces are not present (N=0), and the shear force from the concentrated load for an independent beam is less than 75%:

$$V_{Rd2} = 0.7 f_t b h_0$$

If N is compressive (N < 0)

$$V_{Rd2} = \frac{1.75}{\lambda + 1} f_t b h_0 - 0.07N$$

If N is tensile (N > 0)

$$V_{Rd2} = \frac{1.75}{\lambda + 1} f_t b h_0 - 0.02N$$

The following are given in CivilFEM results:

VRD2 Maximum design shear force resisted by the section without the crushing of the concrete compressive struts.

$$VRD2 = V_{Rd2}$$

CRVRD2 Ratio of the design shear force V to the resistance V_{Rd2}.

$$CRVRD2 = \frac{V}{V_{Rd2}}$$

For sections subjected to an applied tensile axial force so that $V_{Rd2} = 0$, CRVRD2 is taken as 2^{100}.

3) **Checking of elements requiring shear reinforcement.** The shear resistance calculation of a section with reinforcement (V_{Rd3}) will differ according to whether the concentrated load exists.

Conditions below must be verified:

$$V \leq V_{Rd3}$$

where

$$V_{Rd3} = V_{Rd2} + V_s$$

V_s design shear load capacity of reinforcement.

$$V_s = \alpha_{cf} f_{yv} \frac{A_{sv}}{S} h_0$$

$\alpha_{cf} = 1$

$A_{s\,v}$ cross-sectional area of the shear reinforcement.

s spacing of the stirrups measured along the longitudinal axis.

f_{yv} design tensile strength of shear reinforcement.

Results obtained are written for each end in the CivilFEM results file as the following parameters:

V_s Shear strength of the reinforcement.

VRD3 Design shear resistance.

$$VRD3 = V_{Rd3}$$

CRVRD3 Ratio of the design shear force V to the shear resistance V_{Rd3}.

$$CRVRD3 = \frac{V}{V_{Rd3}}$$

If $V_{Rd3} = 0$, CRVRD3 is taken as 2^{100}.

4) **Obtaining the shear criterion.** The shear criterion indicates the validity of the section (if less than 1, the section will be valid; if greater than 1 the section, is not good). Moreover, it provides information with regards to how much more load the section can resist. The shear criterion is defined as follows:

$$CRT\_TOT = Max(CRVRD1; CRVRD3) \leq 1$$

For each element end, calculated results are written in the CivilFEM results file in the parameter CRT_TOT.

A value of 2^{100} in this criterion will indicate that shear resistance (V_{Rd2}) is not been considered, as indicated in the previous step.

6.6.9.3. Shear Check with Seismic Action

Shear checking of elements according to GB50010-2010 and GB50011-2010 follows the steps below:

1) **Determining the factor for seismic fortification, used to adjust the shear capacity and performing the check for shear.** Firstly, this checking method differs from the other typical checking methods:

$$V \leq V_R/Y_{Re}$$

V Design shear force

V_R/γ_{RE} Design shear resistance

γ_{RE} factor for seismic fortification, used to adjust the shear capacity. If the combination of the cases does not include the horizontal seismic action, γ_{RE}=1.

Otherwise, it is selected as illustrated in the following table.

TABLE 10-2 FACTORS FOR SEISMIC FORTIFICATION

Member	Status	γ_{RE}
Beam	Bending	0.75
Column	Eccentric compression and $\frac{N}{f_c A} \leq 0.15$	0.75
	Eccentric compression and $\frac{N}{f_c A} \leq 0.15$	0.8
Shear wall	Eccentric compression	0.85
Other	Shear Eccentric tension	0.85

2) **Checking whether section dimensions meet requirements under the actions of seismic loads.** First, a check is made to ensure the design shear (V) is less than or equal to sectional maximum possible resistance (V_{Rd1}) under the seismic loads:

$$V \leq V_{Rd1}$$

For beam:

$$V_{Rd1} = \frac{1}{\gamma_{RE}} 0.2\beta_c f_c b h_0 \quad \text{for } l/h_0 > 2.5$$
$$V_{Rd1} = \frac{1}{\gamma_{RE}} 0.15\beta_c f_c b h_0 \quad \text{for } l/h_0 \leq 2.5$$

Where:

$\boldsymbol{h_0}$ effective height of the section

$\boldsymbol{l}$ Length between restraints

For column:

$$V_{Rd1} = \frac{1}{\gamma_{RE}} 0.20\beta_c f_c b h_0 \quad \text{for } \lambda > 2$$
$$V_{Rd1} = \frac{1}{\gamma_{RE}} 0.15\beta_c f_c b h_0 \quad \text{for } \lambda \leq 2$$
$$\lambda = M/(Vh_0)$$

VRD1 Maximum possible shear resistance.

$$VRD1 = V_{Rd1}$$

CRVRD1 Ratio of the design shear force V to the resistance V_{Rd1}.

$$CRVRD1 = \frac{V}{V_{Rd1}}$$

3) Checking whether shear reinforcement will be required for the section under actions of seismic loads.

If the member is a beam, axial forces are not present (N=0), and the shear force from the concentrated load is less than 75%:

$$V_{Rd2} = \frac{1}{\gamma_{RE}} 0.42 f_t b h_0$$

If the member is an independent beam and the shear force from concentrated load is more than 75%:

$$V_{Rd2} = \frac{1}{\gamma_{RE}} \frac{1.05}{\lambda + 1} f_t b h_0$$

If the member is a column and N is compressive (N < 0)

$$V_{Rd2} = \frac{1}{\gamma_{RE}} \left[\frac{1.05}{\lambda + 1} f_t b h_0 - 0.056N\right]$$

If N is tensile (N > 0)

$$V_{Rd2} = \frac{1}{\gamma_{RE}} \left[\frac{1.05}{\lambda + 1} f_t b h_0 - 0.2N\right]$$
$$V_{Rd2} \geq 0$$

The following are given in CivilFEM results:

VRD2 Maximum design shear force resisted by the section without crushing of the concrete compressive struts.

$$VRD2 = V_{Rd2}$$

CRVRD2 Ratio of the design shear force V to the resistance V_{Rd2}.

$$CRVRD2 = \frac{V}{V_{Rd2}}$$

For sections subjected to an applied tensile axial force so that $V_{Rd2} = 0$, CRVRD2 is taken as 2^{100}.

4) **Checking of elements that will require shear reinforcement under the actions of seismic loads.** The calculated of the shear resistance of a section with reinforcement (V_{Rd3}) differs according to whether the concentrated load exists.

The following condition is checked:

$$V \leq V_{Rd3}$$

where

$$V_{Rd3} = V_{Rd2} + V_s$$

V_s is the design shear load capacity of reinforcement.

$$V_s = \alpha_{cf} f_{yv} \left(\frac{A_{sv}}{S}\right) h_c$$

$\alpha_{cf} = 1$

$A_{s\,v}$ is the cross-sectional area of the shear reinforcement.

s is the spacing of the stirrups measured along the longitudinal axis.

f_{yv} is the design tensile strength of shear reinforcement.

Results obtained are written for each end in the CivilFEM results file as the following parameters:

V_s Shear strength of the reinforcement.

VRD3 Design shear resistance.

$$VRD3 = V_{Rd3}$$

CRVRD3 Ratio of the design shear force V to the shear resistance V_{Rd3}.

$$CRVRD3 = \frac{V}{V_{Rd3}}$$

If $V_{Rd3} = 0$, CRVRD3 is taken as 2^{100}.

5) **Obtaining the shear criterion.** The shear criterion indicates the validity of the section (if less than 1, the section conforms to code specifications; if greater than 1, the section is not valid). Moreover, it provides information with regards to how much more load section can resist. The shear criterion is defined as follows:

$$\text{CRT\_TOT} = \text{Max}(\text{CRVRD1} \; ; \text{CRVRD3}) \leq 1$$

For each element end, calculated results are written in the CivilFEM results file in the parameter CRT_TOT.

A value of 2^{100} for this criterion indicates that shear resistance (V_{Rd2}) is not considered, as indicated in the previous step.

6.6.9.4. Torsion Check

The torsion checking according to GB50010-2010 follows the steps below:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated to the transverse cross section and for the active time.

   The required data are the following:

f_c	design compressive strength of concrete.
f_t	design tensile strength of concrete.
f_y	design tensile strength for torsion reinforcement.

2) **Obtaining section geometrical data.** Required data for shear checking are the following ones:

A_c	total cross-sectional area of the concrete section.
t_w	thickness of a box section (TWY)

3) **Obtaining geometrical parameters depending on specified code.** The required data are the following:

b	minimum width of the section over the effective depth or section inner diameter for circular section.
h_0	height of the section or section outer diameter for circular section.
h_w	the web height.
W_t	Plastic resistance of torsion moment.
A_{cor}	Core area.
U_{cor}	Core perimeter.

W_{t1} Plastic resistance of torsion moment for branch 1 for T and double T section/I-section.

A_{cor1} Core area for branch 1 for T and double T section/I-section.

U_{cor1} Core perimeter for branch 1 for T and double T section/I-section.

W_{t2} Plastic resistance of torsion moment for branch 2 for T and double T section/I-section.

A_{cor2} Core area for branch 2 for T and double T section/I-section.

U_{cor2} Core perimeter for branch 2 for T and double T section/I-section.

Section 11-A.7 “Previous Considerations to Shear and Torsion Calculation” provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining the reinforcement data section.** The required data are:

Transverse Reinforcement

A_{st}/S transverse reinforcement area per length unit.

Alternatively, the amount of the reinforcement can be calculated from:

A_{st} critical tensile zone.

s spacing between stirrups.

Or from the data below:

s spacing between stirrups.

ϕ_t diameter of the bar of the stirrup.

Longitudinal Reinforcement

A_{sl} Total area of the longitudinal reinforcement.

Alternatively, the amount of the reinforcement can determined from:

ϕ_I Longitudinal bar diameter.

N Longitudinal bar number.

5) **Obtaining section internal forces and moments**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	Description
T	Design torsion moment
N	Axial force

6) Checking if the section dimensions meet the requirement.

$$T \leq T_{Rd1}$$

$$\text{if } h_w/b \leq 4 \text{ or } h_w/t_w \leq 4 \text{ then } T_{Rd1} = 0.2W_t\beta_c f_c$$

$$\text{if } h_w/b \leq 6 \text{ or } h_w/t_w \leq 6 \text{ then } T_{Rd1} = 0.16W_t\beta_c f_c$$

Results are written in the CivilFEM results file for both element ends as the parameters:

TRD1 Maximum possible resistance of torsional moment

$$TRD1 = T_{Rd1}$$

CRTRD1 Ratio of the design torsional moment T to the resistance T_{Td1}.

$$CRTRD1 = \frac{T}{T_{Rd1}}$$

7) Calculating the maximum torsional moment resisted without reinforcements.

$$T \leq T_{Rd2}$$

Where

For rectangular and circular sections:

$$T_{Rd2} = 0.35f_tW_t - 0.07\frac{N}{A_c}W_t$$

N (< 0) is the compressive axial force, if $|N| > 0.3f_cA$, assume $|N| = 0.3f_cA$.

For box sections (axial forces cannot be resisted):

$$T_{Rd2} = 0.35\alpha_h f_t W_t$$

α_h is the influence coefficient of the wall thickness of the box section.

$$\alpha_h = 2.5t_w/b_h, \text{ if } \alpha_h > 1.0, \text{ assume, } \alpha_h = 1.0.$$

For T and double T sections/I-sections, these are divided into rectangle sections and therefore, follow the procedure according to rectangular sections.

Results are written in the CivilFEM results file for both element ends as the parameters:

TRD2 Maximum design torsional moment resisted by the section without crushing the concrete compressive struts.

$$TRD2 = T_{Rd2}$$

CRTRD2 Ratio of the design torsional moment T to the resistance TRd2.

$$CRTRD2 = \frac{T}{T_{Rd2}}$$

8) Calculating the maximum torsional moment resisted by the reinforcement. The design torsional moment T must be less than or equal to the maximum design torsional moment resisted by concrete and the reinforcement (T_{Rd2}); as a result, the following condition must be satisfied:

$$T \leq T_{Rd3} = T_{Rd2} + T_s$$

where

$$T_s = 1.2\sqrt{\xi} f_y \frac{A_{st1} A_{cor}}{S}$$

$\zeta = \frac{A_{stl} S}{A_{st1} U_{cor}}$ the ratio between longitudinal reinforcement and hoop reinforcement strength $0.6 \leq \zeta \leq 1.7$

if $\zeta > 1.7$, assume $\zeta = 1.7$

Calculated results are written in the CivilFEM results file for both element ends as the parameters:

T_s Torsion strength of the reinforcement.

TRD3 Maximum design torsional moment resisted by concrete and the torsion reinforcement.

$$TRD3 = T_{Rd3}$$

CRTRD3 Ratio of the design torsional moment T to the resistance T_{Rd3}.

$$CRTRD3 = \frac{T}{T_{Rd3}}$$

If transverse reinforcement is not defined, $T_{Rd3} = T_{Rd2}$.

9) Obtaining criterion of torsion checking.

CRT_TOT = MAX (CRTRD1, CRTRD3)

6.6.9.5. Combined Shear and Torsion Check

1) Checking for whether section dimensions meet the requirements.

$$V \leq V_{Rd1}$$
$$T \leq T_{Rd1}$$

Where

If $h_w/b \leq 4$ or $h_w/t_w \leq 4$ then $VT_{Rd1} = 0.25\beta_c f_c$

If $h_w/b \leq 6$ or h_w/t_w = 6 then $VT_{Rd1} = 0.2\beta_c f_c$

$$V_{Rd1} = (VT_{Rd1} - \frac{T}{0.8W_t})bh_0$$

$$T_{Rd1} = \left(VT_{Rd1} - \frac{V}{bh_0}\right)0.8W_t$$

Linear interpolation for $4 < h_w/b < 6$ or $4 < t_w/b < 6$

Results are written in the CivilFEM results file for both element ends as the parameters:

VRD1 Maximum shear resistance.

$$VRD1 = V_{Rd1}$$

TRD1 Maximum possible resistance of torsional moment

$$TRD1 = T_{Rd1}$$

CRVRD1 Ratio of the design shear and torsion resistance V to the shear resistance V_{Rd1}.

$$CRVRD1 = \frac{V}{V_{Rd1}}$$

CRTRD1 Ratio of the design shear torsion resistance T to the torsion resistance T_{Rd1}.

$$CRTRD1 = \frac{T}{T_{Rd1}}$$

2) Checking whether the section will require reinforcement.

If $V \leq V_{Rd2}$ where $V_{Rd2} = 0.35 f_t bh_0$ or $V_{Rd2} = 0.875 f_t bh_0/(\lambda + 1)$

No shear reinforcement is necessary.

If $T \leq T_{Rd2}$ where $T_{Rd2} = 0.175 f_t W_t$ or $T_{Rd2} = 0.175\alpha_h f_t W_t$,

No torsion reinforcement is necessary.

Results are written in the CivilFEM results file for both element ends as the parameters:

VRD2 Design shear resistance without considering the reinforcement.

$$\mathrm{VRD2} = V_{Rd2}$$

CRVRD2 Ratio of the design shear force V to the resistance V_{Rd1}.

$$\mathrm{CRVRD2} = \frac{V}{V_{Rd2}}$$

For sections subjected to an axial tensile force so that V_{Rd2}=0, CRVRD1 is taken as 2^{100}.

TRD2 Maximum design torsional moment resisted by the section without crushing the concrete compressive struts.

$$\mathrm{TRD2} = T_{Rd2}$$

CRTRD2 Ratio of the design torsional moment T to the resistance T_{Rd1}.

$$\mathrm{CRTRD2} = \frac{T}{T_{Rd2}}$$

If reinforcement has been defined:

$$V_c = 0.7(1.5 - \beta_t) f_t b h_0$$

$$T_c = 0.35 \alpha_h \beta_t f_t W_t$$

α_h The wall thickness influence coefficient for box sections, $\alpha_h = 2.5 t_w / b_h$, if $\alpha_h > 1$. or for sections other than box, assume $\alpha_h = 1.0$.

$\beta_t = \frac{1.5}{1+0.5\frac{VW_t}{Tbh_0}}$ Torsion reduction coefficient for elements under shear and torsion.

$$\text{if } \beta_t < 0.5, \textit{assume } \beta_t = 0.5;$$
$$\text{if } \beta_t > 1.0, \textit{assume } \beta_t = 1.0;$$

For compressed rectangle section frame columns:

$$V_c = (1.5 - \beta_t)(\frac{1.75}{\lambda+1} f_t b h_0 - 0.07N) \qquad V_s = f_{yv}\frac{A_{sv}}{S} h_0$$

$$T_c = \beta_t(0.35 f_t - 0.07\frac{N}{A})W_t$$

Results obtained are written for each end in the CivilFEM results file as the following parameters:

VRD2 Shear strength of concrete.

$$VRD2 = V_C$$

T_c Torsion strength of concrete.

3) Calculating the maximum load that can be resisted by the reinforcement.

$$V \leq V_{Rd3}$$
$$T \leq T_{Rd3}$$

where

$$V_{Rd3} = V_c + V_s \qquad T_{Rd3} = T_c + T_s$$

$$V_s = 1.25 f_{yv}\frac{A_{sv}}{S} h_0$$

$$T_s = 1.2\sqrt{\zeta}\frac{f_y A_{st1} A_{cor}}{S}$$

For compressed rectangle section frame columns:

$$V_s = f_{yv}\frac{A_{sv}}{S} h_0$$

Results obtained are written for each end in the CivilFEM results file as the following parameters:

VRD3 Design shear resistance.

$$VRD3 = V_{Rd3}$$

CRVRD3 Ratio of the design shear force (V) to the shear resistance V_{Rd3}.

$$CRVRD3 = \frac{V}{V_{Rd3}}$$

If $V_{Rd3} = 0$, CRVRD3 is taken as 2^{100}.

TRD3 Maximum design torsional moment resisted by the torsion reinforcement.

$$TRD3 = T_{Rd3}$$

CRTRD3 Ratio of the design torsional moment T to the resistance T_{Rd3}.

$$CRTRD3 = \frac{T}{T_{Rd3}}$$

If transverse reinforcement is not defined, T_{Rd3}=0, and the criterion would be assigned a value of 2^{100}.

4) **Obtaining the criterion of shear & torsion checking.**

This criterion considers pure shear, pure torsion, shear-torsion and ultimate strength condition of concrete criteria. The criterion determines whether the section is valid and is defined as follows

CRT_TOT= MAX(CRVRD1, CRVRD3, CRTRD1, CRTRD3)

For each end, the value of this criterion is stored in the CivilFEM results file as the parameter CRT_TOT.

6.6.9.6. Shear Design

Elements shear design according to GB50010-2010 follows the steps below:

1) **Obtaining materials strength properties.** The required data are the following:

   f_c design compressive strength of concrete.

   f_t design tensile strength of concrete.

   f_{yv} design tensile strength for of shear reinforcement.

2) **Obtaining geometrical data of the section.** Required data for shear checking are the following:

   A_c total cross-sectional area of the concrete section.

3) **Obtaining geometrical parameters depending on specified code.** Required data are the following:

   b minimum width of the section over the effective depth.

   h_0 effective height of the section.

   h_w the web height.

4) **Obtaining reinforcement data of the section.** Required data are the following:

A_s/S area of reinforcement per unit of length.

α angle between shear reinforcement and the longitudinal axis of the member.

5) **Obtaining the section internal forces and moments**. The shear force that acts on the section as well as the concomitant axial force and bending moment are obtained from the CivilFEM results file (.RCF).

Force	Description
V	Design shear force
N	Axial force

6.6.9.7. Shear Design without Seismic Action

1) **Checking whether the section dimensions meet the requirement.** Firstly, a check is made to ensure the design shear (V) is less than or equal to the maximum resistance of the section (V_{Rd1}):

$$V \leq V_{Rd1}$$

$$\text{If } hw/b \leq 4\text{ , } V_{Rd1} = 0.25\beta_c f_c bh_0$$

$$\text{If } hw/b \geq 6\text{ , } V_{Rd1} = 0.2\beta_c f_c bh_0$$

where β_c is a coefficient depending on the concrete strength:

- For concrete C50 (fc= 23.1 N/mm^2) or under, β_c =1. 0;
- For concrete C80 (fc= 35.9 N/mm^2), β_c =0.8,
- For concrete C55-75, a linear interpolation is made for β_c according to the values of fc.

Results are written for each end in the CivilFEM results file as the following parameters:

VRD1 Maximum possible shear resistance.

$$VRD1 = V_{Rd1}$$

CRVRD1 Ratio of the design shear force V to the resistance V_{Rd1}.

$$CRVRD1 = \frac{V}{V_{Rd1}}$$

2) **Maximum shear force resisted without shear reinforcements.**

If shear reinforcement has not been defined for the section, the design shear force V must be less than the maximum design shear force that can be carried by the concrete without reinforcements (V_{Rd2}):

$$V \leq V_{Rd2}$$

Where:

$$V_{Rd2} = 0.7\beta_h f_t b h_0$$

$\beta_h = (\frac{800}{h_0})^{1/4}$ is the section height factor,

If $h_0 < 800\ mm$, assume $h_0 = 800$ mm;

if $h_0 > 2000\ mm$, assume $h_0 = 2000\ mm$.

If reinforcement has been defined, axial forces are not present (N=0), and the shear force from the concentrated load for an independent beam is less than 75%,

$$V_{Rd2} = 0.7 f_t b h_0$$

If N is compressive (N < 0):

$$V_{Rd2} = \frac{1.75}{\lambda+1} f_t b h_0 - 0.07N$$

If N is tensile (N > 0):

$$V_{Rd2} = \frac{1.75}{\lambda + 1} f_t b h_0 - 0.2N$$

The following are given in CivilFEM results:

VRD2 Maximum design shear force resisted by the section without crushing the concrete compressive struts.

$$VRD2 = V_{Rd2}$$

CRVRD2 Ratio of the design shear force V to the resistance V_{Rd2}.

$$CRVRD2 = \frac{V}{V_{Rd2}}$$

For sections subjected to an axial tensile force so that V_{Rd2}=0, CRVRD2 is taken as 2^{100}.

If design shear force is greater than the shear force required to crush the concrete compressive struts, the reinforcement design will not be feasible; as a result, the parameter pertaining to the reinforcement data will be defined as 2^{100}:

$$\mathrm{ASSH} = \frac{\mathrm{Asv}}{\mathrm{s}} = 2^{100}$$

In this case, the element will be labeled as not designed, and the program will advance to the next element.

If there is no crushing by oblique compression, the calculation process continues.

3) Determining the shear strength contribution of the required transverse reinforcement. The condition for the validity of the section subjected to shear force is:

$$\mathrm{V} \leq \mathrm{V_{Rd3}} = \mathrm{V_{Rd2}} + \mathrm{V_s}$$

V_s shear reinforcement contribution.

Therefore, the reinforcement contribution should be:

$$\mathrm{V_s} = \mathrm{V_{Rd3}} - \mathrm{V_{Rd2}} = \mathrm{V} - \mathrm{V_{Rd2}}$$

For each element end, the V_s value is included in the CivilFEM results file as the parameter:

$$\mathrm{VRD3} = \mathrm{V_s}$$

4) Calculating the required transverse reinforcement ratio. Once the required shear strength of the reinforcement has been obtained, the reinforcement can be calculated from the equation below:

$$\frac{\mathrm{A_{sv}}}{\mathrm{S}} = \frac{\mathrm{V} - \mathrm{V_{Rd2}}}{\alpha_{cf} \mathrm{f_{yv}} \mathrm{h_0}}$$

where:

$\alpha_{cf} = 1$

A_{sv}	cross-sectional area of the shear reinforcement.
s	spacing of the stirrups measured along the longitudinal axis.

The area of designed reinforcement per unit length is stored in the CivilFEM results file as the parameter:

$$\mathrm{ASSH} = \frac{\mathrm{A_{sv}}}{\mathrm{S}}$$

In this case, the element will be labeled as designed (provided that design process is correct for both element sections).

If the section is labeled as not designed, the reinforcement will be defined as 2^{100}.

6.6.9.8. *Shear Design with Seismic Action*

Shear checking of elements according to GB50010-2010 and GB50011-2010 follows the steps below:

1) **Determining the factor for seismic fortification, used to adjust the shear capacity and performing the check for shear.** Firstly, this checking method differs from the other typical checking methods:

$$V \leq V_R/Y_{Re}$$

V Design shear force

V_R/γ_{RE} Design shear resistance

γ_{RE} factor for seismic fortification, used to adjust the shear capacity. If the combination of the cases does not include the horizontal seismic action, γ_{RE}=1.

Otherwise, it is selected as illustrated in the following table.

TABLE 10-3 FACTORS FOR SEISMIC FORTIFICATION

Member	Status	γ_{RE}
Beam	Bending	0.75
Column	Eccentric compression and $\frac{N}{f_c A} \leq 0.15$	0.75
	Eccentric compression and $\frac{N}{f_c A} \leq 0.15$	0.8
Shear wall	Eccentric compression	0.85
Other	Shear Eccentric tension	0.85

2) **Checking whether section dimensions meet requirements under the actions of seismic loads.** First, a check is made to ensure the design shear (V) is less than or equal to the maximum resistance of the section (V_{Rd1}) under the seismic loads:

$$V \leq V_{Rd1}$$

For beams:

$$V_{Rd1} = \frac{1}{\gamma_{RE}} 0.2\beta_c f_c b h_0$$

For columns:

$$V_{Rd1} = \frac{1}{\gamma_{RE}} 0.20\beta_c f_c b h_0 \quad \text{for } \lambda > 2$$
$$V_{Rd1} = \frac{1}{\gamma_{RE}} 0.15\beta_c f_c b h_0 \quad \text{for} \lambda \leq 2$$
$$\lambda = M/(Vh_0)$$

VRD1 Maximum shear resistance.

$$VRD1 = V_{Rd1}$$

CRVRD1 Ratio of the design shear force V to the resistance V_{Rd1}.

$$CRVRD1 = \frac{V}{V_{Rd1}}$$

The design process stops if CRVRD1>1.0

3) Maximum shear force resisted without shear reinforcements under the actions of seismic loads.

If the member is a beam, axial forces are not present (N=0), and the shear force from the concentrated load is less than 75%:

$$V_{Rd2} = \frac{1}{\gamma_{RE}} 0.42 f_t b h_0$$

If the member is an independent beam and the shear force from the concentrated load is more than 75%,

$$V_{Rd2} = \frac{1}{\gamma_{RE}} \frac{1.05}{\lambda + 1} f_t b h_0$$

If the member is a column and N is compressive (N < 0)

$$V_{Rd2} = \frac{1}{\gamma_{RE}} \left[\frac{1.05}{\lambda + 1} f_t b h_0 - 0.056N\right]$$

If N is tensile (N > 0)

$$V_{Rd2} = \frac{1}{\gamma_{RE}} \left[\frac{1.05}{\lambda + 1} f_t b h_0 - 0.2N\right]$$
$$V_{Rd2} \geq 0$$

The following are given in CivilFEM results:

VRD2 Maximum design shear force resisted by the section without crushing of the concrete compressive struts.

$$VRD2 = V_{Rd2}$$

CRVRD2 Ratio of the design shear force V to the resistance V_{Rd2}.

$$CRVRD2 = \frac{V}{V_{Rd2}}$$

For sections subjected to an axial tensile force so that V_{Rd2}=0, CRVRD2 is taken as 2^{100}.

The design process stops if CRVRD2=1.0 because the reinforcement will not be required for the strength (minimum reinforcements are still necessary).

4) **Determining the shear strength contribution of the required transverse reinforcement.** The condition for the validity of the section concerning shear force is:

$$V \leq V_{Rd3} = V_{Rd2} + V_s$$

V_s shear reinforcement contribution.

Therefore, the reinforcement contribution should be:

$$V_s = V_{Rd3} - V_{Rd2} = V - V_{Rd2}$$

For each element end, the V_s value is included in the CivilFEM results file as the parameter:

$$VRD3 = V_s$$

5) **Calculating the required transverse reinforcement ratio.** Once the required shear strength of the reinforcement has been obtained, the reinforcement area per unit length can be calculated:

$$\frac{A_{sv}}{S} = \frac{V - V_{Rd2}}{\alpha_{cf} f_{yv} h_0} \text{x } \gamma_{RE}$$

where:

$\alpha_{cf} = 1.0$

A_{sv} cross-sectional area of the shear reinforcement.

s spacing of the stirrups measured along the longitudinal axis.

The area of the designed reinforcement per unit length is stored in the CivilFEM results file as the parameter:

$$ASSH = \frac{A_{sv}}{S}$$

In this case, the element will be labeled as designed (provided that design process is correct for both element sections).

If the design is not possible, the reinforcement will be assigned the value 2^{100}.

6.6.9.9. Torsion Design

Torsion checking according to GB50010-2010 follows the steps below:

1) **Obtaining materials strength properties.** These properties are obtained from the material properties associated to the transverse cross section and for the active time.

   The required data are the following:

   f_c design compressive strength of concrete.

   f_t design tensile strength of concrete.

   f_y design tensile strength for torsion reinforcement.

2) **Obtaining geometrical data of the section.** Required data for shear checking are the following:

   A_c total cross-sectional area of the concrete section.

   t_w thickness of a box section (TWY)

3) **Obtaining geometrical parameters depending on specified code.** The required data are the following:

   b minimum width of the section over the effective depth or section inner diameter for circular section.

   h_0 height of the section or outer diameter for circular section.

   h_w the web height.

   W_t Plastic resistance of torsion moment

   A_{cor} Core area

   U_{cor} Core perimeter

W_{t1} Plastic resistance of torsion moment for branch 1 for T and double T section/I-section.

A_{cor1} Core perimeter for branch 1 for T and double T section/I-section.

U_{cor1} Core perimeter for branch 1 for T and double T section/I-section.

W_{t2} Plastic resistance of torsion moment for branch 2 for T and double T section/I-section.

A_{cor2} Core perimeter for branch 2 for T and double T section/I-section.

U_{cor2} Core perimeter for branch 2 for T and double T section/I-section.

Section 6.6.1.. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** Required data are the following ones:

Transverse Reinforcement

A_{st}/S Area of transverse reinforcement per unit length.

Longitudinal Reinforcement

A_{sl} total area of the longitudinal reinforcement.

5) **Obtaining section internal forces and moments**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	Description
T	Design torsion moment
N	Axial force

6) **Checking if the section dimensions meet the requirement.**

$$T \leq T_{Rd1}$$

$$\text{If } h_w/b \leq 4(\text{or } h_w/t_w \leq 4)\text{then } T_{Rd1} = 0.2W_t\beta_c f_c$$

$$\text{If } h_w/b = 6(\text{or } h_w/t_w = 6)\text{then } T_{Rd1} = 0.16W_t\beta_c f_c$$

Results are written in the CivilFEM results file for both element ends as the parameters:

TRD1 Maximum resistance of torsional moment

$$TRD1 = T_{Rd1}$$

CRTRD1 Ratio of the design torsional moment T to the resistance T_{Rd1}.

$$CRTRD1 = \frac{T}{T_{Rd1}}$$

7) Calculating the maximum torsional moment resisted without reinforcement.

$$T \leq T_{Rd2}$$

where

For rectangular and circular sections

$$T_{Rd2} = 0.35 f_t W_t - 0.07 \frac{N}{A_c} W_t$$

N (< 0) compressive axial force, if $|N| > 0.3 f_c A$, assume $|N| = 0.3 f_c A$.

For box section (no axial force resistance),

$$T_{Rd2} = 0.35 \alpha_h f_t W_t$$

α_h α_h The influence coefficient of the wall thickness of the box section.

$$\alpha_h = 2.5 t_w / b_h, \text{ if } \alpha_h > 1.0, assume, \alpha_h = 1.0$$

For T and double T sections, these are divided into rectangle sections, following the proceedure according to rectangular sections.

Results are written in the CivilFEM results file for both element ends as the parameters:

TRD2 Maximum design torsional moment resisted by the section without crushing of the concrete compressive struts.

$$TRD2 = T_{Rd2}$$

CRTRD2 Ratio of the design torsional moment T to the resistance T_{Rd1}.

$$CRTRD2 = \frac{T}{T_{Rd2}}$$

8) Calculating the required transverse reinforcement ratio. The design torsional moment T must be less than or equal to the maximum design torsional moment resisted by concrete and the reinforcement (T_{Rd2}); consequently, the following condition must be satisfied:

$$T \leq T_{Rd3} = T_{Rd2} + T_s$$

Where:

$$T_s = 1.2\sqrt{\zeta} f_y \frac{A_{st1} A_{cor}}{S}$$

$\zeta = \frac{A_{stl} S}{A_{st1} U_{cor}}$ is the ratio between longitudinal reinforcement and hoop reinforcement strength $0.6 \leq \zeta \leq 1.7$; $\zeta > 1.7$ if, assume $\zeta = 1.7$

The required transverse reinforcement is given by this expression:

$$\frac{A_{st1}}{S} = \frac{T - T_{Rd2}}{1.2\sqrt{\zeta} f_y A_{cor}}$$

The area of the designed transverse reinforcement per unit length is stored in the CivilFEM results file as the parameter:

$$ASTT = \frac{A_{st}}{S}$$

9) Calculating the required longitudinal reinforcement ratio. The longitudinal reinforcement is calculated from:

$$A_{stl} = \frac{\zeta A_{st1} U_{cor}}{S}$$

where:

A_{stl} area of the designed longitudinal reinforcement.

f_{yv} hoop reinforcements

The area of the designed longitudinal reinforcement is stored in the CivilFEM results file as the parameter:

$$ASLT = A_{stl}$$

If both transverse and longitudinal reinforcements are designed for both element sections, this element will be labeled as designed.

6.6.9.10. Combined Shear and Torsion Design

The check of sections subjected to shear force and concomitant torsional moment we follow the steps below:

1) Obtaining material strength properties. The required data are the following:

f_c design compressive strength of concrete.

f_t design tensile strength of concrete.

f_y design tensile strength for torsion reinforcements

f_{yv} design tensile strength for shear hoop reinforcements

2) **Obtaining geometrical data of the section.**

A_c total cross-sectional area of the concrete section.

3) **Obtaining geometrical parameters depending on code.** Required data are the following:

b minimum width of the section over the effective depth or section inner diameter for circular section.

h_0 effective height of the section or outer diameter for circular section.

h_w the web height.

W_{t1} Plastic resistance of torsion moment for branch 1 for T and double T section/I-section.

A_{cor1} Core perimeter for branch 1 for T and double T section/I-section.

U_{cor1} Core perimeter for branch 1 for T and double T section/I-section.

W_{t2} Plastic resistance of torsion moment for branch 2 for T and double T section/I-section.

A_{cor2} Core perimeter for branch 2 for T and double T section/I-section.

U_{cor2} Core perimeter for branch 2 for T and double T section/I-section.

4) **Obtaining reinforcement data of the section.** Required data are the following:

Shear Reinforcement

A_{sv}/S area of reinforcement per unit length.

Transverse Torsion Reinforcement

A_{st1}/S area of reinforcement per unit length.

Torsion Longitudinal Reinforcement

A_{stl} total area of the longitudinal reinforcement.

5) **Obtaining the section internal forces and moments**. The shear force that acts on the section, as well as the concomitant axial force and bending moment, are obtained from the CivilFEM results file (.RCF).

Force	Description
V	Design shear force
N	Axial force
T	Design torsion moment

1) Checking for whether section dimensions meet the requirements.

$$V \leq V_{Rd1}$$
$$T \leq T_{Rd1}$$

Where

If $h_w/b \leq 4$ or $h_w/t_w \leq 4$ then $VT_{Rd1} = 0.25\beta_c f_c$

If $h_w/b \leq 6$ or h_w/t_w = 6 then $VT_{Rd1} = 0.2\beta_c f_c$

$$V_{Rd1} = (VT_{Rd1} - \frac{T}{0.8W_t})bh_0$$

$$T_{Rd1} = \left(VT_{Rd1} - \frac{V}{bh_0}\right)0.8W_t$$

Linear interpolation for $4 < h_w/b < 6$ or $4 < t_w/b < 6$

Results are written in the CivilFEM results file for both element ends as the parameters:

VRD1 Maximum shear resistance.

$$VRD1 = V_{Rd1}$$

TRD1 Maximum possible resistance of torsional moment

$$TRD1 = T_{Rd1}$$

CRVRD1 Ratio of the design shear and torsion resistance V to the shear resistance V_{Rd1}.

$$CRVRD1 = \frac{V}{V_{Rd1}}$$

CRTRD1 Ratio of the design shear torsion resistance T to the torsion resistance T_{Rd1}.

$$CRTRD1 = \frac{T}{T_{Rd1}}$$

2) Checking whether the section will require reinforcement.

If $V \leq V_{Rd2}$ where $V_{Rd2} = 0.35f_tbh_0$ or $V_{Rd2} = 0.875f_tbh_0/(\lambda + 1)$

Then no shear reinforcement is necessary.

If $T \leq T_{Rd2}$ where $T_{Rd2} = 0.175f_tW_t$ or $T_{Rd2} = 0.175\alpha_hf_tW_t$,

Then no torsion reinforcement is necessary.

Results are written in the CivilFEM results file for both element ends as the parameters:

VRD2 Design shear resistance without considering the reinforcement.

$$VRD2 = V_{Rd2}$$

CRVRD2 Ratio of the design shear force V to the resistance V_{Rd1}.

$$CRVRD2 = \frac{V}{V_{Rd2}}$$

For sections subjected to an axial tensile force so that V_{Rd2}=0, CRVRD1 is taken as 2^{100}.

TRD2 Maximum design torsional moment resisted by the section without crushing the concrete compressive struts.

$$TRD2 = T_{Rd2}$$

CRTRD2 Ratio of the design torsional moment T to the resistance T_{Rd1}.

$$CRTRD2 = \frac{T}{T_{Rd2}}$$

If reinforcement has been defined:

$$V_c = 0.7(1.5 - \beta_t)f_tbh_0$$

$$T_c = 0.35\alpha_h\beta_tf_tW_t$$

α_h The wall thickness influence coefficient for box sections, $\alpha_h = 2.5t_w/b_h$, if $\alpha_h > 1$. or for sections other than box, assume $\alpha_h = 1.0$.

$\beta_t = \frac{1.5}{1+0.5\frac{VW_t}{Tbh_0}}$ Torsion reduction coefficient for elements under shear and torsion.

$$\text{if } \beta_t < 0.5, assume\ \beta_t = 0.5;$$
$$\text{if } \beta_t > 1.0, assume\ \beta_t = 1.0;$$

For compressed rectangle section frame columns:

$$V_c = (1.5 - \beta_t)(\frac{1.75}{\lambda + 1} f_t b h_0 - 0.07N)$$

$$T_c = \beta_t(0.35 f_t - 0.07 \frac{N}{A}) W_t$$

Results obtained are written for each end in the CivilFEM results file as the following parameters:

VRD2 Shear strength of concrete.

$$VRD2 = V_C$$

T_c Torsion strength of concrete.

3) Calculating the maximum load that can be resisted by the reinforcement.

$$V \leq V_{Rd3}$$
$$T \leq T_{Rd3}$$

where

$$V_{Rd3} = V_c + V_s \qquad T_{Rd3} = T_c + T_s$$

$$V_s = 1.25 f_{yv} \frac{A_{sv}}{s} h_0$$

$$T_s = 1.2\sqrt{\zeta} \frac{f_y A_{st1} A_{cor}}{s}$$

For compressed rectangle section frame columns:

$$V_s = f_{yv} \frac{A_{sv}}{s} h_0$$

Results obtained are written for each end in the CivilFEM results file as the following parameters:

VRD3 Design shear resistance.

$$VRD3 = V_{Rd3}$$

CRVRD3 Ratio of the design shear force (V) to the shear resistance V_{Rd3}.

$$CRVRD3 = \frac{V}{V_{Rd3}}$$

If $V_{Rd3} = 0$, CRVRD3 is taken as 2^{100}.

TRD3 Maximum design torsional moment resisted by the torsion reinforcement.

$$TRD3 = T_{Rd3}$$

CRTRD3 Ratio of the design torsional moment T to the resistance T_{Rd3}.

$$CRTRD3 = \frac{T}{T_{Rd3}}$$

If transverse reinforcement is not defined, T_{Rd3}=0, and the criterion would be assigned a value of 2^{100}.

6) Obtaining required shear and torsion reinforcement ratios.

Shear:

$$\frac{A_{sv}}{S} = \frac{V - V_c}{\alpha_{cf} f_{yv} h_0}$$

Torsion:

$$\frac{A_{st1}}{S} = \frac{T - T_c}{1.2\sqrt{\zeta} f_y A_{cor}}$$

where

A_{sv} cross-sectional area of the shear reinforcement.

$\alpha_{cf} = 1.0$

A_{st1} cross-sectional area of the bars used as closed-stirrups.

s spacing of the closed stirrups of the transverse reinforcement.

f_{yv} design yield strength of torsion reinforcement.

$\zeta = \frac{A_{stl} S}{A_{st1} U_{cor}}$ the ratio between longitudinal and hoop reinforcement

reinforcement strength $0.6 \le \zeta \le 1.7$; if $\zeta > 1.7$, assume $\zeta = 1.7$

The area of the designed reinforcement per unit length is stored in the CivilFEM results file as the parameter:

$$ASSH = \frac{A_{sv}}{S}$$

$$ASTT = \frac{A_{st1}}{S}$$

7) **Calculating the required longitudinal requirement ratio.**

$$A_{stl} = \frac{\zeta A_{st1} U_{cor}}{S}$$

The area of the designed longitudinal reinforcement is stored in the CivilFEM results file as the parameter:

$$ASLT = A_{stl}$$

If both transverse and longitudinal reinforcements are designed for both element sections, this element will be labeled as designed.

6.6.11. Shear and Torsion according to AASHTO Standard Specifications for Highway Bridges

6.6.10.1. Shear Check

Shear checking according to AASHTO Standard Specifications for Highway Bridges follows these steps:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated with each transverse cross section and for the active time.

The required data are the following ones:

f'_c specified compressive strength of concrete.

f_{yk} specified yield strength of reinforcement.

2) **Obtaining geometrical data of the section.** Required data for shear checking are the following:

A_g area of concrete section.

3) **Obtaining geometrical parameters depending on specified code.** The required data are the following:

b_w web width or diameter of circular section.

d distance from the extreme compressed fiber to the centroid of the longitudinal tensile reinforcement in the Y direction, (for circular sections, this should be greater than the distance from the extreme compressed fiber to the centroid of the tensile reinforcement in the opposite half of the member).

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining section reinforcement data.** Required data are the following:

α angle between shear reinforcement and the longitudinal axis of the member section.

A_s/S area of reinforcement per unit length.

The reinforcement ratio may also be obtained with the following data:

A_s total area of the reinforcement legs.

s spacing of the stirrups.

or with the following ones:

s spacing of the stirrups.

ϕ diameter of bars.

N number of reinforcement legs.

5) **Obtaining forces and moments acting on the section**. The shear force that acts on the section as well as the concomitant axial force are obtained from the CivilFEM results file (.RCF).

Force	Description
V_u	Design shear force in Y direction for I-section

6) **Calculating the shear strength provided by concrete.** First, the shear strength provided by concrete (V_c) is calculated by the following expression:

$$V_c = 2\sqrt{f_c'} b_w d$$

where:

$\sqrt{f_c'}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force:

$$V_c = 2\left(1 + \frac{N_u}{2000A_g}\right)\sqrt{f_c'} b_w d$$

Where N_u/A_g is expressed in psi.

If section is subjected to a tensile force so that the tensile stress is less than 500 psi:

$$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\sqrt{f_c'}b_w d$$

If section is subjected to a tensile force so that the tensile stress exceeds 500 psi, it is assumed $V_c = 0$.

The calculated result for both element ends is stored in the CivilFEM results file as the parameter VC:

VC Shear strength provided by concrete.

$$VC = V_c$$

7) Calculating the shear strength provided by shear reinforcement. The strength provided by shear reinforcement (V_s) is calculated with the following expression:

$$V_s = \frac{A_v}{S} f_y(\sin\alpha + \cos\alpha)d \leq 8\sqrt{f_c'}b_w d$$

where:

A_v area of the cross-section of shear reinforcement.

s spacing of the stirrups measured along the longitudinal axis.

The calculated result for both element ends is stored in the CivilFEM results file as the parameter VS:

VS Shear strength provided by transverse reinforcement.

$$VS = V_s$$

8) Calculating the nominal shear strength of the section. The nominal shear strength (V_n) is the sum of the provided by concrete and by the shear reinforcement:

$$V_n = V_c + V_s$$

This nominal strength as well as its ratio with the design shear are stored in the CivilFEM results file as the parameters:

VN Nominal shear strength.

$$VN = V_n$$

CRTVN Ratio of the design shear force (V_u) to the resistance V_n.

$$CRTVN = \frac{V_u}{V_n}$$

If the strength provided by concrete is null and the shear reinforcement is not defined in the section, then V_n=0 and the criterion will be equal to –1.

9) Obtaining shear criterion. The section will be valid for shear if the following condition is satisfied:

$$V_u \leq \phi V_n = \phi(V_c + V_s)$$

ϕ strength reduction factor of the section, (0.85 for shear and torsion).

Therefore, the shear criterion for the validity of the section is defined as follows:

$$CRT\_TOT = \frac{V_u}{\phi V_n} \leq 1$$

For each element, this value is stored in the CivilFEM results file as the parameter CRT_TOT.

If the strength provided by concrete is null and the shear reinforcement is not defined in the section, then $V_n = 0$, and the criterion will be equal to 2^{100}.

The $\phi \cdot V_n$ value is stored in CivilFEM results file as the parameter VFI.

6.6.10.2. Torsion Check

Torsion checking of elements is done according to ACI-318, with ϕ=0.85.

6.6.10.3. Combined Shear and Torsion Check

For checking sections subjected to shear force and concomitant torsional moment, the same procedure as for the ACI-318 code is followed, with ϕ=0.85.

6.6.10.4. Shear Design

The shear design according to AASHTO Specific Standards for Highway Bridges follows these steps:

1) Obtaining material strength properties. These properties are obtained from the material properties associated with each transverse cross section and for the active time.

The required data are the following:

f'_c specified compressive strength of concrete.

f_{yk} specified yield strength of reinforcement.

2) Obtaining geometrical data of the section. Required data for shear designing are the following:

A_g area of concrete section.

3) **Obtaining geometrical parameters depending on specified code.** The required data are the following:

b_w web width or diameter of the circular section.

d distance from the extreme compressed fiber to the centroid of the longitudinal tensile reinforcement in Y, (for circular sections, this must not be less than the distance from the extreme compressed fiber to the centroid of the tensile reinforcement in the opposite half of the member).

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** In shear reinforcement designing, it is possible to define the angle α between the reinforcement and the longitudinal axis of the member. This angle must be stored in the shear reinforcement data of each element. If this angle is equal to zero or is not defined, α=90º. Other data concerning the reinforcements are ignored.

5) **Obtaining forces and moments acting on the section.** The shear force that acts on the section, as well as the concomitant axial force, is obtained from the CivilFEM results file (.RCF).

Force	**Description**
V_u	Design shear force in Y

6) **Calculating the shear strength provided by concrete.** First, we calculate the shear strength provided by concrete (V_c) with the following expression:

$$V_c = 2\sqrt{f'_c}b_w d$$

where:

$\sqrt{f'_c}$ square root of specified compressive strength of concrete, in psi (always taken as less than 100 psi).

For sections subject to a compressive axial force:

$$V_c = 2\left(1 + \frac{N_u}{2000\ A_g}\right)\sqrt{f'_c}b_w d$$

Where N_u/A_g is expressed in psi.

If section is subjected to a tensile force so that the tensile stress is less than 500 psi:

$$V_c = 2\left(1 + \frac{N_u}{500\ A_g}\right)\sqrt{f_c'}b_w d$$

If section is subjected to a tensile force so that the tensile stress exceeds 500 psi, it is assumed $V_c = 0$.

The calculated result is stored in the CivilFEM results file for both element ends as the parameter:

VC Shear strength provided by concrete.

$$VC = V_c$$

7) Determining the required reinforcement contribution to the shear strength. The section must satisfy the following condition to resist the shear force:

$$V_u =\leq \phi V_n = \phi(V_c + V_s)$$

Therefore, the required shear force of the reinforcement must be:

$$V_s = \frac{V_u}{\phi} - V_c \leq 8\sqrt{f_c'}b_w\, d$$

If the required shear strength of the reinforcement does not satisfy the expression above, the section will not be designed. Consequently, the parameters for the reinforcement data will be defined as 2^{100}. Therefore:

$$ASSH = \frac{As}{S} = 2^{100}$$

In this case, the element will be labeled as not designed, and the program will then advance to the following element.

Calculated results are stored in the CivilFEM results file for both element ends as the parameter:

VS Shear resistance provided by the transverse reinforcement.

$$VS = V_s \leq 8\sqrt{f_c'}b_w d$$

8) Calculating the required reinforcement ratio. Once the required shear strength of the reinforcement has been obtained, the reinforcement can be calculated with the following expression:

$$\frac{A_v}{s} = \frac{V_s}{f_y(\sin\alpha + \cos\alpha)d}$$

Where:

A_v area of the cross-section of the shear reinforcement.

s spacing of the stirrups measured along the longitudinal axis.

The area of the designed reinforcement per unit length is stored in the CivilFEM results file for both element ends:

$$\mathrm{ASSH} = \frac{\mathrm{A_v}}{\mathrm{s}}$$

In this case, the element will be labeled as designed (providing the design procedure is correct for both element sections).

6.6.10.5. Torsion Design

Torsion reinforcements are designed according to ACI-318.

6.6.10.6. Combined Shear and Torsion Design

The design of sections subjected to shear force and concomitant torsional moment follows the method used for the ACI-318 code.

6.6.12. Shear and Torsion according to NBR6118

6.6.11.1. Shear Check

The checking for shear according to NBR6118 follows these steps:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated with each transverse cross section and for the active time.

   The required data are the following:

   f_{ck} characteristic compressive strength of concrete.

   f_{yk} characteristic yield strength of reinforcement.

   γ_c concrete partial safety factor.

   γ_s steel partial safety factor.

2) **Obtaining geometrical data of the section.** Required data for shear checking are the following ones:

   A_c total area of the concrete section.

3) Obtaining geometrical parameters depending on specified code. Geometrical parameters used for shear calculations must be defined:

b_w	minimum width of the section at a height equal to ¾ the effective depth.
d	effective depth of the section.
θ	Angle of the concrete compressive struts with the longitudinal axis of member

$$30º < \theta < 45º$$

Section 6.6.1. provides detailed information on how to calculate the required data for each code and valid section.

4) Obtaining reinforcement data of the section. Required data are the following:

α	angle between shear reinforcement and the longitudinal axis of the member.
A_s/S	area of reinforcement per unit of length.

The reinforcement ratio may also be obtained with the following data:

A_s	total area of the reinforcement legs.
s	spacing of the stirrups.

Or from the data below:

s	spacing of the stirrups.
ϕ	diameter of bars.
N	number of reinforcement legs.

5) Obtaining forces and moments acting on the section. The shear force that acts on the section is obtained from the CivilFEM results file (.RCF).

Force	**Description**
V_{Sd}	Design shear force

6) Checking failure by compression in the web. First, a check is made to ensure the design shear force (V_{Sd}) is less than or equal to the oblique compression resistance of the concrete in the web (V_{Rd2}). V_{Rd2} is calculated with **Model I** if θ = 45º and with **Model II** if $\theta \neq$ 45º:

$$V_{Sd} \leq V_{Rd2}$$

Model I

$$V_{Rd2} = 0.27 \cdot \alpha_{v2} f_{cd} b_w d$$

Where

$$\alpha_{v2} = 1 - \frac{f_{ck}}{250} \text{ (}f_{ck}\text{ in MPa).}$$

Model II

$$V_{Rd2} = 0.54 \cdot \alpha_{v2} f_{cd} b_w d \cdot \sin^2\theta \cdot (\cot\mathrm{an}\,\alpha + \cot\mathrm{an}\,\theta)$$

Where

$$\alpha_{v2} = 1 - \frac{f_{ck}}{250} \text{ (}f_{ck}\text{ in MPa).}$$

For each element end, calculated results are written in the CivilFEM results file:

VRD2 Ultimate shear strength due to oblique compression of the concrete in web.

$$VRD2 = V_{Rd2}$$

CRTVRD2 Ratio of the design shear (V_{sd}) to the resistance V_{Rd2}.

$$CRTVRD2 = \frac{V_{Sd}}{V_{Rd2}}$$

7) **Checking failure by tension in the web.** The design shear force (V_{sd}) must be less than or equal to the shear force due to tension in the web (V_{Rd3}). V_{Rd3} is calculated with **Model I** if θ = 45º and with **Model II** if θ ≠ 45º:

$$V_{Sd} \leq V_{Rd3}$$

$$V_{Rd3} = V_c + V_{sw}$$

V_{sw} contribution of web shear transverse reinforcement to the shear strength.

V_c contribution of concrete to the shear strength.

Model I

$$V_{sw} = \frac{A_{sw}}{S} 0.9 \cdot d \cdot f_{ywd} (\sin\alpha + \cos\alpha)$$

Where

A_{Sw}/S shear reinforcement area per unit length.

f_{ywd} design strength of reinforcement limited to 435 MPa.

$$V_c = \begin{cases} 0 & \text{Only tension in the section} \\ V_{c0} & \text{Tension and compression in the section} \end{cases}$$

$$V_{c0} = 0.6 \cdot f_{ctd} \cdot b_w \cdot d$$

Model II

$$V_{sw} = \frac{A_{sw}}{S} 0.9 \cdot d \cdot f_{ywd} (\cot an\alpha + \cot an\alpha) \cdot \sin\alpha$$

Where

A_{sw}/S shear reinforcement area per unit of length.

f_{ywd} design strength of reinforcement limited to 435 MPa.

$$V_c = \begin{cases} 0 & \text{Only tension in the section} \\ V_{c1} & \text{Tension and compression in the section} \end{cases}$$

$$V_{c1} = \begin{cases} V_{c0} & V_{Sd} \leq \quad V_{c0} \\ 0 & V_{Sd} = \quad V_{Rd2} \end{cases}$$ Interpolating linearly in between these values.

Where

$$V_{c0} = 0.6 \cdot f_{ctd} \cdot b_w \cdot d$$

For each end, calculated results are written in the CivilFEM results file:

VSW Contribution of the shear reinforcement to the shear strength.

$$VSW = V_{sw}$$

VC Contribution of concrete to the shear strength.

$$VC = V_c$$

VRD3 Ultimate shear strength by tension in the web.

$$VRD3 = V_{Rd3} = V_c + V_{sw}$$

CRTVRD3 Ratio of the design shear force (V_{sd}) to the resistance V_{Rd3}.

$$CRTVRD3 = \frac{V_{Sd}}{V_{Rd3}}$$

If $V_{Rd3} = 0$, the CTRVRD3 criterion is taken as 2^{100}.

8) **Obtaining shear criterion.** The shear criterion indicates whether the section is valid for the design forces (if it is less than 1, the section satisfies the code prescriptions; whereas if it exceeds 1, the section will not be valid). Furthermore, it includes information about how close the design force is to the ultimate section strength. The shear criterion is defined as follows:

$$CRT\_TOT = Max\left(\frac{V_{Sd}}{V_{Rd2}};\frac{V_{Sd}}{V_{Rd3}}\right)$$

For each element end, this value is stored in the CivilFEM results file as CRT_TOT.

A value 2^{100} for this criterion indicates that the shear strength due to tension in the web (V_{Rd3}) is equal to zero, as was indicated in the previous step.

6.6.11.2. Torsion Checking

Checking for torsion according to NBR6118 follows the steps below:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated with each transverse cross section and for the active time.

The required data are the following:

f_{ck}	characteristic strength of concrete
f_{yk}	characteristic yield strength of reinforcement
γ_c	concrete partial safety factor
γ_s	reinforcement steel partial safety factor

2) **Obtaining geometrical parameters depending on specified code.** Geometrical parameters used for torsion calculations must be defined within CivilFEM database. The required data are the following:

h_e	effective thickness.
A_e	area involved by the center-line of the effective hollow section.
U_e	perimeter of the center-line of the effective hollow section.
θ	Angle of the compressive struts of concrete with the longitudinal axis of member:

$$30º < \theta < 45º$$

Section 6.6.1.. provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining reinforcement data of the section.** Required data are the following:

Transverse Reinforcement

A_{St}/S area of transverse reinforcement per unit of length.

The reinforcement ratio can also be obtained with the following data:

A_{St} closed stirrups area for torsion.

s spacing of closed stirrups.

Or from the data below:

s spacing of closed stirrups.

ϕ_t diameter of the closed stirrups bars.

Longitudinal Reinforcement

A_{sl} total area of the longitudinal reinforcement.

The reinforcement ratio can also be obtained from the following data:

⍰ diameter of longitudinal bars.

N number of longitudinal bars.

4) **Obtaining section internal forces and moments**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	**Description**
T_{Sd}	Design torsional moment in the section

5) **Checking compression failure of concrete.** Firstly, a check is made to ensure the design torsional moment (T_{Sd}) is less than or equal to the ultimate torsional moment cause by the compression of the concrete (T_{Rd2}); therefore, the following condition must be satisfied:

$$T_{Sd} \leq T_{Rd2}$$

$$T_{Rd2} = 0.50 \cdot \alpha_{v2} \cdot f_{cd} \cdot A_e \cdot h_e \cdot \sin 20$$

Where:

$$A_{v2} = 1 - \frac{f_{ck}}{250} \text{ (}f_{ck}\text{ in MPa).}$$

Calculated results are stored in the CivilFEM results file as:

TRD2 Maximum torsional moment resisted by the section without crushing the concrete compressive struts due to compression.

$$TRD2 = T_{Rd2}$$

CRTTRD2 Ratio of the design torsional moment (T_{Sd}) to the resistance T_{Rd2}.

$$CRTTRD2 = \frac{T_{Sd}}{R_{Rd2}}$$

6) **Checking transverse reinforcement failure.** The condition for tensile failure of the transverse reinforcement when a torsional moment T_{Sd} is applied is as follows:

$$T_{Sd} \leq T_{Rd3}$$
$$T_{Rd3} = (A_{st}/s) \cdot f_{ywd} \cdot 2A_e \cdot \cot an\theta$$

where:

A_{st} cross-sectional area of one of the bars used as transverse torsional reinforcement.

s spacing of closed stirrups of transverse torsional reinforcement.

f_{ywd} design strength of reinforcement, limited to 435 MPa.

Calculated results are stored in the CivilFEM results file as:

TRD3 Maximum torsional moment resisted by the section without tensile failure of the transverse reinforcement.

$$TRD3 = T_{Rd3}$$

CRTTRD3 Ratio of the design torsional moment (T_{Sd}) to the resistance T_{Rd3}.

$$CRTTRD3 = \frac{T_{Sd}}{T_{Rd3}}$$

If the transverse torsion reinforcement is not defined, the criterion is taken as 2^{100}.

7) **Checking longitudinal reinforcement failure.** The condition of tensile failure for the longitudinal reinforcement when a torsional moment T_{Sd} is applied is as follows:

$$T_{Sd} \leq T_{Rd4}$$
$$T_{Rd4} = (A_{sl}/U_e) \cdot f_{ywd} \cdot 2A_e \cdot \tan\theta$$

where:

A_{sl} area of the longitudinal torsion reinforcement.

f_{ywd} design strength of reinforcement limited to 435 MPa.

Calculated results are stored in the CivilFEM results file as:

TRD4 Maximum torsional moment resisted by the section without tensile failure of transverse reinforcement.

$$\mathrm{TRD4} = \mathrm{T_{Rd4}}$$

CRTTRD4 Ratio of the design torsional moment (T_{Sd}) to the resistance T_{Rd3}.

$$\mathrm{CRTTRD4} = \frac{\mathrm{T_{Sd}}}{\mathrm{T_{Rd4}}}$$

In case the longitudinal reinforcement is not defined, the criterion is taken as 2^{100}.

8) **Obtaining torsion criterion.** The torsion criterion indicates the ratio of the design moment to the section ultimate strength (if it is less than 1, the section is valid; whereas if it exceeds 1, the section is not valid). The criterion for the validity of the section for torsion is defined as follows:

$$\mathrm{CRT\_TOT} = \mathrm{Max}\left(\frac{\mathrm{T_{Sd}}}{\mathrm{T_{Rd2}}};\frac{\mathrm{T_{Sd}}}{\mathrm{T_{Rd3}}};\frac{\mathrm{T_{Sd}}}{\mathrm{T_{Rd4}}}\right)$$

For each element end, this value is stored in the CivilFEM results file as CRT_TOT.

A value 2^{100} for this criterion would indicate the non-definition of one of the torsion reinforcements.

6.6.11.3. *Combined Shear and Torsion Checking*

For checking sections subjected to shear force and concomitant torsional moment, we follow the steps below:

1) **Torsion checking considering a null shear force.** This check is accomplished with the same steps as for the check of elements subjected to pure torsion according to NBR6118.

   Except for this check, the CRT_TOT criterion is stored in the CivilFEM results file as CRTTRS for each element end.

1) **Shear checking assuming a null torsional moment.** This check follows the same procedure as for the checking of elements only subjected to shear according to NBR6118.

Except for this check, the CRT_TOT criterion is stored in the CivilFEM results file as CRTSHR for each element end.

3) **Checking the concrete ultimate strength condition by compression.** The design torsional moment (T_{Sd}) and the design shear force (V_{Sd}) must satisfy the following condition:

$$\left(\frac{V_{Sd}}{V_{Rd2}}\right) + \left(\frac{T_{Sd}}{T_{Rd2}}\right) \leq 1$$

where:

V_{Rd2} ultimate shear force by compression of concrete.

T_{Rd2} ultimate torsional moment due to compression of concrete.

For each element, this criterion value is stored in the CivilFEM results file as CRTCST.

4) **Obtaining the combined shear and torsion criterion.** This criterion considers pure shear, pure torsion and concrete ultimate strength condition criteria. The criterion determines whether the section is valid and is defined as follows:

$$\mathrm{CRT\_TOT} = \mathrm{Max}\left[\frac{V_{Sd}}{V_{Rd2}};\frac{V_{Sd}}{V_{Rd3}};\frac{T_{Sd}}{T_{Rd2}};\frac{T_{Sd}}{T_{Rd3}};\frac{T_{Sd}}{T_{Rd4}};\left(\frac{V_{Sd}}{V_{Rd2}}\right)+\left(\frac{T_{Sd}}{T_{Rd2}}\right)\right]$$

For each element, this criterion value is stored in the CivilFEM results file as CRT_TOT.

A value 2^{100} for this criterion indicates that one of the denominators is null, and therefore, one of the reinforcements is not defined.

6.6.11.4. Shear Design

The shear designing according to NBR6118 follows these steps:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated with each transverse cross section and for the active time.

The required data are the following:

f_{ck} characteristic strength of concrete.

f_{yk} characteristic yield strength of reinforcement.

γ_c concrete safety factor.

γ_s steel safety factor.

2) **Obtaining section geometrical data.** Required data for shear designing are the following:

A_c total area of the concrete section.

3) **Obtaining geometrical parameters depending on specified code.** Required data are the following:

b_w minimum width of the section in a height equal to ¾ the effective depth.

d effective depth of the section.

θ angle of the concrete compressive struts with the longitudinal axis of member.

$$30º < \theta < 45º$$

Section 6.6.1.. provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** With the shear reinforcement design, it is possible to indicate the angle α btetweeen the reinforcement and the longitudinal axis of the member. If this angle is null or it is not defined, α = 90º. Other reinforcement data are ignored.

5) **Obtaining forces and moments acting on the section**. The shear force that acts on the section is obtained from the CivilFEM results file (.RCF).

Force	**Description**
V_{Sd}	Design shear force

6) **Checking compression failure in the web.** Firstly, a check is made to ensure the design shear force (V_{Sd}) is less than or equal to the oblique compression resistance of concrete in the web (V_{Rd2}). V_{Rd2} is calculated with **Model I** if ⍰ = 45º and with **Model II** if ⍰⍰⍰⍰⍰45º:

$$V_{Sd} \leq V_{Rd2}$$

Model I

$$V_{Rd2} = 0.27 \cdot \alpha_{v2} f_{cd} b_w d$$

Where

$$\alpha_{v2} = 1 - \frac{f_{ck}}{250}(f_{ck} \text{ in MPa}).$$

Model II

$$V_{Rd2} = 0.54 \cdot \alpha_{v2} f_{cd} b_w d \cdot \sin^2\theta(\cot an\,\alpha + \cot an\,\theta)$$

Where

$$\alpha_{v2} = 1 - \frac{f_{ck}}{250}(f_{ck} \text{ in MPa}).$$

For each element end, calculated results are written in the CivilFEM results file as:

VRD2 Ultimate shear strength due to oblique compression of the concrete in web.

$$VRD2 = V_{Rd2}$$

CRTVRD2 Ratio of the design shear (V_{sd}) to the resistance V_{Rd2}.

$$CRTVRD2 = \frac{V_{Sd}}{V_{Rd2}}$$

If design shear force is greater than shear force that causes the failure by oblique compression of concrete in the web, the reinforcement design is not feasible. Therefore, the parameter for the reinforcement data is defined as 2^{100}.

$$ASSH = \frac{A_{sw}}{s} 2^{100}$$

In this case, the element is labeled as not designed and the program then advances then to next element.

In the case there is no failure due to oblique compression, the calculation process continues.

7) **Checking if shear reinforcement will be required.** First, a check is made to ensure the design shear force (V_{sd}) is less than or equal to the strength provided by the concrete in members without shear reinforcement (V_c). V_{Rd3} is calculated with **Model I** if ? = 45º and with **Model II** if ?????45º:

$$V_{Sd} \le V_{Rd3}$$

$$V_{Rd3} = V_c$$

Model I

$$V_c = \begin{cases} 0 & \text{Only tension in the section} \\ V_{c0} & \text{Tension and compression in the section} \end{cases}$$

$$V_{c0} = 0.6 \cdot f_{ctd} \cdot b_w \cdot d$$

Model II

$$V_c = \begin{cases} 0 & \text{Only tension in the section} \\ V_{c1} & \text{Tension and compression in the section} \end{cases}$$

$$V_{c1} = \begin{cases} V_{c0} & V_{Sd} \leq V_{c0} \\ 0 & V_{Sd} = V_{Rd2} \end{cases}$$ Interpolating linearly in between these values.

Where

$$V_{c0} = 0.6 \cdot f_{ctd} \cdot b_w \cdot d$$

If the section does not require shear reinforcement, the following parameters are defined (for both element ends):

$$VC = V_c$$

$$VRD3 = V_{Rd3} = V_c$$

$$VSW = 0$$

$$ASSH = \frac{A_{sw}}{s} = 0$$

If the section requires shear reinforcement the calculation process continues.

8) Determining the shear strength contribution of the required transverse reinforcement.

If the section requires shear reinforcement, the condition pertaining to the validity of sections under shear force is as follows:

$$V_{Sd} \leq V_{Rd3}$$
$$V_{Rd3} = V_c + V_{sw}$$

V_{sw} contribution of web shear transverse reinforcement to the shear strength.

V_c contribution of concrete to the shear strength.

V_{sw} is calculated with **Model I** if □ = 45º and with **Model II** if □□□□45º:

Model I

$$V_{sw} = \frac{A_{sw}}{S} 0.9 \cdot d \cdot f_{ywd} (\sin\alpha + \cos\alpha)$$

Where

A_{sw}/S shear reinforcement area per unit length.

f_{ywd} design strength of reinforcement limited to 435 MPa.

Model II

$$V_{sw} = \frac{A_{sw}}{S} 0.9 \cdot d \cdot f_{ywd} (\cot an\alpha + \cot an\theta) \cdot \sin\alpha$$

Where

A_{sw}/S shear reinforcement area per unit length.

f_{ywd} design strength of reinforcement, limited to 435 MPa.

Therefore, the shear reinforcement contribution is given by the equation below:

$$V_{sw} = V_{Rd3} - V_c$$

For each element end, the value of V_c and V_{sw} is stored in the CivilFEM results file:

$$VC = V_c$$

$$VSW = V_{sw}$$

10) **Caculating the required reinforcement ratio.** Once the required shear strength of the reinforcement has been obtained, the reinforcement ratio can be calculated:

$$\frac{A_{sw}}{s} = \frac{V_{sw}}{0.9 \cdot d \cdot f_{ywd} (\sin\alpha + \cos\alpha)} \text{ for Model I}$$

$$\frac{A_{sw}}{s} = \frac{V_{sw}}{0.9 \cdot d \cdot f_{ywd} (\cot an\,\alpha + \cot an\,\alpha) \cdot \sin\alpha} \text{ for Model II}$$

Where:

A_{sw}/S cross-sectional area of the designed shear reinforcement per unit length.

f_{ywd} design strength of reinforcement, limited to 435 MPa.

The area of designed reinforcement per unit length is stored in the CivilFEM results file for both ends:

$$ASSH = \frac{A_{sw}}{S}$$

In this case the element is labeled as designed (provided that the design process is correct for both element sections).

6.6.11.5. Torsion Design

Torsion reinforcement design according to NBR6118 follows the following steps:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated with each transverse cross section and for the active time.

   The required data are the following:

f_{ck}	characteristic strength of concrete
f_{yk}	characteristic yield strength of reinforcement
γ_c	concrete partial safety factor
γ_s	reinforcement partial safety factor

2) **Obtaining geometrical parameters depending on specified code.** Geometrical parameters used for torsion designing must be defined for each code in data at member level according to chapter 5 of this manual. The required data are the following:

A_e	area enclosed by the center-line of the effective hollow section.
U_e	perimeter of the center-line of the effective hollow section.
⍰	angle of the concrete compressive struts with the longitudinal axis of member: 30º ⍰ ⍰ ⍰ 45º

   Section 6.6.1.. provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining forces and moments acting on the section**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCF).

Moment	**Description**
T_{Sd}	Design torsional moment

4) **Checking compression failure of concrete.** First, the design torsional moment (T_{Sd}) must be less than or equal to the ultimate torsional moment due to compression in the concrete (T_{Rd2}); therefore, the following condition must be satisfied:

$$T_{Sd} \leq T_{Rd2}$$
$$T_{Rd2} = 0.50 \cdot \alpha_{v2} \cdot f_{cd} \cdot A_e \cdot h_e \cdot \sin 2\,\theta$$

Where:

$$\alpha_{v2} = 1 \frac{f_{ck}}{250} \text{ (}f_{ck}\text{ in MPa).}$$

Calculated results are stored in the CivilFEM results file:

TRD2 Maximum torsional moment resisted by the section without crushing the concrete compressive struts due to compression.

$$TRD2 = T_{Rd2}$$

CRTTRD2 Ratio of the design torsional moment (T_{Sd}) to the resistance T_{Rd2}.

$$CRTTRD2 = \frac{T_{Sd}}{T_{Rd2}}$$

If design torsional moment is greater than the torsional moment that causes the compression failure of concrete, the reinforcement design is not feasible. Therefore, the parameters for reinforcement data are assigned a value of 2^{100}.

$$ASTT = \frac{A_{st}}{S} = 2^{100} \text{ for transverse reinforcement}$$
$$ASLT = A_{sl} = 2^{100} \text{ for longitudinal reinforcement}$$

In this case, the element is labeled as not designed, and the program then advances to the next element.

In the case there is no failure due to oblique compression, the calculation process continues.

5) **Calculating the transverse reinforcement required.** The ultimate strength condition of the transverse reinforcement is:

$$T_{Sd} \leq T_{Rd3} = (A_{st}/S) \cdot f_{ywd} \cdot 2A_e \cdot \cot an\theta$$

where:

A_{St} cross-sectional area of one of the bars used as transverse torsional reinforcement.

s spacing of closed stirrups of transverse torsional reinforcement.

f_{ywd} design strength of reinforcement, limited to 435 MPa.

Therefore, the required transverse reinforcement is:

$$\frac{A_{St}}{S} = \frac{T_{Sd}}{f_{ywd} \cdot 2A_e} \tan\theta$$

The area per unit length of the designed transverse reinforcement is stored in the CivilFEM results file for both element ends as:

$$ASTT = \frac{A_{st}}{S}$$

6) **Calculating the longitudinal reinforcement required.** The ultimate strength condition of the longitudinal reinforcement is:

$$T_{Sd} \leq T_{Rd4} = (A_{sl}/U_e) \cdot f_{ywd} \cdot 2A_e \cdot \tan\theta$$

where:

A_{sl} area of the longitudinal torsion reinforcement.

f_{ywd} design strength of reinforcement limited to 435 MPa.

Consequently, the longitudinal reinforcement required is:

$$A_{Sl} = \frac{U_e}{f_{ywd} \cdot 2A_e} \cdot T_{Sd} \cdot \cot an\,\theta$$

The area of the designed longitudinal reinforcement is stored in the CivilFEM results file for both element ends as:

$$ASLT = A_{sl}$$

If the design for both element sections is completed for both transverse and longitudinal reinforcements, the element will be labeled as designed.

6.6.11.6. Combined Shear and Torsion Design

The design of sections subjected to shear force and concomitant torsional moment follows the steps below:

1) **Torsion design considering a null shear force.** This design is accomplished with the same steps as for the designing of elements subjected to pure torsion according to NBR6118.

2) **Shear design assuming a null torsional moment.** This design follows the same procedure as for the design of elements only subjected to shear force according to NBR6118.

3) **Checking the failure condition by compression in the concrete.** The design torsional moment (T_{Sd}) and the design shear force (V_{Sd}) must satisfy the following condition:

$$\left(\frac{V_{Sd}}{V_{Rd2}}\right)+\left(\frac{T_{Sd}}{T_{Rd2}}\right)\leq 1$$

where:

V_{Rd2} ultimate shear force by compression of concrete.

T_{Rd2} ultimate torsional moment due to compression of concrete.

For each element end, this criterion value is stored in the CivilFEM results file as CRTCST.

4) **Obtaining required shear and torsion reinforcement ratios.** If the concrete ultimate strength condition is satisfied (i.e. the concrete can resist the combined shear and torsion action), the reinforcements calculated are taken as the designed reinforcements. The element will be labeld as designed.

If the concrete ultimate strength condition is not satisfied, the parameters corresponding to each reinforcement group take the value of 2^{100}.

6.6.13. Shear and Torsion according to EHE-08

6.6.12.1. Shear Check

The checking for shear according to EHE-08 follows the steps below:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated to each transverse cross section and for the active time.

   The required data are the following:

   f_{ck} characteristic compressive strength of concrete.

   f_{yk} characteristic yield strength of reinforcement.

   $f_{ct,m}$ mean tensile strength of concrete.

   $f_{ct,k}$ characteristic tensile strength of concrete (fctk_005).

   γ_c concrete partial safety factor.

   γ_s steel partial safety factor.

2) **Obtaining section geometrical data.** Section geometrical requirements must be defined within CivilFEM. Required data for shear checking are the following:

   A_c total area of the concrete section.

3) **Obtaining geometrical parameters depending on specified code.** Geometrical parameters used for shear calculations must be defined. Required data are the following ones:

   b_w Width of element equal to the total width in solid sections or in case of box sections, the width equals the sum of the width of both webs.

   d effective depth of the section.

   ρ_1 geometric ratio of the tensile longitudinal reinforcement anchored at a distance greater than or equal to d from the considered section:

$$\rho_1 = \frac{A_s}{b_w\ d} \leq 0.02$$

   θ angle of the concrete compressive struts with the longitudinal axis of member:

$$0.5 \leq \cot\theta \leq 2.0$$

Section “Previous Considerations to Shear and Torsion Calculation” provides detailed information on how to calculate the required data for each code and valid section.

4) Obtaining reinforcement data of the section. Data concerning reinforcements of the section must be included within CivilFEM. Required data are the following ones:

α angle between shear reinforcement and the longitudinal axis of the member.

A_s/S area of reinforcement per unit length.

The reinforcement ratio may also be obtained with the following data:

A_s total area of the reinforcement legs.

s spacing of the stirrups.

Or with the data below:

s spacing of the stirrups.

ϕ diameter of bars.

N number of reinforcement legs.

5) **Obtaining forces and moments acting on the section**. The shear force that acts on the section, as well as the concomitant axial force and bending moment, are obtained from the CivilFEM results file.

Force	**Description**
V_{rd}	Design shear force in Y
N_d	Axial force

6) **Checking failure by compression in the web.** First, a check is made to ensure the design shear force (V_{rd}) is less than or equal to the oblique compression resistance of concrete in the web (V_{u1}):

$$V_{rd} \leq V_{u1}$$

$$V_{u1} = K\, f_{1cd} b_w \cdot d \frac{\cot\theta + \cot\alpha}{1 + \cot^2\theta}$$

where:

f_{1cd} design compressive strength of concrete.

$$f_{1cd}\begin{cases} 0.60\cdot f_{cd} & f_{ck}<60\,MPa \\ \left(0.90-\dfrac{f_{ck}[MPa]}{200}\right)f_{cd}\geq 0.5\cdot f_{cd} & f_{ck}\geq 60\text{ MPa}\end{cases}$$

K reduction factor by axial forces effect

$$K\begin{cases} 1 & \sigma'_{cd}=0 \\ 1+\dfrac{\sigma'_{cd}}{f_{cd}} & 0<\sigma'_{cd}\leq 0.25\cdot f_{cd} \\ 1.25 & 0.25\cdot f_{cd}<\sigma'_{cd}\leq 0.50\cdot f_{cd} \\ 2.5\left(1-\dfrac{\sigma'_{cd}}{f_{cd}}\right) & 0.50\cdot f_{cd}<\sigma'_{cd}\leq f_{cd}\end{cases}$$

σ'_{cd} effective axial stress in concrete (compression positive) considering the axial stress taken by compressed reinforcement.

For each element end, calculated results are written in the CivilFEM results file:

VU1 Ultimate shear strength due to oblique compression of the concrete in web.

$$VU1=V_{u1}$$

CRTVU1 Ratio of the design shear (V_{rd}) to the resistance V_{u1}.

$$CRTVU1=\frac{V_{rd}}{V_{u1}}$$

7) **Checking failure by tension in the web.** A check is made to ensure the design shear force (V_{rd}) is less than or equal to the shear force due to tension in the web (V_{u2}):

$$V_{rd}\leq V_{u2}$$

$$V_{u2}=V_{su}+V_{cu}$$

V_{su} contribution of web shear transverse reinforcement to the shear strength.

V_{cu} contribution of concrete to the shear strength.

Members without shear reinforcement

If shear reinforcement has not been defined:

$$V_{su}=0$$

$$V_{u2}=V_{cu}=\left[\frac{0.18}{\gamma_c}\xi(100\rho_1 f_{ck})^{1/3}+0.15\sigma'_{cd}\right]b_w d$$

$$V_{u2}>\left[\frac{0.075}{\gamma_c}\xi^{3/2}\sqrt{f_{ck}}+0.15\sigma'_{cd}\right]b_w d$$

where:

$$\sigma'_{cd} = \frac{N_d}{A_c} < 0.30 \cdot f_{cd} \leq 12 \text{ MPa (Compression positive)}$$

$$\xi = 1 + \sqrt{\frac{200}{d}} < 2, \text{ d in mm}$$

f_{ck} limited to 60 MPa

Members with shear reinforcement

If shear reinforcement has been defined:

$$V_{su} = 0.9\, d\; \sin\alpha(\cot\alpha + \cot\theta)\frac{A_s}{S} f_{yd}$$

where:

A_s/S shear reinforcement area per unit of length

f_{yd} design strength of reinforcement $\left(f_{yd} \leq 400\ \ N/mm^2\right)$

In this case, the concrete contribution to shear strength is:

$$V_{cu} = \left[\frac{0.15}{\gamma_c}\xi(100\,\rho_1 f_{ck})^{1/3} + 0.15\sigma'_{cd}\right] b_w\ \ d\ \ \beta$$

where:

$$\beta = \frac{2\cot\theta - 1}{2\cot\theta_e - 1} \text{ if } 0.5 \leq \cot\ \theta < \cot\theta_e$$

$$\beta = \frac{\cot\theta - 2}{\cot\theta_e - 2} \text{ if } \cot\theta_e \leq \cot\theta \leq 2.0$$

θ_e reference angle of cracks inclination, obtained from:

$$\cot\theta_e = \frac{\sqrt{f_{ct,m}^2 - f_{ct,m}\left(\sigma_{xd} + \sigma_{yd}\right) + \sigma_{xd}\sigma_{yd}}}{f_{ct,m} - \sigma_{yd}} \begin{cases} \geq 0.5 \\ \leq 2.0 \end{cases}$$

σ_{xd}, σ_{yd} design normal stresses, at the section’s center of gravity, parallel to the longitudinal axis of member and the shear force V_d respectively (tension positive)

Taking $\sigma_{yd} = 0 \;\rightarrow\; \cot\theta_e = \sqrt{1 - \frac{\sigma_{xd}}{f_{ct,m}}}$

In addition, the increment in tensile force due to shear force is calculated with the following equation:

$$\Delta T = V_{rd} \cdot \cot\theta - \frac{V_{su}}{2} \cdot \cot\theta + \cot\alpha$$

For each end, calculated results are written in the CivilFEM results file as:

VSU Contribution of the shear reinforcement to the shear strength.

$$VSU = V_{su}$$

VCU Contribution of concrete to the shear strength.

$$VCU = V_{cu}$$

VU2 Ultimate shear strength by tension in the web.

$$VU2 = V_{u2} = V_{su} + V_{cu}$$

CRTVU2 Ratio of the design shear force (V_{rd})to the resistance V_{u2} .

$$CRTVU2 = \frac{V_{rd}}{V_{u2}}$$

If $V_{u2} = 0$, the CTRVU2 criterion is taken as 2^{100}.

The tension increment due to shear force is stored in the CivilFEM results file as INCTENS.

8) **Obtaining shear criterion.** The shear criterion indicates whether the section is valid or not for the design forces (if it is less than 1, the section satisfies the code provisions; whereas if it exceeds 1, the section will not be valid). Furthermore, it includes information about how close the design force is to the ultimate section strength. The shear criterion is defined as follows:

$$CRT\_TOT = \text{Max}\left(\frac{V_{rd}}{V_{u1}}; \frac{V_{rd}}{V_{u2}}\right) \leq 1$$

For each element end, this value is stored in the CivilFEM results file as CRT_TOT.

A value 2^{100} for this criterion indicates that the shear strength for tension in the web (V_{u2}) is equal to zero, as was described in the previous step.

6.6.12.2. Torsion Checking

The torsion checking according to EHE-08 follows the steps below:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated to each transverse cross section and for the active time.

The required data are the following:

f_{ck} characteristic strength of concrete

f_{yk} characteristic yield strength of reinforcement

γ_c concrete partial safety factor

γ_s reinforcement steel partial safety factor

2) **Obtaining geometrical parameters depending on specified code.** Geometrical parameters used for torsion calculations must be defined within CivilFEM. The required data are the following:

h_e effective thickness.

A_e area enclosed by the center-line of the effective hollow section.

U_e perimeter of the center-line of the effective hollow section.

KEYAST indicator of the position of torsional reinforcement in the section:

= 0 if closed stirrups are placed in both faces of the equivalent hollow section wall or of the real hollow section (value by default for hollow sections).

= 1 if there are closed stirrups only along the periphery of the member (value by default for solid sections).

θ Angle of the compressive struts of concrete with the longitudinal axis of member:

$$0.50 \leq \cot\theta \leq 2.00$$

"Previous Considerations to Shear and Torsion Calculation" provides detailed information on how to calculate the required data for each code and valid section.

3) **Obtaining section reinforcement data.** Data concerning reinforcements of the section must be included within CivilFEM database. Required data are the following:

Transverse Reinforcement

A_{st}/S area of transverse reinforcement per unit of length.

The reinforcement ratio can also be obtained with the following data:

A_{st} closed stirrups area for torsion.

s spacing of closed stirrups.

Or with the data below:

s spacing of closed stirrups.

Φ_t diameter of the closed stirrups bars.

Longitudinal Reinforcement

A_{sl} total area of the longitudinal reinforcement.

The reinforcement ratio can also be obtained with the following data:

Φ diameter of longitudinal bars.

N number of longitudinal bars.

4) **Obtaining the section's internal forces and moments**. The torsional moment that acts on the section is obtained from the CivilFEM results file (.RCV).

Moment	Description
T_d	Design torsional moment in the section

5) **Checking compression failure of concrete.** First, a check is made to ensure the design torsional moment (T_d) is less than or equal to the ultimate torsional moment due to compression in the concrete (T_{u1}); as a result, the following condition must be satisfied:

$$T_d \leq T_{u1}$$

$$T_{u1} = 2K\alpha\, f_{1cd}\, A_e\, h_e \frac{\cot\theta}{1+\cot^2\theta}$$

Where:

f_{1cd} design compressive strength of concrete

$$f_{1cd} = \begin{cases} 0.60 \cdot f_{cd} & f_{ck} < 60\ MPa \\ \left(0.90 - \dfrac{f_{ck}[\text{MPa}]}{200}\right) f_{cd} \geq 0.5 \cdot f_{cd} & f_{ck} \geq 60\ \text{MPa} \end{cases}$$

K reduction factor by axial forces effect

$$K = \begin{cases} 1 & \sigma'_{cd} = 0 \\ 1 + \dfrac{\sigma'_{cd}}{f_{cd}} & 0 < \sigma'_{cd} \leq 0.25 \cdot f_{cd} \\ 1.25 & 0.25 \cdot f_{cd} < \sigma'_{cd} \leq 0.50 \cdot f_{cd} \\ 2.5\left(1 - \dfrac{\sigma'_{cd}}{f_{cd}}\right) & 0.50 \cdot f_{cd} < \sigma'_{cd} \leq f_{cd} \end{cases}$$

α 0.60 if only there are stirrups along the periphery of the member;

0.75 if closed stirrups are placed at both faces of the wall of the effective hollow section or real hollow section.

Calculated results are stored in the CivilFEM results file as:

TU1 — Maximum torsional moment that can be resisted by the section without crushing due to compression of concrete compressive struts.

$$TU1 = T_{u1}$$

CRTTU1 — Ratio of the design torsional moment (T_d) to the resistance T_{u1}.

$$CRTTU1 = \frac{T_d}{T_{u1}}$$

6) **Checking transverse reinforcement failure.** The tensile failure condition of the transverse reinforcement in a section subjected to a torsional moment T_d is:

$$T_d \leq T_{u2}$$

$$T_{u2} = \frac{2\,A_e\,A_t}{S} f_{yd} \cot\theta$$

where:

A_t — cross-sectional area of one of the bars used as transverse torsional reinforcement.

s — spacing of closed stirrups of transverse torsional reinforcement.

f_{td} — design yield strength of torsion reinforcement ($f_{yd} \leq 400$ N/mm^2). The same steel type will be used for both transverse and longitudinal torsion reinforcement.

Calculated results are stored in the CivilFEM results file as:

TU2 — Maximum torsional moment resisted by the section so without causing failure in the transverse reinforcement due to tension.

$$TU2 = T_{u2}$$

CRTTU2 — Ratio of the design torsional moment (T_d) to the resistance T_{u2}.

$$CRTTU2 = \frac{T_d}{T_{u2}}$$

If the torsion transverse reinforcement is not defined, the criterion is taken as 2^{100}.

7) **Checking longitudinal reinforcement failure.** The tensile failure condition of the longitudinal reinforcement in a section subjected to a torsional moment T_d is:

$$T_d \leq T_{u3}$$

$$T_{u3} = \frac{2\,A_e}{U_e} A_{sl}\, f_{yd}\, \tan\theta$$

Where A_{sl} is the area of the longitudinal torsion reinforcement.

Calculated results are stored in the CivilFEM results file as:

TU3 Maximum torsional moment resisted by the section without causing tensile failure in the longitudinal reinforcement.

$$TU3 = T_{u3}$$

CRTTU3 Ratio of the design torsional moment (T_d) to the resistance T_{u3}.

$$CRTTU3 = \frac{T_d}{T_{u3}}$$

In the case the longitudinal reinforcement is not defined, the criterion is taken as 2^{100}.

8) **Obtaining torsion criterion.** The torsion criterion identifies the ratio of the design moment to the section's ultimate strength (if it is less than 1, the section is valid; whereas if it exceeds 1, the section is not valid). The criterion concerning the validity for torsion is defined as follows:

$$CRT\_TOT = \mathrm{Max}\left(\frac{T_d}{T_{u1}};\frac{T_d}{T_{u2}};\frac{T_d}{T_{u3}}\right) \leq 1$$

For each element end, this value is stored in the CivilFEM results file as CRT_TOT.

A value 2^{100} for this criterion indicates that any one of the torsion reinforcements are not defined.

6.6.12.3. Combined Shear and Torsion Checking

Checking sections subjected to shear force and concomitant torsional moment follows the steps below:

1) **Torsion checking considering a null shear force.** This check is accomplished with the same steps as for the check of elements subjected to pure torsion according to EHE-08.

   For each element end, this value is stored in the CivilFEM results file as CRTTRS.

2) **Shear checking assuming a null torsional moment.** Follows the same procedure as for the check of elements only subjected to shear according to EHE-08.

   For each element end, this value is stored in the CivilFEM results file as CRTSHR.

3) **Checking the ultimate compressive strength condition of concrete.** The design torsional moment (T_d) and the design shear force (V_{rd}) must satisfy the following condition:

$$\left(\frac{T_{Sd}}{T_{u1}}\right)^{\beta}+\left(\frac{V_{Sd}}{V_{u1}}\right)^{\beta}\le 1$$

Where:

$$\beta=2\left(1-\frac{h_e}{b_w}\right)$$

T_{u1} ultimate torsional moment due to compression of concrete, calculated in step No. 1.

V_{u1} ultimate shear force by compression of concrete, calculated in step No. 2.

For each element, this criterion value is stored in the CivilFEM results file as CRTCST.

4) **Obtaining the combined shear and torsion criterion.** This criterion comprehends pure shear, pure torsion and concrete ultimate strength condition criteria. The criterion determines whether the section is valid or not, and it is defined as follows:

$$\mathrm{CRT\_TOT}=\mathrm{Max}\left[\frac{V_{rd}}{V_{u1}};\frac{V_{rd}}{V_{u2}};\frac{T_d}{T_{u1}};\frac{T_d}{T_{u2}};\frac{T_d}{T_{u3}}\left(\frac{V_{rd}}{V_{u1}}\right)^{\beta}+\left(\frac{T_d}{T_{u1}}\right)^{\beta}\right]\le 1$$

For each element, this criterion value is stored in the CivilFEM results file as CRT_TOT.

A value 2^{100} for this criterion indicates that one of the denominators is null, because one of the reinforcements is not defined.

6.6.12.4. Shear Design

The shear designing according to EHE-08 follows these steps:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated to each transverse cross section and for the active time.

The required data are the following ones:

f_{ck} characteristic strength of concrete.

f_{yk} characteristic yield strength of reinforcement.

$f_{ct,m}$ mean tensile strength of concrete.

γ_c concrete safety factor.

γ_s steel safety factor.

2) **Obtaining geometrical data of the section.** Section geometrical requirements must be defined within CivilFEM database:

A_c total area of the concrete section.

3) Obtaining geometrical parameters depending on specified code. Geometrical parameters used for shear design must be defined within the CivilFEM. Required data are the following ones:

b_w Width of element equal to the total width in solid sections or in case of box sections, the width equals the sum of the width of both webs.

d effective depth of the section.

ρ_1 geometric ratio of the tension longitudinal reinforcement anchored at a distance greater than or equal to d from the considered section.

$$\rho_1 = \frac{A_s}{b_w\, d} \leq 0.02$$

θ angle of the concrete compressive struts with the longitudinal axis of member:

$$0.5 \leq \cot\theta \leq 2.0$$

Section “Previous Considerations to Shear and Torsion Calculation” provides detailed information on how to calculate the required data for each code and valid section.

4) Obtaining reinforcement data of the section. In the shear reinforcement design, it is possible to define the angle between the reinforcement and the longitudinal axis of the member. If this angle is null or it is not defined, it’s defined as 90º. Other reinforcement data are ignored.

5) Obtaining forces and moments acting on the section. The shear force that acts on the section as well as the concomitant axial force are obtained from the CivilFEM results file.

Force	**Description**
V_{rd}	Design shear force in Y
N_d	Design axial force

6) Checking compression failure in the web. First, a check is made to ensure the design shear force (V_{rd}) is less than or equal to the oblique compression resistance of concrete in the web (V_{u1}):

$$V_{rd} \leq V_{u1}$$

$$V_{u1} = K\, f_{1cd} b_w \cdot d \frac{\cot\theta + \cot\alpha}{1+\cot^2\theta}$$

where:

f_{1cd} design compressive strength of concrete

$$f_{1cd}\begin{cases} 0.60\cdot f_{cd} & f_{ck} < 60\,MPa \\ \left(0.90-\frac{f_{ck}[MPa]}{200}\right) f_{cd} \geq 0.5\cdot f_{cd} & f_{ck} \geq 60\ \mathrm{MPa} \end{cases}$$

K reduction factor by axial forces effect

$$K=\begin{cases} 1 & \sigma'_{cd}=0 \\ 1+\frac{\sigma'_{cd}}{f_{cd}} & 0<\sigma'_{cd}\leq 0.25\cdot f_{cd} \\ 1.25 & 0.25\cdot f_{cd}<\sigma'_{cd}\leq 0.50\cdot f_{cd} \\ 2.5\left(1-\frac{\sigma'_{cd}}{f_{cd}}\right) & 0.50\cdot f_{cd}<\sigma'_{cd}\leq f_{cd} \end{cases}$$

σ'_{cd} effective axial stress in concrete (compression positive) considering the axial stress taken by reinforcement in compression.

For each element end, calculated results are written in the CivilFEM results file a*s*:

VU1 Ultimate shear strength due to oblique compression of the concrete in web.

$$VU1 = V_{u1}$$

CRTVU1 Ratio of the design shear force (V_{Rd}) to the resistance V_{u1}.

$$CRTVU1=\frac{V_{rd}}{V_{u1}}$$

If the design shear force is greater than the shear force that causes failure due to oblique compression in the concrete of the web, the reinforcement design will not be feasible. The parameter where the reinforcement data is stored will be defined as 2^{100}.

$$ASSH=\frac{As}{s}=2^{100}$$

In this case, the element is labeled as not designed, and the program then advances to next element.

In the case there is no failure due to oblique compression, the calculation process continues.

7) Checking if section requires shear reinforcement. First, a check is made to ensure the design shear force V_d is less than the strength provided by the concrete in members without shear reinforcement (V_{cu}):

$$V_{rd}\leq V_{u2}$$

$$V_{u2}=V_{cu}=\left[\frac{0.18}{\gamma_c}\xi(100\rho_1\,f_{ck})^{1/3}+0.15\sigma'_{cd}\right]b_w d$$

$$V_{u2}>\left[\frac{0.075}{\gamma_c}\xi^{3/2}\sqrt{f_{ck}}+0.15\sigma'_{cd}\right]b_w d$$

where:

$$\sigma'_{cd} = \frac{N_d}{A_c} < 0.30 \cdot f_{cd} \leq 12 \text{ MPa (Compression positive)}$$

$$\xi = 1 + \sqrt{\frac{200}{d}} < 2, \text{ d in mm}$$

f_{ck} limited to 60 MPa

If the section does not require shear reinforcement, the following parameters are defined (for both element ends):

$$VCU = V_{cu}$$

$$VU2 = V_{cu}$$

$$VSU = 0$$

$$ASSH = \frac{As}{s} = 0$$

If section requires shear reinforcement, the calculation process continues.

8) Determining the contribution of the required transverse reinforcement to the shear strength. If the section requires shear reinforcement, the condition for the validity of the sections under shear force is the following:

$$V_{rd} \leq V_{u2}$$

$$V_{u2} = V_{su} + V_{cu}$$

V_{su} contribution of transverse shear reinforcement in the web to the shear strength.

V_{cu} contribution of concrete to the shear strength.

$$V_{cu} = \left[\frac{0.15}{\gamma_c}\xi(100\rho_1 f_{ck})^{1/3} + 0.15\sigma'_{cd}\right] b_w \, d \, \beta$$

where:

$$\beta = \frac{2\cot\theta - 1}{2\cot\theta_e - 1} \text{ if } 0.5 \leq \cot\theta < \cot\theta_e$$

$$\beta = \frac{\cot\theta - 2}{\cot\theta_e - 2} \text{ if } \cot\theta_e \leq \cot\theta \leq 2.0$$

θ_e reference angle of cracks inclination, obtained from the following expression:

$$\cot\theta_e = \frac{\sqrt{f_{ct,m}^2 - f_{ct,m}(\sigma_{xd} + \sigma_{yd}) + \sigma_{xd}\sigma_{yd}}}{f_{ct,m} - \sigma_{yd}} \quad \begin{cases} \geq 0.5 \\ \leq 2.0 \end{cases}$$

σ_{xd}, σ_{yd} design normal stresses, at the center of gravity of the section, parallel to the longitudinal axis of the member or to the shear force V_d, respectively (tension positive)

$$\text{Taking } \sigma_{yd} = 0 \rightarrow \cot\theta_e = \sqrt{1 - \frac{\sigma_{xd}}{f_{ct,m}}}$$

Therefore, the shear reinforcement contribution is given by:

$$V_{su} = V_{u2} - V_{cu} = V_{rd} - V_{cu}$$

For each element end, the value of V_{cu} and V_{su} is stored in the CivilFEM results file:

$$VCU = V_{cu}$$

$$VSU = V_{su}$$

9) Required reinforcement ratio. Once the required shear strength of the shear reinforcement has been obtained, the reinforcement ratio can be calculated from the equation below:

$$\frac{A_s}{S} = \frac{V_{su}}{f_{yd} 0.9\, d\, (\cot\alpha + \cot\theta)}$$

Where:

A_s/S cross-sectional area of the designed shear reinforcement per unit length.

The area of the designed reinforcement per unit length is stored in the CivilFEM results file for both ends:

$$ASSH = \frac{As}{S}$$

In this case, the element is labeled as designed (provided that the design process is correct for both element sections).

6.6.12.5. Torsion Design

Torsion reinforcement design according to EHE-08 follows the following steps:

1) Obtaining material strength properties. These properties are obtained from the material properties associated to each transverse cross section and for the active time.

The required data are the following ones:

f_{ck} characteristic strength of concrete

f_{yk} characteristic yield strength of reinforcement

γ_c concrete partial safety factor

γ_s reinforcement partial safety factor

2) Obtaining geometrical parameters depending on specified code. Geometrical parameters utilized used for torsion design must be defined for each code at member level according to chapter 5 of this manual. The required data are the following ones:

A_e area enclosed by the center-line of the effective hollow section.

U_e perimeter of the center-line of the effective hollow section.

KEYAST indicator of the position of the torsion reinforcement in the section.

= 0 if closed stirrups are placed in both faces of the equivalent hollow section wall or of the real hollow section (value by default for hollow sections).

= 1 if closed stirrups are only placed along the periphery of the member (value by default for solid sections).

θ angle of the concrete compressive struts with the longitudinal axis of member:

$$0.50 \leq \cot\theta \leq 2.00$$

Section “Previous Considerations to Shear and Torsion Calculation” provides detailed information on how to calculate the required data for each code and valid section.

3) Obtaining forces and moments acting on the section. The torsional moment that acts on the section is obtained from the CivilFEM results file.

Moment	**Description**
T_d	Design torsional moment

4) Checking compression failure of concrete. First, a check is made to ensure the design torsional moment (T_d) is less than or equal to the ultimate torsional moment for compression in concrete (T_{u1}); therefore, the following condition must be satisfied:

$$T_d \leq T_{u1}$$

$$T_{u1} = 2K\alpha\, f_{1cd}\, A_e\, h_e \frac{\cot\theta}{1+\cot^2\theta}$$

where:

f_{1cd} concrete compressive strength

$$f_{1cd} = \begin{cases} 0.60 \cdot f_{cd} & f_{ck} < 60\ MPa \\ \left(0.90 - \frac{f_{ck}[\mathrm{MPa}]}{200}\right) f_{cd} \geq 0.5 \cdot f_{cd} & f_{ck} \geq 60\ \mathrm{MPa} \end{cases}$$

K reduction factor by axial forces effect

$$K = \begin{cases} 1 & \sigma'_{cd} = 0 \\ 1 + \frac{\sigma'_{cd}}{f_{cd}} & 0 < \sigma'_{cd} \leq 0.25 \cdot f_{cd} \\ 1.25 & 0.25 \cdot f_{cd} < \sigma'_{cd} \leq 0.50 \cdot f_{cd} \\ 2.5\left(1 - \frac{\sigma'_{cd}}{f_{cd}}\right) & 0.50 \cdot f_{cd} < \sigma'_{cd} \leq f_{cd} \end{cases}$$

α 1.20 if stirrups are only placed along the periphery of the member.

1.50 if closed stirrups are placed at both faces of the wall of the effective hollow section or of the real hollow section.

Calculated results are stored in the CivilFEM results file:

TU1 Maximum torsional moment resisted by the section without causing crushing due to compression of concrete compressive struts.

$$TU1 = T_{U1}$$

CRTTU1 Ratio of the design torsional moment (T_d) to the resistance T_{u1}.

$$CRTTU1 = \frac{T_d}{T_{u1}}$$

If design torsional moment is greater than the torsional moment that causes the compression failure of concrete, the reinforcement design will not be feasible. Therefore, the parameters for the reinforcement data will be defined as 2^{100}.

$$AST/ST = \frac{Ast}{S} = 2^{100} \text{for transverse reinforcement}$$

$$AST/ST = Asl = 2^{100} \text{for longitudinal reinforcement}$$

In this case, the element is labeled as not designed, and the program then advances to the next element.

In the case there is no failure due to oblique compression, the calculation process continues.

5) Calculating the transverse reinforcement required. The ultimate strength condition of the transverse reinforcement is:

$$T_d \leq T_{u2} = \frac{2\,A_e\,A_t}{S} f_{yd}\ \cot\alpha$$

where:

A_t area of the section of one of the bars used as transverse reinforcement for torsion.

s spacing of the closed stirrups of the transverse reinforcement for torsion.

Therefore, the required transverse reinforcement is:

$$\frac{A_t}{s} = \frac{T_d}{2\,A_e f_{yd}} \tan\theta$$

The area per unit length of the designed transverse reinforcement is stored in the CivilFEM results file for both element ends as:

$$ASTT = \frac{A_t}{s}$$

6) Calculating the longitudinal reinforcement required. The ultimate strength condition of the longitudinal reinforcement is:

$$T_d \leq T_{u3} = \frac{2\,A_e}{U_e} A_{sl}\, f_{yd}\, \tan\theta$$

Where A_{sl} is the area of the torsional longitudinal reinforcement.

Consequently, the longitudinal reinforcement required is:

$$A_{sl} = \frac{U_e}{2\,A_e f_{yd}} T_d \cot g\,\theta$$

The area of the designed longitudinal reinforcement is stored in the CivilFEM results file for both element ends as:

$$ASLT = A_{sl}$$

If design for both element sections is done for both transverse and longitudinal reinforcements, and the element will be labeled as designed.

6.6.12.6. Combined Shear and Torsion Design

The design of sections subjected to shear force and concomitant torsional moment follows the steps below:

1) **Torsion design considering a null shear force.** This design is accomplished with the same steps as for the design of elements subjected to pure torsion according to EHE-08.

2) **Shear design assuming a null torsional moment.** This design follows the same procedure as for the design of elements only subjected to shear force according to EHE-08.

3) **Checking the failure condition by compression in the concrete.** The design torsional moment (T_d) and the design shear force (V_{rd}) must to satisfy the following condition:

$$\left(\frac{T_{Sd}}{T_{u1}}\right)^{\beta} + \left(\frac{V_{Sd}}{V_{u1}}\right)^{\beta} \leq 1$$

where:

$$\beta = 2\left(1 - \frac{h_e}{b_w}\right)$$

T_{u1} ultimate torsional moment due to compression of concrete, calculated in step 1.

V_{u1} ultimate shear strength due to compression of concrete, calculated in step 2.

For each element end, this criterion value is stored in the CivilFEM results file as CRTCST.

4) **Obtaining required shear and torsion reinforcement ratios.** If the concrete ultimate strength condition is satisfied (i.e. the concrete can resist the combined shear and torsion action), the reinforcements calculated in steps 1 and 2 are taken as the designed reinforcements. The element will be labeled as designed.

If the concrete ultimate strength condition is not satisfied, the parameters corresponding to each reinforcement group will take the value 2^{100}.

6.6.14. Shear and Torsion according to IS 456

6.6.13.1 Shear Checking

Shear checking of elements according to IS 456 follow the steps below:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated with each transverse cross section and for the active time. Those material properties should be previously defined. The required data are the following:

f_{ck} characteristic compressive strength of concrete.

f_y characteristic yield strength of reinforcement.

γ_c partial safety factor for concrete.

γ_s partial safety factor for reinforcement.

2) **Obtaining geometrical data of the section.** Section geometrical requirements must be defined within the CivilFEM. Required data for shear checking are the following:

A_c	total cross-sectional area of the concrete section.

3) **Obtaining geometrical parameters depending on code.** Geometrical parameters used for shear calculations must be defined within CivilFEM. Required data are the following:

b_w	effective width of the section.
d	effective depth of the section.
ρ_1	ratio of the longitudinal tensile reinforcement extending beyond the effective depth of the considered section, except in supports where the total area of the tensile reinforcement is used.:

$$\rho_1 = \frac{A_{sl}}{b_w d}$$

Section "Previous Considerations to Shear and Torsion Calculation" provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** Data concerning reinforcements of the element section must be included within the CivilFEM database. Required data are the following:

α	angle between shear reinforcement and the longitudinal axis of the member section.
A_s/s	area of reinforcement per unit length.

The reinforcement ratio may also be obtained with the following data:

A_s	total area of the reinforcement legs.
s	spacing of the stirrups.

or with the data below:

s	spacing of the stirrups.
ϕ	diameter of bars.
N	number of reinforcement legs.

5) **Obtaining the section internal forces and moments**. The shear force that acts on the section as well as the concomitant axial force are obtained from the CivilFEM results file.

Force	**Description**
V_u	Design shear force
P_u	Concomitant axial force

6) **Calculating the nominal shear stress.** The nominal shear stress is calculated by the following expression:

$$\tau_v \leq \frac{V_u}{b_w \cdot d}$$

This stress is written for each end of the element in the CivilFEM results file as:

TAOV Shear strength

$$TAOV = \tau_v$$

7) **Checking of the maximum shear stress.** The nominal shear stress must be less than or equal to the maximum shear stress:

$$\tau_v \leq \tau_{c\,max}$$

where $\tau_{c\,max}$ is given in Table 20 according to the concrete type:

Table 20 Maximum Shear Stress, $\tau_{c\,max}$, N/mm²
(*Clauses* 40.2.3, 40.2.3.1, 40.5.1 *and* 41.3.1)

Concrete Grade	M 15	M 20	M 25	M 30	M 35	M 40 and above
$\tau_{c\,max}$, N/mm²	2.5	2.8	3.1	3.5	3.7	4.0

Results are stored for each end in the CivilFEM results file as the following parameters:

TCMAX Maximum shear stress.

$$TCMAX = \tau_{c\,max}$$

CRTCMAX Ratio of the nominal shear stress to the shear maximum stress.

$$CRTCMAX = \frac{\tau_v}{\tau_{c\,max}}$$

8) **Calculating the shear resistance of the section.** The shear resistance is calculated as the sum of the resistance provided by the concrete and the shear reinforcement:

$$V_{ut} = V_{uc} + V_{us}$$

where:

V_{ut} shear resistance of the section

V_{uc} concrete contribution to the shear resistance

V_{us} shear reinforcement contribution to the shear resistance

The concrete contribution to the resistance is:

$$V_{uc} = \tau_c \cdot b_w \cdot d$$

where τ_c is given in Table 19 according to the concrete type and the amount of the longitudinal tension reinforcement:

Table 19 Design Shear Strength of Concrete, τ_c, N/mm²

(*Clauses* 40.2.1, 40.2.2, 40.3, 40.4, 40.5.3, 41.3.2, 41.3.3 *and* 41.4.3)

$100\frac{A_s}{bd}$	Concrete Grade					
	M 15	M 20	M 25	M 30	M 35	M 40 and above
(1)	(2)	(3)	(4)	(5)	(6)	(7)
≤ 0.15	0.28	0.28	0.29	0.29	0.29	0.30
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.75	0.71	0.82	0.90	0.94	0.96	0.98
3.00 and above	0.71	0.82	0.92	0.96	0.99	1.01

NOTE — The term A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at support where the full area of tension reinforcement may be used provided the detailing conforms to **26.2.2** and **26.2.3**

For members subjected to axial compression P_u, the design shear strength of concrete, given in Table 19, shall be multiplied by the following factor:

$$\delta = 1 + \frac{3P_u}{A_c f_{ck}} \leq 1.5$$

The reinforcement contribution to the shear resistance shall be calculated as:

$$V_{us} = 0.87 f_y \frac{A_{sw}}{s} d(\sin\alpha + \cos\alpha)$$

A_{sw}	total cross sectional area of the shear reinforcement
s	spacing of the stirrups along the axis of the member

Results are stored for each end in the CivilFEM results file as the following parameters:

TC Design shear stress.

$$TC = \tau_c$$

VUC Contribution of concrete to the shear resistance.

$$VUC = V_{uc}$$

VUS Contribution of shear reinforcement to the shear resistance.

$$VUS = V_{us}$$

VUT Design shear resistance of the section.

$$VUT = V_{ut} + V_{us}$$

CRVUT Ratio of the design shear force (V_u) to the shear resistance V_{ut}.

$$CRVUT = \frac{V_u}{V_{ut}}$$

If $V_{ut} = 0$, CRVUT is taken as 2^{100}.

9) **Obtaining shear criterion.** The shear criterion indicates whether the section is valid or not for the design forces (if it is less than 1, the section satisfies the code prescriptions, whereas if it exceeds 1, the section will not be valid). Furthermore, it includes information about how close is the design force from the ultimate section strength. The shear criterion is defined as follows:

$$CRT\_TOT = Max\left(\frac{\tau_v}{\tau_{c\,max}}; \frac{V_u}{V_{ut}}\right) \leq 1$$

For each end, this value is stored in the CivilFEM results file as the parameter CRT_TOT.

A value of 2^{100} for this criterion would mean that V_{ut} are equal to zero.

6.6.13.2 Axial and Bending with combined Shear and Torsion Checking

The axial and bending with combined shear and torsion checking according to IS 456 follows the steps below:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated with the transverse cross section and for the active time.

2) **Obtaining of the geometrical parameters of the section.** Geometrical parameters of the section must be defined within the CivilFEM database.

3) **Obtaining geometrical parameters depending on specified code.** The required data are the following:

b_w effective width of the section.

d effective depth of the section.

ρ_1 ratio of the longitudinal tensile reinforcement extending beyond the effective depth of the considered section, except in supports where the total area of the tensile reinforcement is used:

$$\rho_1 = \frac{A_{sl}}{b_w\, d}$$

b_1, d_1 center to center distances between corner bars situated between transversal stirrups, measured along the width and the flange of the section respectively.

Section "Previous Considerations to Shear and Torsion Calculation" provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** Data concerning reinforcements of the section must be included within CivilFEM. Required data are the following:

Longitudinal Bending Reinforcement

It is obtained from the bending reinforcement distribution of the section

Transverse Shear Reinforcement

α angle between the shear reinforcement and the longitudinal axis of the member section.

A_s/s area of transverse reinforcement per unit length.

The reinforcement ratio may also be obtained with the following data:

A_s total area of the reinforcement legs.

s spacing of the stirrups.

or with the data below:

s spacing of the stirrups.

θ diameter of bars.

N number of reinforcement legs.

TransverseTorsional Reinforcement

A_{st}/s area of transverse reinforcement per unit length.

The reinforcement ratio can also be obtained with the following data:

A_s closed stirrups area for torsion.

s spacing of closed stirrups.

Or with the data below:

s spacing of closed stirrups.

ϕ_t diameter of the closed stirrups.

Longitudinal Shear Reinforcement

This reinforcement will be ignored.

5) **Obtaining section internal forces and moments**. The forces and moments that acts on the section are obtained from the CivilFEM results file.

Force/Moment	Description
V_u	Design shear force
T_u	Design torsional moment
P_u	Concomitant axial force
M_u	Concomitant bending moment

6) **Calculating the equivalent shear.** Equivalent shear shall be calculated from the following formula:

$$V_e = V_u + 1.6\frac{T_u}{b_w}$$

Where V_e is the equivalent shear force.

7) **Calculating the equivalent nominal shear stress**. The equivalent nominal shear stress shall be calculated from the following formula:

$$\tau_{ve} \leq \frac{V_e}{b_w \cdot d}$$

Results are written in the CivilFEM results file for both element ends as the parameters:

TAOVE Nominal shear stress

$$\mathrm{TAOVE} = \tau_{ve}$$

8) **Checking with the maximum shear stress.** The equivalent nominal shear stress must be less than or equal to the maximum shear stress:

$$\tau_{ve} \leq \tau_{c\,max}$$

where $\tau_{c\,max}$ is given in Table 20 according to the type of concrete:

Table 20 Maximum Shear Stress, $\tau_{c\,max}$, N/mm²
(*Clauses* 40.2.3, 40.2.3.1, 40.5.1 *and* 41.3.1)

Concrete Grade	M 15	M 20	M 25	M 30	M 35	M 40 and above
$\tau_{c\,max}$, N/mm²	2.5	2.8	3.1	3.5	3.7	4.0

Results are stored for each end in the CivilFEM results file as the following parameters:

TCMAX Maximum shear stress.

$$\mathrm{TCMAX} = \tau_{c\,max}$$

CRTCMAX Ratio of the nominal shear stress to the maximum shear stress.

$$\mathrm{CRTCMAX} = \frac{\tau_{ve}}{\tau_{c\,max}}$$

9) **Checking whether the section will require transverse reinforcement.** Transverse reinforcement will not be required if the equivalent nominal shear stress is less than or equal to the maximum shear stress:

$$\tau_{ve} \leq \tau_{c}$$

where τ_c is given in Table 19 according to the concrete type and the amount of the longitudinal tension reinforcement:

Table 19 Design Shear Strength of Concrete, τ_c, N/mm²
(*Clauses* 40.2.1, 40.2.2, 40.3, 40.4, 40.5.3, 41.3.2, 41.3.3 *and* 41.4.3)

$100\frac{A_s}{bd}$	Concrete Grade					
	M 15	M 20	M 25	M 30	M 35	M 40 and above
(1)	(2)	(3)	(4)	(5)	(6)	(7)
≤ 0.15	0.28	0.28	0.29	0.29	0.29	0.30
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.75	0.71	0.82	0.90	0.94	0.96	0.98
3.00 and above	0.71	0.82	0.92	0.96	0.99	1.01

NOTE — The term A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at support where the full area of tension reinforcement may be used provided the detailing conforms to **26.2.2** and **26.2.3**

Results are stored for each end in the CivilFEM results file as the following parameters:

ATT Area of the necessary transverse reinforcement.

$$ATT = \left(\frac{A_{sv}}{S_v}\right)_{nec} = 0$$

CRTATT Ratio of the area of the necessary transverse reinforcement to the area of the defined transverse reinforcement (sum of shear and torsional transverse reinforcement).

$$CRTATT = \frac{\left(\frac{A_{sv}}{S_v}\right)_{nec}}{\frac{A_{ss}}{S} + \frac{A_{st}}{S}} = 0$$

10) Calculating the transverse reinforcement required. If the equivalent nominal stress exceeds the maximum shear stress, the necessary transverse reinforcement will be calculated with the following expression:

$$\frac{A_{sv}}{S_v} = \frac{T_u}{b_1 d_1 (0.87 f_y)} + \frac{V_u}{2.5 d_1 (0.87 f_y)}$$

ATT Area of the necessary transverse reinforcement

$$ATT = \left(\frac{A_{sv}}{S_v}\right)_{nec}$$

CRTATT Ratio of the area of the necessary transverse reinforcement to the area of the defined transverse reinforcement (sum of shear and torsional transverse reinforcement).

$$CRTATT = \frac{\left(\frac{A_{sv}}{S_v}\right)_{nec}}{\frac{A_{ss}}{S} + \frac{A_{st}}{S}}$$

If the shear and torsional transverse reinforcement is zero, $A_{ss}/s+A_{st}/s=0$, the criterion is taken as 2^{100}.

11) Checking of the longitudinal reinforcement. We check if the defined longitudinal bending reinforcement resists an equivalent bending moment given by the formula:

$$M_{el} = M_u + M_t$$

where

M_{el} equivalent bending moment

M_t increment due to torsional moment:

$$M_t = T_u \left(\frac{1 + {}^{D}/_{b_w}}{1.7} \right)$$

D overall depth

This equivalent moment is used in the axial bending checking (in the direction defined in the command argument). For further information about this calculation procedure, see chapters about axial load and biaxial bending of the Theory Manual.

The calculation results are stored in the CivilFEM results file for both element ends as the parameters:

MT increment of the bending moment due to torsional moment

$$MT = M_t$$

MEL equivalent bending moment

$$MEL = M_{el}$$

CRTASL Ratio of the forces and moments that acts on the section to the ultimate forces and moments.

$$CRTASL = \frac{(N_u, M_{el})}{(N_h, M_h)}$$

12) **Obtaining total criterion.** The criterion of the combined axial, bending, shear and torsional checking is obtained from the enveloping of the partial criterions. If it is less than 1, the section is valid; if it exceeds 1, the section is not valid:

CRT_TOT = Max (CRTCMAX; CRTATT; CRTASL)

This value is stored in the CivilFEM results file for both element ends as the parameter CRT_TOT.

A value of 2^{100} for this criterion indicates that the shear and torsion transverse reinforcements have not been defined.

6.6.13.3 Shear Design

Shear reinforcement design according to IS 456 follows the steps below:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated with the transverse cross section and for the active time. The required data are the following:

f_{ck} characteristic compressive strength of concrete.

f_y characteristic yield strength of reinforcement.

γ_c partial safety factor for concrete.

γ_s partial safety factor for reinforcement.

2) Obtaining section geometrical data. Section geometrical requirements must be defined within CivilFEM database. Required data for shear designing are the following:

A_c total cross-sectional area of the concrete section.

3) Obtaining geometrical parameters depending on specified code. Geometrical parameters used for shear designing must be defined within the CivilFEM. Required data are the following ones:

b_w effective width of the section.

d effective depth of the section.

ρ_1 ratio of the tensile reinforcement extending beyond the effective depth of the considered section, except in supports where the total area of the tensile reinforcement is used.:

$$\rho_1 = \frac{A_{sl}}{b_w\, d}$$

Section "Previous Considerations to Shear and Torsion Calculation" provides detailed information on how to calculate the required data for each code and valid section.

4) Obtaining reinforcement data of the section. In shear reinforcement design, it is possible to define the angle between the reinforcement and the longitudinal axis of the member. This angle should be included in the reinforcement definition of each element. If this angle is null or it is not defined, =90º. Other reinforcement data are ignored.

5) Obtaining forces and moments acting on the section. The shear force that acts on the section as well as the concomitant axial force and bending moment are obtained from the CivilFEM results file.

Force	Description
V_u	Design shear force
P_u	Concomitant axial force

6) Calculating the nominal shear stress. The nominal shear stress is calculated from the following expression:

$$\tau_v \leq \frac{V_u}{b_w \cdot d}$$

This stress is written for each end in the CivilFEM results file as:

TAOV Shear strength

$$\mathrm{TAOV} = \tau_v$$

7) Checking of the maximum shear stress. The nominal shear stress must be less than or equal to the maximum shear stress:

$$\tau_v \leq \tau_{c\,max}$$

where $\tau_{c\,max}$ is given in Table 20 according to the concrete type:

Table 20 Maximum Shear Stress, $\tau_{c\,max}$, N/mm²
(*Clauses* 40.2.3, 40.2.3.1, 40.5.1 *and* 41.3.1)

Concrete Grade	M 15	M 20	M 25	M 30	M 35	M 40 and above
$\tau_{c\,max}$, N/mm²	2.5	2.8	3.1	3.5	3.7	4.0

Results are stored for each end in the CivilFEM results file as the following parameters:

TCMAX Maximum shear stress.

$$\mathrm{TCMAX} = \tau_{c\,max}$$

CRTCMAX Ratio of the nominal shear stress to the shear maximum stress.

$$\mathrm{CRTCMAX} = \frac{\tau_v}{\tau_{c\,max}}$$

If the nominal shear stress is greater than the maximum shear stress, the reinforcement design will not be possible; therefore, the parameter where the reinforcement amount is stored will be defined as 2^{100}.

$$\mathrm{ASSH} = \frac{A_s}{S} = 2^{100}$$

In this case, the element will be labeled as not designed, advancing then to the following element end.

8) Determining the required transverse reinforcement contribution to the shear strength. The shear resistance is calculated as the sum of the resistance provided by the concrete and the resistance provided by the shear reinforcement:

$$V_u \leq V_{ut} = V_{uc} + V_{us}$$

where:

V_u design shear force

V_{ut} shear resistance of the section

V_{uc} concrete contribution to the shear strength

V_{us} shear reinforcement contribution to the shear strength

Therefore, the shear reinforcement contribution shall be:

$$V_{us} = V_u - V_{uc}$$

The concrete contribution to the strength is:

$$V_{uc} = \tau_c \cdot b_w \cdot d$$

where τ_c is given in Table 19 according to the concrete type and the amount of the longitudinal tension reinforcement:

Table 19 Design Shear Strength of Concrete, τ_c, N/mm²

(*Clauses* 40.2.1, 40.2.2, 40.3, 40.4, 40.5.3, 41.3.2, 41.3.3 *and* 41.4.3)

$100\frac{A_s}{bd}$	Concrete Grade					
	M 15	M 20	M 25	M 30	M 35	M 40 and above
(1)	(2)	(3)	(4)	(5)	(6)	(7)
≤ 0.15	0.28	0.28	0.29	0.29	0.29	0.30
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.75	0.71	0.82	0.90	0.94	0.96	0.98
3.00 and above	0.71	0.82	0.92	0.96	0.99	1.01

NOTE — The term A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at support where the full area of tension reinforcement may be used provided the detailing conforms to **26.2.2** and **26.2.3**

For members subjected to axial compression P_u, the design shear strength of concrete, given in Table 19, shall be multiplied by the following factor:

$$\delta = 1 + \frac{3P_u}{A_c f_{ck}} \leq 1.5$$

For each element end, the V_{us} value is included in the CivilFEM results file as the parameter:

VUS reinforcement design shear force

$$VUS = V_{us}$$

9) Calculating the required reinforcement ratio. The resistance contribution of the shear reinforcement is calculated with the following expression:

$$V_{us} = 0.87 f_y \frac{A_{sw}}{s} d(\sin\alpha + \cos\alpha)$$

A_{sw} area of the cross-section of the shear reinforcement

s spacing of the stirrups measured along the longitudinal axis

Therefore:

$$\frac{A_{sw}}{s} = \frac{V_{us}}{0.87 f_y d(\sin\alpha + \cos\alpha)}$$

The area of the designed reinforcement per unit length is stored in the CivilFEM results file for both element ends:

$$ASSH = \frac{Asw}{s}$$

In this case, the element will be labeled as designed (providing the design procedure is correct for both element sections).

If the reinforcement design is not possible, the reinforcement value is taken as 2^{100} and the element will be considered not designed.

DSG_CRT Design criterion (Ok the element is designed and NotOk the element is not designed).

10.6.11.4 Axial and Bending with Combined Shear and Torsion Design

Axial and bending with shear and torsion longitudinal and transverse reinforcement design according to IS 456 follows the following steps:

1) **Obtaining material strength properties.** These properties are obtained from the material properties associated to each transverse cross section and for the active time,.

   The required data are the following:

   f_{ck} characteristic compressive strength of concrete.

   f_y characteristic yield strength of reinforcement.

   γ_c partial safety factor for concrete.

   γ_s partial safety factor for reinforcement.

2) **Obtaining geometrical parameters.** Geometrical parameters must be defined within CivilFEM database.

A_c	gross area of the concrete section.

3) **Obtaining geometrical parameters depending on specified code.** Geometrical parameters used for torsion designing must be defined within the CivilFEM. The required data are the following:

b_w	effective width of the section.
d	effective depth.
ρ_1	ratio of the tensile reinforcement extending beyond the effective depth of the considered section, except in supports where the total area of the tensile reinforcement is used.:

$$\rho_1 = \frac{A_{sl}}{b_w\, d}$$

Section "Previous Considerations to Shear and Torsion Calculation" provides detailed information on how to calculate the required data for each code and valid section.

4) **Obtaining reinforcement data of the section.** In longitudinal reinforcement design, it is necessary to define the distribution of bending reinforcement. In transversal reinforcement design, it is possible to define the angle α between the reinforcement and the longitudinal axis of member can be indicated. This angle should be stored in the section data of each element. If this angle is null or it is not defined, α=90º. Other reinforcement data will be ignored.

5) **Obtaining forces and moments acting on the section**. The forces and moments that act on the section are obtained from the CivilFEM results file:

Force/Moment	**Description**
V_u	Design shear force
T_u	Design torsional moment
P_u	Concomitant axial force
M_u	Concomitant bending moment

6) **Calculating the equivalent shear.** Equivalent shear shall be calculated from the following formula:

$$V_e = V_u + 1.6\frac{T_u}{b_w}$$

Where V_e is the equivalent shear force.

7) **Calculating the equivalent nominal shear stress**. The equivalent nominal shear stress shall be calculated from the following formula:

$$\tau_{ve} \leq \frac{V_e}{b_w \cdot d}$$

Results are written in the CivilFEM results file for both element ends as the parameters:

TAOVE Nominal shear stress

$$\mathrm{TAOVE} = \tau_{ve}$$

8) **Checking with the maximum shear stress.** The equivalent nominal shear stress must be less than or equal to the maximum shear stress:

$$\tau_{ve} \leq \tau_{c\ max}$$

where $\tau_{c\ max}$ is given in Table 20 according to the type of concrete:

Table 20 Maximum Shear Stress, $\tau_{c\ max}$, N/mm²

(*Clauses* 40.2.3, 40.2.3.1, 40.5.1 *and* 41.3.1)

Concrete Grade	M 15	M 20	M 25	M 30	M 35	M 40 and above
$\tau_{c\ max}$, N/mm²	2.5	2.8	3.1	3.5	3.7	4.0

Results are stored for each end in the CivilFEM results file as the following parameters:

TCMAX Maximum shear stress.

$$\mathrm{TCMAX} = \tau_{c\ max}$$

CRTCMAX Ratio of the nominal shear stress to the shear maximum stress.

$$\mathrm{CRTCMAX} = \frac{\tau_{ve}}{\tau_{c\ max}}$$

If the nominal shear stress is greater than the maximum shear stress, the reinforcement design will not be possible; therefore the parameter for the area per unit length of the reinforcement will be taken as 2^{100}.

$$\mathrm{ASSH} = \frac{A_s}{S} = 2^{100}$$

9) Checking whether the section will require transverse reinforcement. This reinforcement is not required if the equivalent nominal shear stress is less than or equal to the maximum shear stress:

$$\tau_{ve} \leq \tau_c$$

where τ_c is given in Table 19 according to the concrete type and the amount of the longitudinal tension reinforcement:

Table 19 Design Shear Strength of Concrete, τ_c, N/mm²

(*Clauses* 40.2.1, 40.2.2, 40.3, 40.4, 40.5.3, 41.3.2, 41.3.3 *and* 41.4.3)

$100\frac{A_s}{bd}$	Concrete Grade					
	M 15	M 20	M 25	M 30	M 35	M 40 and above
(1)	(2)	(3)	(4)	(5)	(6)	(7)
≤ 0.15	0.28	0.28	0.29	0.29	0.29	0.30
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.75	0.71	0.82	0.90	0.94	0.96	0.98
3.00 and above	0.71	0.82	0.92	0.96	0.99	1.01

NOTE — The term A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at support where the full area of tension reinforcement may be used provided the detailing conforms to **26.2.2** and **26.2.3**

Results are stored for each end in the CivilFEM results file as the following parameters:

ATT Area of the required transverse reinforcement.

$$ATT = \frac{A_{sv}}{S_v} = 0$$

10) Calculating the required transverse reinforcement. If the equivalent nominal stress exceeds the maximum shear stress, the required transverse reinforcement will be calculated by:

$$\frac{A_{sv}}{S_v} = \frac{T_u}{b_w d_1 (0.87 f_y)} + \frac{V_u}{2.5\, d_1 (0.87 f_y)}$$

ATT Area of the necessary transverse reinforcement

$$ATT = \left(\frac{A_{sv}}{S_v}\right)_{nec}$$

11) **Calculating the longitudinal reinforcement amount.** A check is made to ensure the defined longitudinal bending reinforcement resists an equivalent bending moment given by the formula:

$$M_{el} = M_u + M_t$$

where

M_{el} equivalent bending moment

M_t increment due to torsional moment:

$$M_t = T_u\left(\frac{1 + {}^{D}/_{b_w}}{1.7}\right)$$

D overall depth

This equivalent moment is used in the axial bending design (in the direction defined in the command argument). For further information on the calculation procedure, see chapters 11-A.3 and 11-A.4 of the Theory Manual.

The calculated results are stored in the CivilFEM results file for both element ends as the parameters:

MT increment of the bending moment due to torsional moment

$$MT = M_t$$

MEL equivalent bending moment

$$MEL = M_{el}$$

REINFACT Factor to multiply the scalable longitudinal bending reinforcement to satisfy the code provisions.

If the reinforcement factor is greater than the upper reinforcement limit established by the command, the design will not be possible; therefore, the reinforcement factor is defined as 2^{100}.

$$REINFACT = 2^{100}$$

If the reinforcement design is not possible at both ends, the reinforcement value is taken as 2^{100} and the element will be considered not designed.

DSG_CRT Design criterion (Ok the element is designed and NotOk the element is not designed).

6.7. Cracking Checking

6.7.1 Cracking according to Eurocode 2 (EN 1992-1-1:2004/AC:2008)

10.7.2.10 Cracking Checking

The cracking check calculates the crack width and checks the following condition:

$$W_k \leq W_{max}$$

where:

W_k Design crack width.

W_{max} Maximum crack width

The design crack width is obtained from the following expression (Art. 7.3.4):

$$W_k = S_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$$

$S_{r,max}$ Maximum spacing between cracks.

ε_{sm} Mean strain in the reinforcement.

ε_{cm} Mean strain in the concrete between bars.

$$S_{r,max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \frac{\phi}{\rho_{p,eff}}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}}(1+\alpha_e \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

ϕ Reinforcement bar size in mm.

$\rho_{p,eff} = \frac{A_s + \xi_1^2 \cdot A_p'}{A_{c,eff}}$ Effective reinforcement ratio, where $A_{c,eff}$ is the effective area of concrete in tension, A_s is the area of reinforcement contained within the effective concrete area and A_p' is the area of pre- or post-tensioned tendons within $A_{c,eff}$.

k_1 Coefficient accounting for the influence of the bond properties of the bonded reinforcement.

k_2 Coefficient accounting for the influence of the form of the strain distribution:

$$k_2 = \frac{\varepsilon_1 + \varepsilon_2}{2\varepsilon_1}$$

Where ε_1 is the larger tensile strain and ε_2 is the smaller tensile strain at the boundary of a section subjected to eccentric tension.

k_3, k_4 Constants defined in the National Annexes.

c Cover to the longitudinal reinforcement.

σ_s Stress in the tensile reinforcement calculated for a cracked section.

E_s Elastic modulus of the longitudinal reinforcement.

k_t Coefficient accounting for the influence of the duration of the loading.

α_e Ratio between steel-concrete elastic modulus (E_s/E_{cm}).

10.7.1.2 Reinforcement Stress Calculation

During the calculation process, it is necessary to determine the reinforcement stress under service loads (σ_s) with the assumption the section is cracked.

The calculation of these stresses is an iterative process in which CivilFEM searches for the deformation plane that causes a stress state that is in equilibrium with the external loads. The reinforcement stress is obtained from this deformation plane and from the reinforcement position.

The design loads are taken as external loads for the case of serviceability stress calculation. For the stress calculation at the instant the crack appears, the external loads are taken as homothetic to the design loads that cause a stress equivalent to the concrete tensile strength in the fiber under the greatest amount of tension.

If the loads acting on the cross section cause collapse under axial plus bending checking, the cross section and the associated element are labeled as non-checked.

10.7.2.41 Reinforcement Stress Calculation

Checking results are stored in the corresponding alternative in the CivilFEM results file.

The following results are available:

CRT_TOT Cracking criterion.

SIGMA Maximum tensile stress.

WK Design crack width. (Not valid for decompression checking).

SRMAX Maximum spacing between cracks. (Not valid for decompression checking).

EM Difference between the mean strain in the reinforcement and the mean strain in concrete $\varepsilon_{sm} - \varepsilon_{cm}$. (Not valid for decompression checking).

POS Cracking position inside the section. (Not valid for decompression checking).

1 Upper fiber.

-1 Lower fiber.

0 Upper and lower fibers.

For the cracking check ($w_{max} > 0$) the total criterion is defined as:

$$\mathrm{CRT\_TOT} = \frac{w_k}{w_{max}}$$

Therefore, values for the total criterion larger than one indicate that the section does not pass as valid for this code.

6.8. Cracking Checking

6.7.2 Cracking according to Structural Code (Spanish code)

10.7.2.10 Cracking Checking

The cracking check calculates the crack width and checks the following condition:

$$W_k \leq W_{max}$$

where:

W_k Design crack width.

W_{max} Maximum crack width

The design crack width is obtained from the following expression (Art. 7.3.4):

$$W_k = S_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$$

$S_{r,max}$ Maximum spacing between cracks.

ε_{sm} Mean strain in the reinforcement.

ε_{cm} Mean strain in the concrete between bars.

$$S_{r,max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \frac{\phi}{\rho_{p,eff}}$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}}(1+\alpha_e \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

ϕ Reinforcement bar size in mm.

$\rho_{p,eff} = \frac{A_s + \xi_1^2 \cdot A_p'}{A_{c,eff}}$ Effective reinforcement ratio, where $A_{c,eff}$ is the effective area of concrete in tension, A_s is the area of reinforcement contained within the effective concrete area and A_p’ is the area of pre- or post-tensioned tendons within $A_{c,eff}$.

k_1 Coefficient accounting for the influence of the bond properties of the bonded reinforcement.

k_2 Coefficient accounting for the influence of the form of the strain distribution:

$$k_2 = \frac{\varepsilon_1 + \varepsilon_2}{2\varepsilon_1}$$

Where ε_1 is the larger tensile strain and ε_2 is the smaller tensile strain at the boundary of a section subjected to eccentric tension.

k_3, k_4 Constants defined in the National Annexes.

c Cover to the longitudinal reinforcement.

σ_s Stress in the tensile reinforcement calculated for a cracked section.

E_s	Elastic modulus of the longitudinal reinforcement.
k_t	Coefficient accounting for the influence of the duration of the loading.
α_e	Ratio between steel-concrete elastic modulus (E_s/E_{cm}).

10.7.1.2 Reinforcement Stress Calculation

During the calculation process, it is necessary to determine the reinforcement stress under service loads (σ_s) with the assumption the section is cracked.

The calculation of these stresses is an iterative process in which CivilFEM searches for the deformation plane that causes a stress state that is in equilibrium with the external loads. The reinforcement stress is obtained from this deformation plane and from the reinforcement position.

The design loads are taken as external loads for the case of serviceability stress calculation. For the stress calculation at the instant the crack appears, the external loads are taken as homothetic to the design loads that cause a stress equivalent to the concrete tensile strength in the fiber under the greatest amount of tension.

If the loads acting on the cross section cause collapse under axial plus bending checking, the cross section and the associated element are labeled as non-checked.

10.7.2.41 Reinforcement Stress Calculation

Checking results are stored in the corresponding alternative in the CivilFEM results file.

The following results are available:

CRT_TOT	Cracking criterion.
SIGMA	Maximum tensile stress.
WK	Design crack width. (Not valid for decompression checking).
SRMAX	Maximum spacing between cracks. (Not valid for decompression checking).
EM	Difference between the mean strain in the reinforcement and the mean strain in concrete $\varepsilon_{sm} - \varepsilon_{cm}$. (Not valid for decompression checking).
POS	Cracking position inside the section. (Not valid for decompression checking).

1 Upper fiber.

-1 Lower fiber.

0 Upper and lower fibers.

For the cracking check ($w_{max} > 0$) the total criterion is defined as:

$$CRT\_TOT = \frac{w_k}{w_{max}}$$

Therefore, values for the total criterion larger than one indicate that the section does not pass as valid for this code.

6.7.3 Cracking according to ACI 318-05

10.7.2.1 Cracking Checking

Checking of the Cracking Limit State according to ACI 318-05 consists of the following condition:

$$s_d \leq s$$

Where:

s_d Reinforcement spacing closest to the fiber in tension

s Design reinforcement spacing

CivilFEM checks this condition by applying the general calculation method for the reinforcement spacing (Art. 10.6.4):

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c$$

where:

f_s Calculated stress in reinforcement at service loads.

c_c Geometrical cover

10.7.2.2 Reinforcement Stress Calculation

During the calculation process, it’s necessary to determine the reinforcement stress under service loads (f_s).

The calculation of the stresses is an iterative process in which the program searches for the deformation plane that causes a stress state that is in equilibrium with the external loads.

The reinforcement stress is obtained from this deformation plane and from the reinforcement position.

The design loads are taken as external loads.

If the loads acting on the cross section cause collapse under axial plus bending checking, the cross section and the element to which it belongs are marked as non checked.

10.7.2.3 Checking Results

The following results are available:

CRT_TOT Cracking criterion.

S Design reinforcement spacing. (Not valid for decompression checking).

FS Reinforcement stress. (Not valid for decompression checking).

SIGMA Maximum tensile stress.

POS Cracking position inside the section. (Not valid for decompression checking).

- 1 Upper fiber.
- -1 Lower fiber.
- 0 Upper and lower fibers.

ELM_OK Plots Ok and not Ok elements.

For the cracking check ($s_d > 0$) the total criterion is defined as:

$$CRT\_TOT = \frac{s_d}{s}$$

For decompression checking ($s_d = 0$) the total criterion is defined as:

$$CRT\_TOT = \frac{f_c + \sigma_{max}}{f_c}$$

where

f_c concrete design compressive strength.

σ_{max} Maximum section stress (positive tension), corresponding to the SIGMA result. (If CRT_TOT is negative, it is taken as zero)

Therefore, the values for the total criterion larger than one indicate that the section is not considered valid for this code.

Chapter 7
Code Check for Structural Steel Members

7.1. Steel Structures According to Eurocode 3

For checking steel structures according to Eurocode 3 in CivilFEM, it is possible to check structures composed by welded or rolled shapes under axial forces, shear forces and bending moments in 3D. The calculations made by CivilFEM correspond to the recommendations of **Eurocode 3: Design of steel structures Part 1-1: General rules and rules for buildings** (EN 1993-1-1:2005).

With CivilFEM it is possible to accomplish the following check and analysis types:

Check steel sections subjected to	
- Tension	Art. 6.2.3
- Compression	Art. 6.2.4
- Bending	Art. 6.2.5
- Shear force	Art. 6.2.6
- Bending and Shear	Art. 6.2.8
- Bending and axial force	Art. 6.2.9
- Bending, shear and axial force	Art. 6.2.10
Check for buckling	
- Compression members with constant cross-section	Art. 6.3.1
- Lateral-torsional buckling of beams	Art. 6.3.2
- Members subjected to bending and axial tension	N/A
- Members subjected to bending and axial compression	Art. 6.3.3

Valid cross-sections supported by CivilFEM for checks according to Eurocode 3 are the following:

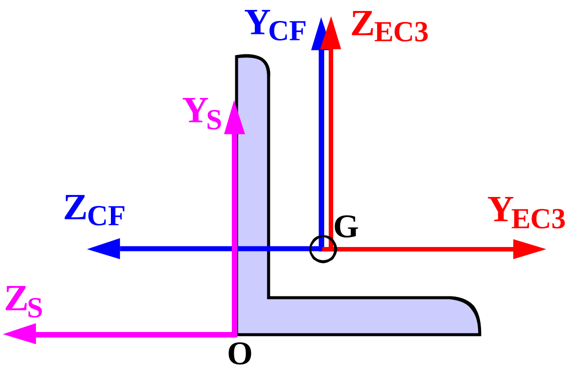
- All rolled shapes included in the program libraries (see the hot rolled shapes library).
- The following welded beams: double T shapes, U or channel shapes, T shapes, box, equal and unequal legs angles and pipes.
- Structural steel sections defined by plates.

CivilFEM considers the above sections as sections composed of plates; for example, an I-section is composed by five plates: four flanges and one web. These cross sections are therefore adapted to the method of analysis of Eurocode 3. Obviously circular sections cannot be decomposed into plates, so these sections are analyzed separately.

7.1.1. Reference axis

With checks according to Eurocode 3, CivilFEM includes three different coordinate reference systems. All of these systems are right-handed:

1. CivilFEM Reference Axis. (X_{CF}, Y_{CF}, Z_{CF}).
2. Cross-Section Reference Axis. (X_S, Y_S, Z_S).
3. Eurocode 3 Reference Axis. (Code axis). (X_{EC3}, Y_{EC3}, Z_{EC3}).



For the Eurocode 3 axes system:

- The origin matches to the CivilFEM axes origin.
- X_{EC3} axis coincides with CivilFEM X-axis.
- Y_{EC3} axis is the relevant axis for bending and its orientation is defined by the user (in steel check process).
- Z_{EC3} axis is perpendicular to the plane defined by X and Y axis, to ensure a right-handed system.

To define this reference system, the user must indicate which direction of the CivilFEM axis (-Z, -Y, +Z or +Y) coincides with the relevant axis for positive bending. The user may define this reference system when checking according to this code. In conclusion, the code reference system coincides with that of CivilFEM, but it is rotated a multiple of 90 degrees, as shown in table below.

Relevant Axis for Bending in CivilFEM Reference System	Angle of Rotation (clockwise) of Eurocode 3 Reference System respect to the CivilFEM Reference System
$- Z_{CF}$	90 º (Default value)
$- Y_{CF}$	180 º
$+ Z_{CF}$	270 º
$+ Y_{CF}$	0 º

7.1.2. Material properties

For Eurocode 3 checking, the following material properties are used:

Description	Property
Steel yield strength	Fy(th)
Ultimate strength	Fu(th)
Partial safety factors	γ_{M0} γ_{M1} γ_{M2}
Elasticity modulus	E
Poisson coefficient	ν
Shear modulus	G

*th =thickness of the plate

7.1.3. Section data

Eurocode 3 considers the following data set for the section:

- Gross section data
- Net section data
- Effective section data
- Data belonging to the section and plates class.

Gross section data correspond to the nominal properties of the cross-section. For the net section, only the area is considered. This area is calculated by subtracting the holes for screws, rivets and other holes from the gross section area. The area of holes is introduced within the structural steel code properties.

Effective section data and section and plates class data are obtained in the checking process according to the effective width method. For class 4 cross-sections, this method subtracts the non-resistance zones for local buckling. However, for cross-sections of a lower class, the sections are not reduced for local buckling.

In the following tables, the section data used in Eurocode 3 are shown:

Description	Data
Input data:	
1.- Height	H
2.- Web thickness	Tw
3.- Flanges thickness	Tf
4.- Flanges width	B
5.- Distance between flanges	Hi
6.- Radius of fillet (Rolled shapes)	r_1
7.- Toe radius (Rolled shapes)	r_2
8.- Weld throat thickness (Welded shapes)	a
9.- Web free depth	d
Output data	(None)

Description	Data	Reference axis
Input data:		
1.- Depth in Y	Tky	CivilFEM
2.- Depth in Z	tkz	CivilFEM
3.- Cross-section area	A	
4.- Moments of inertia for torsion	It	CivilFEM
5.- Moments of inertia for bending	Iyy, Izz	CivilFEM
6.- Product of inertia	Izy	CivilFEM
7.- Elastic resistant modulus	Wely, Welz	CivilFEM
8.- Plastic resistant modulus	Wply, Wplz	CivilFEM
9.- Radius of gyration	iy, iz	CivilFEM
10.- Gravity center coordinates	Ycdg, Zcdg	Section
11.- Extreme coordinates of the perimeter	Ymin, Ymax, Zmin, Zmax	Section
12.- Distance between GC and SC in Y and in Z	Yms, Zms	Section
13.- Warping constant	Iw	

Description	Data	Reference axis
14.- Shear resistant areas	Yws, Zws	CivilFEM
15.- Torsional resistant modulus	Xwt	CivilFEM
16.- Moments of inertia for bending about U, V	Iuu, Ivv	Principal
17.- Angle Y->U or Z->V	α	CivilFEM
Output data:	(None)	

The effective section depends on the section geometry and on the forces and moments that are applied to it. Consequently, for each element end, the effective section is calculated.

Description	Data	Reference axis
Imput data:	(None)	
Output data:		
1.- Cross-section area	Aeff	
2.- Moments of inertia for bending	Iyyeff, Izzeff	CivilFEM
3.- Product of inertia	Izyeff	CivilFEM
4.- Elastic resistant modulus	Wyeff, Wzeff	CivilFEM
5.- Gravity center coordinates	Ygeff, Zgeff	Section
6.- Distance between GC and SC in Y and in Z	Ymseff, Zmseff	Section
7.- Warping constant	Iw	
8.- Shear resistant areas	Yws, Zws	CivilFEM

7.1.4. Structural steel code properties

For Eurocode 3 checking, besides the section properties, more data are needed for buckling checks. These data are shown in the following table.

Description	EN 1993-1-1:2005
Input data:	
1.- Unbraced length of member (global buckling). Length between lateral restraints (lateral-torsional buckling).	L
2.- Buckling effective length factors in XY, XZ planes YZ (Effective	K XY, K XZ

Description	EN 1993-1-1:2005
buckling length for plane XY =L*K XY) (Effective buckling length for plane XZ =L*K XZ).	
3.- Lateral buckling factors, depending on the load and restraint conditions.	C1, C2, C3
4.- Equivalent uniform moment factors for flexural buckling.	CMy, CMz
5.- Equivalent uniform moment factors for lateral-torsional buckling.	CMLt
6.- Effective length factor regarding the boundar conditions.	K
7.- Warping effective factor.	KW

7.1.5. Check Process

The checking process includes the evaluation of the following expression:

$$\frac{N_{Ed}}{N_{c.Rd}} + \frac{M_{y.Ed}}{M_{y.Rd}} + \frac{M_{z.Ed}}{M_{z.Rd}} \leq 1$$

Evaluation steps:

1. Read the loading check requested by the user.
2. Read the CivilFEM axis to be considered as the relevant axis for bending so that it coincides with the Y axis of Eurocode 3. In CivilFEM, by default, the principle bending axis that coincides with the +Y axis of Eurocode 3 is the –Z.
3. The following operations are necessary for each selected element:
   a. Obtain material properties of the element stored in CivilFEM database and calculate the rest of the properties needed for checking:
   Properties obtained from CivilFEM database:

   Calculated properties:

   Epsilon, material coefficient:

$$\varepsilon = \sqrt{235/f_y(th)} \qquad (f_y \text{ in } N/mm^2)$$

   b. Obtain the cross-section data corresponding to the element.
   c. Initialize values of the effective cross-section.
   d. Initialize reduction factors of section plates and the rest of plate parameters necessary for obtaining the plate class.

e. If necessary for the type of check (check for buckling), calculate the critical forces and moments of the section for buckling: elastic critical forces for the XY and XZ planes and elastic critical moment for lateral-torsional buckling. (See section: Calculation of critical forces and moments).

f. Obtain internal forces and moments: N_{Ed}, $V_{y.Ed}$, $V_{z.Ed}$, $M_{x.Ed}$, $M_{y.Ed}$, $M_{z.Ed}$ within the section.

g. Specific section checking according to the type of external load. The specific check includes:

1. If necessary, selecting the forces and moments considered for the determination of the section class and used for the checking process.
2. Obtaining the cross-section class and calculating the effective section properties.
3. Checking the cross-section according to the external load and its class by calculating the check criterion.

h. Store the results.

7.1.6. Section Class and Reduction Factors Calculation

Sections, according to Eurocode 3, are made up by plates. These plates can be classified according to:

1. Plate function: webs and flanges in Y and Z axis, according to the considered relevant axis of bending.
2. Plate union condition: internal plates or outstand plates.

For sections included in the program libraries, the information above is defined for each plate. CivilFEM classifies plates as flanges or webs according to their axis and provides the plate union condition for each end. Ends can be classified as fixed or free (a fixed end is connected to another plate and free end is not).

For checking the structure for safety, Eurocode 3 classifies sections as one of four possible classes:

Class 1	Cross-sections which can form a plastic hinge with the rotation capacity required for plastic analysis.
Class 2	Cross-sections which can reach their plastic moment resistance, but have limited rotation capacity.

Class 3	Cross-sections for which the stress in the extreme compression fiber of the steel member can reach the yield strength, but local buckling is liable to prevent the development of the plastic moment resistance.
Class 4	Cross-sections for which it is necessary to make explicit allowances for the effects of local buckling when determining their moment resistance or compression resistance.

The cross-section class is the highest (least favorable) class of all of its elements: flanges and webs (plates). First, the class of each plate is determined according to the limits of Eurocode 3. The plate class depends on the following:

1. The geometric width to thickness ratio with the plate width properly corrected according to the plate and shape type.

GeomRat = Corrected_Width / thickness

The width correction consists of subtracting the zone that does not contribute to buckling resistance in the fixed ends. This zone depends on the shape type of the section. Usually, the radii of the fillet in hot rolled shapes or the weld throats in welded shapes determine the deduction zone. The values of the corrected width that CivilFEM uses for each shape type include:

- **Welded Shapes:**

Double T section:

Internal webs or flanges:

Corrected width = d

d	Web free depth

Outstand flanges:

Corrected width	$\frac{B}{2} - \frac{T_w}{2} - r_1$

Where:

B	Flanges width
T_w	Web thickness
r_1	Radius of fillet

T section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width = B/d

C section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width	$B - T_w - r_1$

L section:

Corrected width = $\sqrt{I_1^2 + I_2^2}$

l_1, l_2 Angle flange width

Box section:

Internal webs:

Corrected width = H

H Height

Internal flanges:

$\text{Corrected width} = B - 2 \cdot T_w$

T_w Web thickness

Circular hollow section

Corrected width = H

- **Rolled Shapes:**

Double T section:

Internal webs or flanges:

Corrected width = d

d Web free depth

Outstand flanges:

Corrected width = B/2

B Flanges width

T Section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width = B/2

C Section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width = B

L Section:

Corrected width = $= \sqrt{l_1^2 + l_2^2}$

l_1, l_2 Angle flange width

Box section:

Internal webs:

Corrected width = d

Internal flanges:

$$\text{Corrected width} = \mathrm{B} - 3 \cdot \mathrm{T_f}$$

T_f Flanges thickness

Pipe section:

Corrected width = H

2. The limit listed below for width to thickness ratio. This limit depends on the material parameter ε and the normal stress distribution in the plate section. The latter value is given by the following parameters: α, Ψ and k_0, and the plate type, internal or outstand; the outstand case depends on if the free end is under tension or compression.

$$\text{Limit (class)} = f(\varepsilon, \alpha, \Psi, k_0)$$

$$\varepsilon = \sqrt{235/f_y}\ (f_y \text{ in } N/mm^2)$$

where:

α Compressed length / total length

ψ σ_2/σ_1

k_0 Buckling factor

σ_2 The higher stress in the plate ends.

σ_1 The lower stress in the plate ends.

A linear stress distribution on the plate is assumed.

The procedure to determine the section class is as follows:

1. Obtain stresses at first plate ends from the stresses applied on the section, properly filtered according to the check type requested by the user.
2. Calculate the parameters: α, Ψ and k_0

For internal plates:

	ENV 1993-1-1:1992	EN 1993-1-1:2005
$1 \geq \Psi \leq 0$	$k_0 = \frac{16}{\sqrt{(1+\Psi)^2 + 0.112 \cdot (1-\Psi)^2} + (1+\Psi)}$	$k_0 \frac{8.2}{1.05 + \Psi}$
$0 > \Psi > -1$		$k_0 = 7.81 - 6.29 \cdot \Psi + 9.78 \cdot \Psi^2$

$-1 \geq \Psi - 2$	$k_0 = 5.98 \cdot (1 - \Psi)^2$
$\Psi \leq -2$	k_0 = infinite

For outstand plates with an absolute value of the stress at the free end greater than the corresponding value at the fixed end:

For $1 \geq \Psi \geq -1$

$$k_0 = 0.57 - 0.21 \cdot \Psi + 0.07 \cdot \Psi^2$$

For $-1 > \Psi$

k_0= infinite

For outstand plates with an absolute value of the stress at the free end lower than the corresponding value at the fixed end:

For $1 \geq \Psi \geq 0$

$$k_0 = \frac{0.578}{\Psi + 0.34}$$

For $0 > \Psi \geq -1$

$$k_0 = 1.7 - 5 \cdot \Psi + 17.1 \cdot \Psi^2$$

For $-1 > \Psi$

k_0= infinite

Cases in which $k_0 = infinite$ are not included in Eurocode 3. With these cases, the plate is considered to be practically in tension and it will not be necessary to determine the class. These cases have been included in the program to avoid errors, and the value $k_0 = infinite$ has been adopted because the resultant plate class is 1 and the plate reduction factor is ρ = 1 (the same values as if the whole plate was in tension). The reduction factor is used later in the effective section calculation.

3. Obtain the limiting proportions as functions of: α, Ψ and k_0 and the plate characteristics (internal, outstand: free end in compression or tension).

EN 1993-1-1:2005:

Internal plates:

$\text{Limit}(1) = 396\,\varepsilon\,/\,(13\,\alpha - 1)$ for $\alpha < 0.5$

$\text{Limit}(1) = 36\,\varepsilon\,/\,\alpha$ for $\alpha < 0.5$

$\text{Limit}(2) = 456\,\varepsilon\,/\,(13\,\alpha - 1)$ for $\alpha \geq 0.5$

$\text{Limit}(2) = 41.5\,\varepsilon\,/\,\alpha$ for $\alpha < 0.5$

$\text{Limit}(3) = 42\,\varepsilon\,/\,(0.67 + 0.33\,\Psi\,)$ for $\Psi > -1$

$\text{Limit}(3) = 62\,\varepsilon\,(1 - \Psi)\sqrt{(-\Psi)}$ for $\Psi \leq -1$

Outstand plates, free end in compression:

$$\text{Limit}(1) = 9\,\varepsilon\,/\,\alpha$$

$$\text{Limit}(2) = 10\,\varepsilon\,/\,\alpha$$

$$\text{Limit}(3) = 21\,\varepsilon\,\sqrt{K_0}$$

Outstand plates, free end in tension:

$$\text{Limit}(1) = \frac{9\,\varepsilon}{\alpha\sqrt{\alpha}}$$

$$\text{Limit}(2) = \frac{10\,\varepsilon}{\alpha\sqrt{\alpha}}$$

$$\text{Limit}(3) = 21\,\varepsilon\sqrt{K_0}$$

Above is the general equation used by the program to obtain the limiting proportions for determining plate classes. In addition, plates of Eurocode 3 may be checked according to special cases.

For example:

In sections totally compressed:

α= 1; Ψ = 1 for all plates

In sections under pure bending:

α = 0.5; Ψ = -1 for the web

α = 1; Ψ = 1 for compressed flanges

4. Obtain the plate class:

If		GeomRat	< Limit(1)	Plate Class = 1
If	Limit(1) ≤	GeomRat	< Limit(2)	Plate Class = 2
If	Limit(2) ≤	GeomRat	< Limit(3)	Plate Class = 3
If	Limit(3) ≤	GeomRat		Plate Class = 4

Repeat these steps (1,2,3,4) for each section plate.

5. Assign of the highest class of the plates to the entire section.
   In tubular sections, the section class is directly determined as if it were a unique plate, with GeomRat and the Limits calculated as follows:
   6. GeomRat = outer diameter/ thickness.
      $\mathrm{Limit}(1) = 50\ \varepsilon^2$
      $\mathrm{Limit}(2) = 70\ \varepsilon^2$
      $\mathrm{Limit}(3) = 90\ \varepsilon^2$

For class 4 sections, the section resistance is reduced, using the effective width method.

For each section plate, the effective lengths at both ends of the plate and the reduction factors ρ_1 and ρ_2 are calculated. These factors relate the length of the effective zone at each plate end to its width.

Effective_length_end 1 = $\mathrm{plate\_width}^*\rho_1$

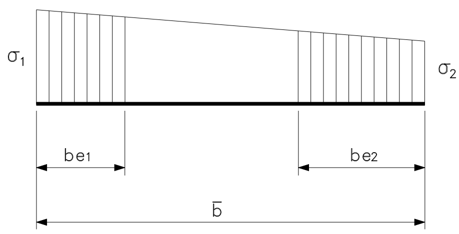
Effective_length_end 2 = $\mathrm{plate\_width}^*\rho_2$

The following formula from Eurocode 3 has been implemented for this process:

$$\Psi = \sigma_2/\sigma_1$$

1. Internal plates:

For $0 \leq \Psi \leq 1$ (Both ends compressed)



$$b_{eff} = \rho\ \bar{b}$$

$$b_{e1} = 2\ b_{eff}/\ (5 - \Psi)$$

$$b_{e2} = b_{eff} - b_{e1}$$

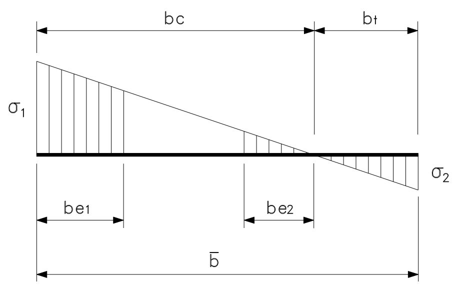
$$\rho_1 = \frac{b_{e1}}{plate\_width}$$

$$\rho_2 = \frac{b_{e2}}{plate\_width}$$

$\bar{b} =$ corrected plate width

plate_width = real plate width

For $\Psi < 0$ (end 1 in compression and end 2 in tension)



$$b_{eff} = \rho\ b_c = \rho\ \bar{b}/(1 - \Psi)$$

$$b_{e1} = 0.4\ b_{eff}$$

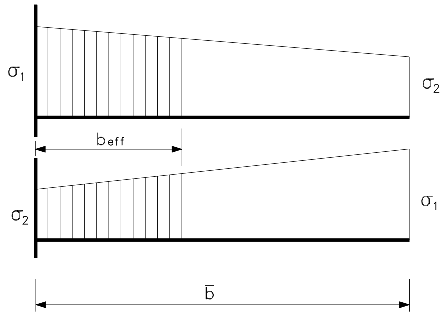
$$b_{e2} = 0.6\ b_{eff}$$

$$\rho_1 = \frac{b_{e1}}{plate\_width}$$

$$\rho_2 = \frac{b_{e2} + bt}{plate\_width}$$

2. Outstand plates:

For $0 \le \Psi \le 1$ (Both ends in compression: end 1 fixed, end 2 free)

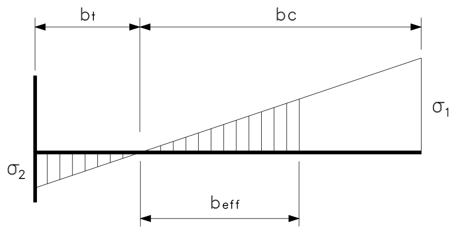


$$b_{eff} = \rho\,\bar{b}$$

$$\rho_1 = \frac{b_{eff}}{plate\_width}$$

$$\rho_2 = 0$$

For $\Psi < 0$(end 1 fixed and in tension, end 2 free and in compression)

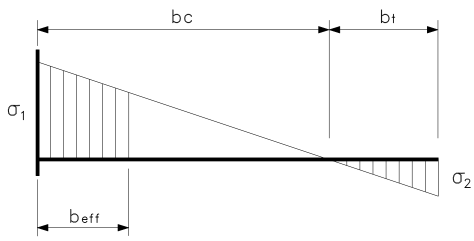


$$b_{eff} = \rho b_c = \rho c/(1-\Psi)$$

$$\rho_1 = \frac{b_{eff} + bt}{plate\_width}$$

$$\rho_2 = 0$$

For $\Psi < 0$ (end 1 fixed and in compression, end 2 free and in tension)



$$b_{eff} = \rho b_c = \rho c/(1-\Psi)$$

$$\rho_1 = \frac{b_{eff}}{plate\_width}$$

$$\rho_2 = \frac{b_t}{plate\_width}$$

If end 2 is the fixed end, the values ρ_1 and ρ_2 are switched.

The global reduction factor ρ is obtained by as follows:

<u>EN 1993-1-1:2005:</u>

For internal compression elements

For $\bar{\lambda}_p > 0.673$

$$\rho = \frac{\bar{\lambda}_p - 0.055(3+\Psi)}{\bar{\lambda}_p^{\ 2}}$$

For $\bar{\lambda}_p \leq 0.673$

$$\rho = 1$$

For outstands compression elements:

For $\bar{\lambda}_p > 0.748$

$$\rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^{\ 2}}$$

For $\bar{\lambda}_p \leq 0.748$

$$\rho = 1$$

Both Eurocode define as the plate slendernesss given by:

$$\bar{\lambda}_p = \frac{\bar{b}/t}{28.4\varepsilon\sqrt{k_0}}$$

where:

$\bar{b}$ = corrected plate width

t = relevant thickness

ε = material parameter

k_0 = buckling factor

To determine effective section properties, three steps are followed:

1. *Effective widths of flanges* are calculated from factors α and Ψ these factors are determined from the gross section properties. As a result, an intermediate section is obtained with reductions taken in the flanges only.
2. The resultant section properties are obtained and factors α and Ψ are calculated again.
3. *Effective widths of webs* are calculated so that the finalized effective section is determined. Finally, the section properties are recalculated once more.

The recalculated section properties are included in the effective section data table.

Checking can be accomplished with the gross, net or effective section properties, according to the section class and checking type.

Each checking type follows a specific procedure that will be explained in the following sections.

7.1.7. Checking of Members in Axial Tension

Corresponds to chapter 6.2.3 in EN 1993-1-1:2005.

1. Forces and moments selection.
   The forces and moments considered for this checking type are:

   N_{Ed}= FX Design value of the axial force (positive if tensile, element not processed if compressive).

2. Class definition and effective section properties calculation.
   For this checking type, the section class is always 1 and the considered section is either the gross or net section.
3. Criteria calculation.
   For members under axial tension, the general criterion *Crt_TOT* is checked at each section. This criterion coincides with the axial criterion *Crt_N*.

$$N_{Ed} \leq N_{t.Rd} \rightarrow \quad Crt\_TOT = Crt\_N = \frac{N_{Ed}}{N_{t.Rd}} \leq 1$$

where $N_{t.Rd}$ is the design tension resistance of the cross-section, taken as the smaller value of:

$N_{Pl,Rd} = Af_y/\Upsilon_{M0}$ plastic design strength of the gross cross-section

$N_{u.Rd} = 0.9A_{net}f_u/\Upsilon_{M2}$ ultimate design strength of the net cross-section

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table:

Result	**Concepts**	**Description**
NED	N_{Ed}	Design value of the tensile force (EN 1993-1-1:2005).
NTRD	$N_{t.Rd}$	Design tensile strength of the cross-section.
CRT_N	$N_{Ed}/N_{t.Rd}$	Axial criterion.
CRT_TOT	$N_{Ed}/N_{t.Rd}$	Eurocode 3 global criterion.
NPLRD	$N_{pl.Rd}$	Design plastic strength of the gross cross-section.
NURD	$N_{u.Rd}$	Ultimate design strength

7.1.8. Checking of Members in Axial Compression

Corresponds to chapter 6.2.4 in EN 1993-1-1:2005.

1. Forces and moments selection.
   The forces and moments considered for this checking type are:

   N_{Ed}= FX Design value of the axial force (positive if compressive, element not processed if tensile).

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of the section with the previously selected forces and moments if the selected option is partial or with all the forces and moments if the selected option is full. The entire calculation process is accomplished with the gross section properties..

3. Criteria calculation.
   For members in axial compression, the general criterion *Crt_TOT* is checked at each section. This criterion coincides with the axial criterion *Crt_N:*

$$N_{Ed} \leq N_{c.Rd} \rightarrow \text{Crt\_TOT} = \text{Crt\_N} = \frac{N_{Ed}}{N_{c.Rd}} \leq 1$$

where $N_{c.Rd}$ is the design compression resistance of the cross-section

Class 1,2 or 3 cross-sections:

$N_{c.Rd} = Af_y/Y_{M\,0}$ design plastic resistance of the gross section

Class 4 cross sections:

EN 1993-1-1:2005:

$N_{c.Rd} = A_{eff}f_y/Y_{M\,0}$

4. Output results written in the CivilFEM results file (.CRCF) . Checking results: criteria and variables are described at the following table.

Result	Concepts	Description
NED	N_{Ed}	Design axial force (EN 1993-1-1:2005).
NCRD	$N_{c.Rd}$	Design compression strength of the cross-section.
CRT_N	$N_d/N_{c.Rd}$	Axial criterion.
CRT_TOT	$N_d/N_{c.Rd}$	Eurocode 3 global criterion.
CLASS		Section Class.
AREA	A, A_{eff}	Area of the section (Gross or Effective).

7.1.9. Checking of Members under Bending Moment

Corresponds to chapter 6.2.5 in EN 1993-1-1:2005.

1. Forces and moments selection.
   The forces and moments considered for this checking type are:
   $M_{Ed} = MY \text{ or } MZ$ Design value of the bending moment along the relevant axis for bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of the section with the previously selected forces and moments if the selected option is partial or with all the forces and moments if the selected option is full. The entire calculation process is accomplished with the gross section properties.

3. Criteria calculation.
   For members subjected to a bending moment in the absence of shear force, the following condition is checked at each section:
   where:

$$|M_{Ed}| \leq M_{c.Rd} \rightarrow Crt\_TOT = Crt\_My = \left|\frac{M_{Ed}}{M_{c.Rd}}\right| \leq 1$$

M_{Ed} = design value of the bending moment

$M_{c.Rd}$ = design moment resistance of the cross-section

Class 1 or 2 cross-sections:

$$M_{c.Rd} = W_{pl} \cdot f_y \;/\; Y_{M\,0}$$

Class 3 cross sections:

$$M_{c.Rd} = W_{el} \cdot f_y \;/\; Y_{M\,0}$$

Class 4 cross sections:

<u>EN 1993-1-1:2005:</u>

$$M_{c.Rd} = W_{eff} \cdot f_y \;/\; Y_{M\,0}$$

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
MED	M_{Ed}	Design value of the bending moment (EN 1993-1-1:2005).
MCRD	$M_{c.Rd}$	Design moment resistance of the cross-section.
CRT_M	$M_d/M_{c.Rd}$	Bending criterion.
CRT_TOT	$M_d/M_{c.Rd}$	Eurocode 3 global criterion.
CLASS		Section Class.
W	$W_{el}, W_{pl} W_{eff}$	Used section modulus (Elastic, Plastic or Effective).

7.1.10. Checking of Members under Shear Force

Corresponds to chapter 6.2.6 in EN 1993-1-1:2005.

1. Forces and moments selection.
   The forces and moments considered for this checking type are:
   $V_{Ed} = FZ \text{ or } FY$ Design value of the shear force perpendicular to the relevant axis of bending.

2. Class definition and effective section properties calculation.
   For this checking type, the section class is always 1 and the effective section is the gross section.

3. Criteria calculation.
   With members under shear force, the following condition is checked at each section:

$$|V_{Ed}| \le V_{Pl.Rd} \rightarrow \text{Crt\_TOT} = \text{Crt\_S} = \left|\frac{V_{Ed}}{V_{Pl.Rd}}\right| \le 1$$

where:

V_{Ed} design value of the shear force

$V_{Pl.Rd}$ design plastic shear resistance: $V_{Pl.Rd} = A_v \left(f_y/\sqrt{3}\right)/\Upsilon_{M0}$

A_v shear area, obtained subtracting from the gross area the summation of the flanges areas: $A_v = A - \sum \text{Flanges\_Area}$

Modifications to the previous computation of A_v are as follows:

- Rolled I and H sections, load parallel to web:
  $A_v = A_v + (t_w + 2r)t_f$

- Rolled channel sections, load parallel to web:
  $A_v = A_v + (t_w + r)t_f$

EN 1993-1-1:2005 specifies additional cases for the calculation of A_v:

- Rolled I and H sections with load parallel to web:

  $A_v = A_v + (t_w + 2r)t_f$ but not less than $\eta\, h_w t_w$

- Rolled T shaped sections with load parallel to web:
  $A_v = 0.9 \cdot (A - b \cdot t_r)$

Where:

η η = 1.2 for steels with fy = 460 MPa

η= 1.0 for steels with fy > 460 MPa

h_w Web depth

t_w Web thickness

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
VED	V_{Ed}	Design value of the shear force (EN 1993-1-1:2005).
VPLRD	$V_{pl.Rd}$	Design plastic shear resistance.
CRT_S	$V_d/V_{pl.Rd}$	Shear criterion.
CRT_TOT	$V_d/V_{pl.Rd}$	Eurocode 3 global criterion.
CLASS		Section Class.
S_AREA	Av	Shear area.

7.1.11. Checking of Members under Bending Moment and Shear Force

Corresponds to chapter 6.2.8 in EN 1993-1-1:2005.

1. Forces and moments selection.

The forces and moments considered for this checking type are:

$V_{Ed} = \text{FZ or FY}$ Design value of the shear force perpendicular to the relevant axis of bending.

$M_{Ed} = \text{MY or MZ}$ Design value of the bending moment along the relevant axis of bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of the sections with the previously selected forces and moments if the selected option is partial or with all the forces and moments if the selected option is full. The entire calculation is accomplished with gross section properties.
3. Criteria calculation.
   For members subjected to bending moment and shear force, the following condition is checked at each section:

$$|M_{Ed}| \leq M_{V.Rd} \rightarrow \text{Crt\_TOT} = \text{Crt\_BS} = \left|\frac{M_d}{M_{V.Rd}}\right| \leq 1$$

Where:

$M_{V.Rd}$ = design resistance moment of the cross-section, reduced by the presence of shear.

The reduction for shear is applied if the design value of the shear force exceeds 50% of the design plastic shear resistance of the cross-section; written explicitly as:

$$V_{Ed} > 0.5\ V_{pl.Rd}$$

The design resistance moment is obtained as follows:

EN 1993-1-1:2005:

a. For double T cross-sections with equal flanges, bending about the major axis:

$$M_{V.Rd} = \left(W_{pl} - \frac{\rho A_v^2}{4t_w}\right) {f_y}/{\gamma_{M0}}$$

$$\rho = \left(\frac{2V_{Ed}}{V_{pl.Rd}} - 1\right)^2$$

$$A_w = h_w t_w$$

b. For other cases the yield strength is reduced as follows:

$$f_y = f_y(1 - \rho)$$

Note: This reduction of the yield strength fy is applied to the entire section. Eurocode 3 only requires the reduction to be applied to the shear area, and therefore, it is a conservative simplification.

For both cases, $M_{V.Rd}$ is the smaller value of either $M_{V.Rd}$ or $M_{C.Rd}$.

$M_{C.Rd}$ is the design moment resistance of the cross-section, calculated according to the class.

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description

MED	M_{Ed}	Design value of the bending moment (EN 1993-1-1:2005).
VED	V_{Ed}	Design value of the shear force (EN 1993-1-1:2005).
MVRD	$M_{v.Rd}$	Reduced design resistance moment of the cross-section.
CRT_BS	$M_d/M_{v.Rd}$	Bending and Shear criterion.
CRT_TOT	$M_d/M_{v.Rd}$	Eurocode 3 global criterion.
CLASS		Section Class.
S_AREA	A_v	Shear area.
W	W_{el}, W_{pl}, W_{eff}	Used section modulus (Elastic, Plastic or Effective).
VPLRD	$V_{pl.Rd}$	Design plastic shear resistance.
RHO	ρ	Reduction factor.

7.1.12. Checking of Members under Bending Moment and Axial Force

Corresponds to chapter 6.2.9 in EN 1993-1-1:2005.

1. Forces and moments selection.

The forces and moments considered for this checking type are:

$N_{Ed} = FX$ Design value of the axial force.

$M_{y.Ed} = MY \text{ or } MZ$ Design value of the bending moment along the relevant axis of bending.

$M_{z.Ed} = MZ \text{ or } MY$ Design value of the bending moment about the secondary axis of bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of the sections with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. These calculations are accomplished with the gross section properties.
3. Criteria calculation.
   For members subjected to bi-axial bending and in absence of shear force, the following conditions at each section are checked:

Class 1 and 2 sections:

$$\left(\frac{M_{y.Ed}}{M_{Ny.Rd}}\right)^{\alpha} + \left(\frac{M_{z.Ed}}{M_{Nz.Rd}}\right)^{\beta} \leq 1$$

This condition is equivalent to:

$$Crt\_TOT = (Crt\_My)^{\alpha} + (Crt\_Mz)^{\beta} \leq 1$$

$$Crt\_My = \left(\frac{M_{y.Ed}}{M_{Ny.Rd}}\right)$$

$$Crt\_Mz = \left(\frac{M_{z.Ed}}{M_{Nz.Rd}}\right)$$

Where $M_{Ny.Rd}$ and $M_{Nz.Rd}$ are the design moment resistance of the cross-section, reduced by the presence of the axial force:

$$M_{Ny.Rd} = My_{pl.Rd}\left[1 - \left({N_{Ed}}/{N_{pl.Rd}}\right)^2\right]$$

$$M_{Nz.Rd} = Mz_{pl.Rd}\left[1 - \left({N_{Ed}}/{N_{pl.Rd}}\right)^2\right]$$

Where α and β are constants, which may take the following values:

For I and H sections:

$$\alpha = 2 \quad \text{and } \beta = 5n \quad \beta \geq 1$$

For circular tubes:

$$\alpha = 2 \quad \text{and } \beta = 2$$

For rectangular hollow sections:

$$\alpha = \beta = \frac{1.66}{1-1.13n^2}$$

$$\text{but} \quad \alpha = \beta \leq 6$$

For solid rectangles and plates (the rest of sections):

$$n = \left(\frac{N_{Ed}}{N_{pl.Rd}}\right)$$

Furthermore, the code specifies that in the case of rolled shapes for I or H sections or other sections with flanges, it is not necessary to reduce the design plastic strength for bending around the y-y axis due to the axial force if the following two conditions are fulfilled:

$$N_d \leq 0.25 \cdot N_{pl.Rd} \cdot y$$

$$N_d \leq \frac{0.5 \cdot h_w \cdot t_w \cdot f_y}{\gamma_{M0}}$$

(if it does not reach half the tension strength of the web)

The same is applicable for bending around the z-z axis due to the axial force. There is no reduction when the following condition is fulfiled:

$$N_d \leq \frac{h_w \cdot t_w \cdot f_y}{\gamma_{M0}}$$

In absence of $M_{z.d}$, the previous check can be reduced to:

$$\left(\frac{M_{y.Ed}}{M_{Ny.Rd}}\right) \leq 1$$

Condition equivalent to:

$$Crt\_TOT = Crt\_My = \left(\frac{M_{y.Ed}}{M_{Ny.Rd}}\right)$$

Class 3 sections (without holes for fasteners):

$$\left(\frac{N_{Ed}}{Af_{yd}}\right) + \left(\frac{M_{y.Ed}}{W_{el.y}f_{yd}}\right) + \left(\frac{M_{z.Ed}}{W_{el.z}f_{yd}}\right) \leq 1$$

Condition equivalent to:

$$Crt\_TOT = Crt\_N + Crt\_My + Crt\_Mz \leq 1$$

$$Crt\_N = \left(\frac{N_{Ed}}{Af_{yd}}\right)$$

$$Crt\_My = \left(\frac{M_{y.Ed}}{W_{el.y}f_{yd}}\right)$$

$$Crt\_Mz = \left(\frac{M_{z.Ed}}{W_{el.z}f_{yd}}\right) f_{yd} = {}^{f_y}/_{\gamma_{M0}}$$

Where $M_{el.y}$ is the elastic resistant modulus about the y axis and $W_{el.z}$ is the elastic resistant modulus about the z axis.

In absence of $M_{z.d}$, the above criterion becomes:

$$\left(\frac{N_{Ed}}{Af_{yd}}\right)+\left(\frac{M_{y.Ed}}{W_{el.y}f_{yd}}\right)\leq 1$$

Which is equivalent to:

Crt_TOT = Crt_N + Crt_My ≤ 1

$$\text{Crt\_N}\left(\frac{N_{Ed}}{Af_{yd}}\right)$$

$$\text{Crt\_My}=\left(\frac{M_{y.Ed}}{W_{el.y}f_{yd}}\right)$$

Class 4 sections:

$$\left(\frac{N_{Ed}}{A_{eff}f_{yd}}\right)+\left(\frac{M_{y.Ed}+N_{Ed}e_{Nz}}{W_{eff.y}f_{yd}}\right)+\left(\frac{M_{z.Ed}+N_{Ed}e_{Ny}}{W_{eff.z}f_{yd}}\right)$$

Condition equivalent to:

Crt_TOT = Crt_N + Crt_My + Crt_Mz ≤ 1

$$\text{Crt\_N}=\left(\frac{N_{Ed}}{A_{eff}f_{yd}}\right)$$

$$\text{Crt\_My}=\left(\frac{M_{Ey.d}+N_{Ed}e_{Nz}}{W_{eff.y}f_{yd}}\right)$$

$$\text{Crt\_Mz}=\left(\frac{M_{z.Ed}+N_{Ed}e_{Ny}}{W_{eff.z}f_{yd}}\right)$$

Where:

A_{eff}	effective area of the cross-section
$W_{eff.y}$	effective section modulus of the cross-section when subjected to a moment about the y axis
$W_{eff.z}$	effective section modulus of the cross-section when subjected to a moment about the z axis
e_{Ny}	shift of the center of gravity along the y axis
e_{Nz}	shift of the center of gravity along the z axis

Without $M_{z.d}$, the above criterion becomes:

$$\left(\frac{N_{Ed}}{A_{eff}f_{yd}}\right)+\left(\frac{M_{y.Ed}+N_{Ed}e_{Nz}}{W_{eff.y}f_{yd}}\right)+\left(\frac{N_{Ed}e_{Ny}}{W_{eff.z}f_{yd}}\right)\leq 1$$

which is equivalent to:

$$Crt\_TOT = Crt\_N + Crt\_My + Crt\_Mz \leq 1$$

$$Crt\_N=\left(\frac{N_{Ed}}{A_{eff}f_{yd}}\right)$$

$$Crt\_My=\left(\frac{M_{y.Ed}+N_{d}e_{Nz}}{W_{eff}f_{yd}}\right)$$

$$Crt\_Mz=\left(\frac{N_{Ed}e_{Ny}}{W_{eff.z}f_{yd}}\right)$$

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
NED	N_{Ed}	Design value of the axial force (EN 1993-1-1:2005).
MYED	$M_{y.Ed}$	Design value of the bending moment about Y axis (EN 1993-1-1:2005).
MZED	$M_{z.Ed}$	Design value of the bending moment about Z axis (EN 1993-1-1:2005).
NCRD	$A \cdot f_{yd,}$ $A_{eff} \cdot f_{yd}$	Design compression resistance of the cross-section
MNYRD	$M_{Ny\cdot Rd,} W_{el.y} \cdot f_{yd,}$ $W_{eff\cdot y} \cdot f_{yd}$	Reduced design moment resistance of the cross-section about Y axis
MNZRD	$M_{Nz.Rd,} W_{el.z} \cdot f_{yd,}$ $W_{eff\cdot}Z \cdot f_{yd}$	Reduced design moment resistance of the cross-section about Z axis
CRT_N	N_{Ed}/N_{cRd}	Axial criterion
CRT_MY	M_{yEd}/M_{NyRd}	Bending criterion along Y
CRT_MZ	M_{zEd}/M_{NzRd}	Bending criterion along Z

Result	Concepts	Description
ALPHA	α	Alpha constant
BETA	β	Beta constant
CRT_TOT	Crt_tot ≤ 1	Eurocode 3 global criterion
CLASS		Section Class
AREA	A, A_{eff}	Area of the section utilized (Gross or Effective)
WY	$W_{el.y,} W_{pl.y,} W_{eff.y}$	Used section Y modulus (Elastic, Plastic or Effective)
WZ	$W_{el.z,} W_{pl.z,} W_{eff.z}$	Used section Z modulus (Elastic, Plastic or Effective)
SIGXED	$\sigma_{X.Ed}$	Maximum longitudinal stress
ENY	e_{Ny}	Shift of the Z axis in Y direction
ENZ	e_{Nz}	Shift of the Y axis in Z direction
USE_MY	$M_{y.Ed} + N_{Ed} \cdot e_{Nz}$	Modified design value of the bending moment about Y axis
USE_MZ	$M_{z.Ed} + N_{Ed} \cdot e_{Ny}$	Modified design value of the bending moment about Z axis
PARM_N	n	Parameter n

7.1.13. Checking of Members under Bending, Shear and Axial Force

Corresponds to chapter 6.2.10 in EN 1993-1-1:2005.

1. Forces and moments selection. The forces and moments considered for this checking type are:

$N_{Ed} = FX$ Design value of the axial force.

$V_{y.Ed} = FY \text{ or } FZ$ Design value of the shear force perpendicular to the secondary axis of bending.

$V_{z.Ed} = FY \text{ or } FZ$ Design value of the shear force perpendicular to the relevant axis of bending.

$M_{y.Ed} = MY \text{ or } MZ$ Design value of the bending moment about the relevant axis of bending.

$M_{z.Ed} = MZ \text{ or } MY$ Design value of the bending moment about the secondary axis of bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of the sections with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. The entire calculation is accomplished with the gross section properties.
3. Criteria calculation.
   For members subjected to bending, axial and shear force, the same conditions of the bending +axial force and bi-axial bending are checked at each section, reducing the design plastic resistance moment for the presence of shear force.
   The shear force effect is taken into account when it exceeds 50% of the design plastic resistance of the cross-section. In this case, both the axial and the shear force are taken into account.
   The axial force effects are included as stated in the previous section, and the shear force effects are taken into account considering a yield strength for the cross-section, reduced by the factor (1-ρ), as follows:

$$f_{yd} = f_y(1-\rho)/\Upsilon_{M\,0}$$

   where:

$$\rho = \left(2V_{Ed}/V_{pl.Rd} - 1\right)^2 \quad \text{for} \quad V_{Ed}/V_{pl.Rd} > 0.5$$

$$\rho = 0 \quad \text{for} \quad V_{Ed}/V_{pl.Rd} < 0.5$$

   This yield strength reduction is selectively applied to the resistance of the cross-section along each axis, according to the previous conditions.

   Note: The yield strength reduction is applied to the entire cross-section; however, Eurocode only requires the reduction to be applied to the shear area. Thus, it is a conservative simplification.

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
NED	N_{Ed}	Design value of the axial force (EN 1993-1-1:2005).
VZED	$V_{z,Ed}$	Design value of the shear force (EN 1993-1-1:2005).
VYED	$V_{y,Ed}$	Design value of the shear force (EN 1993-1-1:2005).
MYED	$M_{y,Ed}$	Design value of the bending moment about Y axis (EN 1993-1-1:2005).
MZED	$M_{z,Ed}$	Design value of the bending moment about Z axis (EN 1993-1-1:2005).
NCRD	$A \cdot f_{yd,}$ $A_{eff} \cdot f_{yd}$	Design compression resistance of the cross-section.
MNYRD	$M_{Ny.Rd,}$ $W_y \cdot f_{yd} \cdot (1-\rho)$	Reduced design moment Y resistance of the cross-section.
MNZRD	$M_{Nz\cdot Rd,}$ $W_z \cdot f_{yd} \cdot (1-\rho)$	Reduced design moment Z resistance of the cross-section.
CRT_N	N_{Ed}/N_{cRd}	Axial criterion.
CRT_MY	M_{yEd}/M_{NyRd}	Bending Y criterion.
CRT_MZ	M_{zEd}/M_{NzRd}	Bending Z criterion.
ALPHA	α	Alpha constant.
BETA	β	Beta constant.
RHO_Y	ρ	Reduction factor for MNYRD.
RHO_Z	ρ	Reduction factor for MNZRD.
CRT_TOT	$Crt\_tot \leq 1$	Eurocode 3 global criterion.
AREA	A, A_{eff}	Used area of the section (Gross or Effective).
WY	$W_{el.y,}W_{pl.y,}W_{eff.y}$	Used section Y modulus (Elastic, Plastic or Effective).
WZ	$W_{el.z,}W_{pl\cdot z,}W_{eff\cdot z}$	Used section Z modulus (Elastic, Plastic or Effective).

Result	Concepts	Description
SIGXED	$\sigma_{x.Ed}$	Maximum longitudinal stress.
ENY	e_{Ny}	Shift of the Z axis in Y direction.
ENZ	e_{Nz}	Shift of the Y axis in Z direction.
USE_MY	$M_{y \cdot Ed} + N_{Ed} . e_{Nz}$	Modified design value of the bending moment about Y axis.
USE_MZ	$M_{Z.Ed} + N_{Ed} \cdot e_{Ny}$	Modified design value of the bending moment about Z axis.
SHY_AR	A_v	Shear Y area.
SHZ_AR	A_v	Shear Z area.
PARM_N	n	Parameter n.

7.1.14. Checking for Buckling of Members in Compression

Corresponds to chapter 6.3.1 in EN 1993-1-1:2005.

1. Forces and moments selection.

   The forces and moments considered in this checking type are:

   $N_{Ed} = FX$ Design value of the axial force (positive if compressive, otherwise element is not processed).

2. Class definition and effective section properties calculation.
   The section class is determined by the sections general processing with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. The entire calculation is accomplished with the gross section properties.
3. Criteria calculation.
   When checking the buckling of compression members, the criterion is given by:

$$N_{Ed} \leq N_{b.Rd} \quad \rightarrow \quad Crt\_TOT = Crt\_CB = \frac{N_{Ed}}{N_{b.Rd}} \leq 1$$

where:

$N_{b.Rd}$ Design buckling resistance. $N_{b.Rd} = \chi \ \beta \ A \, f_y / \gamma_{M1}$

β = 1 for class 1, 2 or 3 sections.

β = A_{eff}/A for class 4 sections.

χ	Reduction factor for the relevant buckling mode, the program does not consider the torsional or the lateral-torsional buckling.

The χ calculation in members of constant cross-section may be determined from:

$$\chi = \frac{1}{\emptyset + \left(\emptyset^2 - \bar{\lambda}^2\right)^{1/2}} \leq 1$$

$$\emptyset = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

where α is an imperfection factor that depends on the buckling curve. This curve depends on the cross-section type, producing the following values for α:

Section type	**Limits**	**Buckling axis**	**Steel f_y**	**Buckling curve**	α
Rolled I	h/b>1.2 and t ≤ 40mm	y – y	< 460 MPa	a	0.21
			≥ 460 MPa	a0	0.13
Rolled I	h/b>1.2 and t ≤ 40mm	z – z	< 460 MPa	b	0.34
			≥ 460 MPa	a0	0.13
Rolled I	h/b>1.2 and 40mm<t ≤ 100mm	y – y	< 460 MPa	b	0.34
			≥ 460 MPa	a	0.21
Rolled I	h/b>1.2 and 40mm<t ≤ 100mm	z – z	< 460 MPa	c	0.49
			≥ 460 MPa	a	0.21
Welded I	h/b ≤ 1.2 and t ≤ 100mm	y – y	< 460 MPa	b	0.34
			≥ 460 MPa	a	0.21
Welded I	h/b ≤ 1.2 and t ≤ 100mm	z – z	< 460 MPa	c	0.49
			≥ 460 MPa	a	0.21
Rolled I	t>100mm	y – y	< 460 MPa	d	0.76
			≥ 460 MPa	c	0.49
Rolled I	t>100mm	z – z	< 460 MPa	d	0.76
			≥ 460 MPa	c	0.49
Welded I	t ≤ 40mm	y – y	all	b	0.34
Welded I	t ≤ 40mm	z – z	all	c	0.49
Welded I	t >40mm	y – y	all	c	0.49

Welded I	t >40mm	z – z	all	d	0.76
Pipes	Hot finished	all	< 460 MPa	a	0.21
			≥ 460 MPa	a0	0.13
	Cold formed	all	all	c	0.49
Reinforced box sections	Thick weld: a/t>0.5 b/t<30 h/tw<30	all	all	c	0.49
	In other case	all	all	b	0.34
U, T, plate	-	all	all	c	0.49
L	-	all	all	b	0.34

$$\bar{\lambda} = \left[\beta_A A f_y / N_{cr}\right]^{1/2}$$

Where N_{cr} is the elastic critical force for the relevant buckling mode. (See section for Critical Forces and Moments Calculation).

In the case of angular sections, the buckling length will be taken as the highest among the buckling lengths on the Y and Z axis.

4. The elastic critical axial forces are calculated in the planes XY (Ncr_{xy}) and XZ (Ncr_{xz}) and the corresponding values of χ_{xy} and χ_{xz}, and the correspondent to the principal axis Ncr_u and Ncr_v and the values for χ_u and χ_v taking the smaller one as the final value for χ.

$$\chi = \min\left(\chi_{xy}, \chi_{xz}, \chi_u, \chi_v\right)$$

5. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
NED	N_{Ed}	Design value of the compressive force (EN 1993-1-1:2005).
NBRD	$N_{b.Rd}$	Design buckling resistance of a compressed member.

Result	Concepts	Description
CRT_CB	$N_d/N_{b.Rd}$	Compression buckling criterion.
CRT_TOT	$N_d/N_{b.Rd}$	Eurocode 3 global criterion.
CHI	$Min\{\chi_y, \chi_z\}$	Reduction factor for the relevant buckling mode.
BETA_A	β_A	Ratio of the used area to gross area.
AREA	A	Area of the gross section.
CHI_Y	χ_y	Reduction factor for the relevant My buckling mode.
CHI_Z	χ_z	Reduction factor for the relevant Mz buckling mode.
CHI_V	χ_v	Reduction factor for the principal axis V.
CHI_U	χ_u	Reduction factor for the principal axis U.
CLASS		Section Class.
PHI_Y	$\emptyset_y$	Parameter Phi for bending My.
PHI_Z	$\emptyset_z$	Parameter Phi for bending Mz.
PHI_V	$\emptyset_v$	Parameter Phi for the principal axis V.
PHI_U	$\emptyset_u$	Parameter Phi for the principal axis U.
LAM_Y	λ_y	Non-dimensional reduced slenderness for bending My.
LAM_Z	λ_z	Non-dimensional reduced slenderness for bending Mz.
LAM_V	λ_v	Non-dimensional reduced slenderness for the principal axis V.
LAM_U	λ_u	Non-dimensional reduced slenderness for the principal axis U.
NCR_Y	N_{cr}	Elastic critical force for the relevant My buckling mode.
NCR_Z	N_{cr}	Elastic critical force for the relevant Mz buckling mode.
NCR_V	N_{cr}	Elastic critical force for the principal axis V.

Result	Concepts	Description
NCR_U	N_{cr}	Elastic critical force for the principal axis U.
ALP_Y	α_y	Imperfection factor for bending My.
ALP_Z	αz	Imperfection factor for bending Mz.

7.1.15. Checking for Lateral-Torsional Buckling of Beams Subjected to Bending

Corresponds to chapter 6.3.2 in EN 1993-1-1:2005.

1. Forces and moments selection.

The forces and moments considered for this checking type are:

$M_{Ed} = \text{MY or MZ}$ Design value of the bending moment about the relevant axis of bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of sections with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. The entire calculation is accomplished with the gross section properties.
3. Criteria calculation.
   When checking for lateral-torsional buckling of beams, the criterion shall be taken as:

$$|M_{Ed}| \leq M_{b.Rd} \rightarrow \text{Crt\_TOT} = \text{Crt\_LT} = \left|\frac{M_{Ed}}{M_{b.Rd}}\right| \leq 1$$

where:

$M_{b.Rd}$ Design buckling resistance moment of a laterally unrestrained beam. $M_{b.Rd} = \chi_{LT}\beta_w W_{pl.y} f_y \ / \ \gamma_{M1}$

β_w = 1 for class 1and 2 sections.

$\beta_w = W_{el.y}/W_{pl.y}$ for class 3 sections.

$\beta_w = W_{eff.y}/W_{pl.y}$ for class 4 sections.

χ_{LT} Reduction factor for lateral-torsional buckling.

The value of χ_{LT} is calculated as:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \left(\phi_{LT}^2 - \bar{\lambda}_{LT}^2\right)^{1/2}} \leq 1$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right]$$

$$\bar{\lambda}_{LT} = \left[\beta_w W_{pl.y} f_y / M_{cr}\right]^{1/2}$$

Where:

α_{LT} is the imperfection factor for lateral-torsional buckling:

Section type	**Limits**	**Buckling curve**	**α**
Rolled I	h/b≤2 h/b>2	a b	0.21 0.34
Welded I	h/b≤2 h/b>2	c d	0.49 0.76
Others			0.76

M_{cr} is the elastic critical moment for lateral-torsional buckling.

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	**Concepts**	**Description**
MED	M_{Ed}	Design value of the bending moment (EN 1993-1-1:2005).
MBRD	$M_{b.Rd}$	Buckling resistance moment of a laterally unrestrained beam.
CRT_LT	$M_d/M_{b.Rd}$	Lateral-torsional buckling criterion.
CRT_TOT	$M_d/M_{b \cdot Rd}$	Eurocode 3 global criterion.
CLASS		Section Class.
CHI_LT	χ_{LT}	Reduction factor for lateral-torsional buckling.

BETA_W	β_W	Ratio of the used modulus to plastic modulus.
WPL	$W_{pl.y}$	Plastic modulus.
PHI_LT	$\emptyset_{LT}$	Parameter Phi for lateral-torsional buckling.
LAM_LT	λ_{LT}	Non-dimensional reduced slenderness.
MCR	Mcr	Elastic critical moment for lateral-torsional buckling.
ALP_LT	α_{LT}	Imperfection factor for lateral-torsional buckling.

7.1.16. Checking for Lateral-Torsional Buckling of Members Subjected to Bending and Axial Compression

Corresponds to chapter 6.3.3 in EN 1993-1-1:2005.

1. Forces and moments selection.

The forces and moments considered in this checking type are:

N_{Ed}= FX — Design value of the axial compression (positive if compressive, otherwise element not processed if tensile).

$M_{y.Ed}$= MY or MZ — Design value of the bending moment about the relevant axis of bending.

$M_{z.Ed}$ = MZ or MY — Design value of the bending moment about the secondary axis of bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of sections with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. The entire calculation is accomplished with the gross section properties.
3. Criteria calculation.

EN 1993-1-1:2005 and Annex B (method 2)

The following criterion will always be calculated:

$$\left(\frac{N_{Ed}}{N_{b,Rd\,1}}\right)+\left(K_y C_{my}\frac{M_{y,Ed}+e_{N,y}N_{Ed}}{M_{yb,Rd\,1}}\right)+\left(\alpha_z K_y C_{mz}\frac{M_{z,Ed}+e_{N,z}N_{Ed}}{M_{zb,Rd\,1}}\right)\leq 1$$

Crt_1 = Crt_N1 + Crt_My1 + Crt_Mz1 ≤ 1

Elements without torsional buckling:

$$\left(\frac{N_{Ed}}{N_{b,d\,2}}\right) + \left(\alpha_y K_y C_{my} \frac{M_{y,Ed} + e_{N,y} N_{Ed}}{M_{yb,Rd\,2}}\right) + \left(K_z C_{mz} \frac{M_{z,Ed} + e_{N,z} N_{Ed}}{M_{zb,Rd\,2}}\right) \leq 1$$

Elements which may have torsional buckling:

$$\left(\frac{N_{Ed}}{N_{b.Rd\,2}}\right) + \left(K_{yLT} \frac{M_{y,Ed} + e_{N,y} N_{Ed}}{M_{yb,Rd\,2}}\right) + \left(K_z C_{mz} \frac{M_{z,Ed} + e_{N,z} N_{Ed}}{M_{zb,Rd\,2}}\right) \leq 1$$

→ Crt_2 = Crt_N2 + Crt_My2 + Crt_Mz2 ≤ 1

→ Crt_TOT = Max (Crt_1, Crt_2)

Where:

$\text{Crt\_N1} = \left(\frac{N_{Ed}}{N_{b,Rd\,1}}\right)$	Axial force criterion 1.
$\text{Crt\_My1} = \left(K_y C_{my} \frac{M_{y,Ed} + e_{N,y} N_{Ed}}{M_{yb,Rd\,1}}\right)$	Bending moment criterion for principal axis 1.
$\text{Crt\_Mz1} = \left(K_z C_{mz} \frac{M_{z,Ed} + e_{N,z} N_{Ed}}{M_{zb,Rd\,2}}\right)$	Bending moment criterion for secondary axis 1
Crt_TOT1	General criterion 1.
$\text{Crt\_N2} = \left(\frac{N_{Ed}}{N_{b,Rd\,2}}\right)$	Axial force criterion 2.
$\text{Crt\_My2} \left(\alpha_y K_y C_{my} \frac{M_{y,Ed} + e_{N,y} N_{Ed}}{M_{yb,Rd\,2}}\right)$	Bending moment criterion 2 for principal axis without torsional buckling
$\text{Crt\_My2} = \left(K_{yLT} \frac{M_{y,Ed} + e_{N,y} N_{Ed}}{M_{yb,Rd\,2}}\right)$	Bending moment criterion 2 for principal axis when torsional buckling is considered.
$\text{Crt\_Mz2} = \left(K_z C_{mz} \frac{M_{z,Ed} + e_{N,z} N_{Ed}}{M_{zb,Rd\,2}}\right)$	Bending moment criterion 2 for secondary axis.
Crt_TOT2	Criterion 2
Crt_TOT=max (Crt_TOT1, Crt_TOT2)	Global criterion.

Where:

$N_{b,Rd1} = \chi_y A f_y/\gamma_{M1}$ $M_{b,Rdy1} = \chi_{LT} W_y f_y/\gamma_{M1}$ $M_{b,Rdz1} = W_z f_y/\gamma_{M1}$

$N_{b,Rd2} = \chi_y A f_y/\gamma_{M1}$ $M_{b,Rdy2} = \chi_{LT} W_y f_y/\gamma_{M1}$ $M_{b,Rdz2} = W_z f_y/\gamma_{M1}$

($\chi_{LT} = 1.0$ when torsional buckling is not considered).

χ_y and χ_z are the reduction factors defined for the section corresponding to the check for Buckling of Compression Members.

χ_{LT} lateral buckling factor according to 6.3.2.2. Assumes the value of 1 for members not susceptible to torsional deformations.

$e_{N,y}$ and $e_{N,z}$ shifts of the centroid of the effective area relative to the centre of gravity of the gross section in class 4 members for y, z axes.

$C_{m,y}$, $C_{m,z}$ and $C_{m,LT}$ are equivalent uniform moment factors for flexural bending. These factors are entered as member properties at member level. (See C_{My}, C_{My} and C_{Mz}). These factors may be taken from Table B.3 from Annex B of code EN 1993-1-1:2005.

Checking Parameters:

Class	A	W_Y	W_z	α_Y	α_z	$e_{N,y}$	$e_{N,z} e_{N,z}$
1	A	$W_{pl,y}$	$W_{pl,z}$	0.6	0.6	0	0
2	A	$W_{pl,y}$	$W_{pl,z}$	0.6	0.6	0	0
3	A	$W_{el,y}$	$W_{el,z}$	0.8	1	0	0
4	A_{eff}	$W_{eff,y}$	$W_{eff,z}$	0.8	1	Depending on members and stresses	Depending on members and stresses

Interaction Factors:

Class	Section type	K_y	K_z	K_{yLT}
1 y 2	I, H	$1 + (\bar{\lambda}_y - 0.2)\frac{N_{Ed}}{\chi_y N_{C,Rd}}$	$1 + (2\bar{\lambda}_z - 0.6)\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	$1 - \frac{0.1.\bar{\lambda}_z}{(C_{mLT} - 0.25)} . \frac{N_{Ed}}{\chi_z N_{C,Rd}} \leq 0.6 + \bar{\lambda}_z$

	RHS		$1+(\bar{\lambda}_z-0.2)\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	
3 y 4	All sections	$1+0.6\bar{\lambda}_y\frac{N_{Ed}}{\chi_y N_{C,Rd}}$	$1+0.6\bar{\lambda}_z\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	$1-\frac{0.5.\bar{\lambda}_z}{(C_{mLT}-0.25)}\cdot\frac{N_{Ed}}{\chi_z N_{C,Rd}}$

where:

λ_y y λ_z Limited slenderness values for y-y and z-z axes, less than 1.

$$N_{C,Rd}=A^*\frac{f_y}{\gamma_{M1}}$$

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	**Concepts**	**Description**
NED	N_{Ed}	Design value of the axial compression force.
MYED	$M_{y.Ed}$	Design value of the bending moment about Y axis.
MZED	$M_{z.Ed}$	Design value of the bending moment about Z axis.
NBRD1	$\chi_y \cdot A \cdot f_y/\gamma_{M1}$	Design compression resistance of the cross-section.
MYRD1	$\chi_{LT} \cdot W_y \cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Y axis.
MZRD1	$W_z \cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Z axis.
NBRD2	$\chi_z \cdot A \cdot f_y/\gamma_{M1}$	Design compression resistance of the cross-section.
MYRD2	$\chi_{LT} \cdot W_y \cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Y axis.

Result	Concepts	Description
MZRD2	$W_z \cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Z axis.
K_Y	K_y	Parameter K_y.
K_Z	K_z	Parameter K_z.
K_LT	K_{LT}	Parameter K_{LT}.
CRT_N1	N_{Ed}/N_{cRd1}	Axial criterion.
CRT_MY1	$K_y C_{my}\left(M_{y,Ed} + N_{Ed} \cdot e_{Ny}\right)/M_{yb,Rd1}$	Bending Y criterion.
CRT_MZ1	$\alpha_z \cdot K_z \cdot C_{mz}\left(M_{z,Ed} + N_{Ed}e_{Nz}\right)/M_{zb,Rd1}$	Bending Z criterion.
CRT_1	CRT_N1+CRT_MY1+CRT_MZ1	Criterion 1
CRT_N2	N_{Ed}/N_{cRd2}	Axial criterion.
CRT_MY2	$KC_{my}\left(M_{y,Ed} + N_{Ed} \cdot e_{Ny}\right)/M_{yRd2}$	Bending Y criterion. K=K_{LT}/C_{my} if torsion exists and if not present K=$\alpha_y K_y$
CRT_MZ2	$K_z \cdot C_{mz}\left(M_{z,Ed} + N_{Ed}e_{Nz}\right)/M_{zb,Rd2}$	Bending Z criterion.
CRT_2	CRT_N2+CRT_MY2+CRT_MZ2	Criterion 2
CRT_TOT	Crt_tot $\leq$ 1	Eurocode 3 global criterion.
CLASS		Section Class.
CHIMIN	$\mathrm{Min}\{\chi_y, \chi_z\}$	Reduction factor for the relevant buckling mode.
CHI_Y	χ_y	Reduction factor for the relevant My buckling mode.
CHI_Z	χ_z	Reduction factor for the relevant Mz buckling mode.
CHI_LT	χ_{LT}	Reduction factor for lateral-torsional buckling.
AREA	A, A_{eff}	Used area of the section (Gross or Effective).

Result	Concepts	Description
WY	$W_{el.y}, W_{pl,y}, W_{eff.y}$	Used section Y modulus (Elastic, Plastic or Effective).
WZ	$W_{el.z}, W_{pl,z}, W_{eff.z}$	Used section Z modulus (Elastic, Plastic or Effective).
ENY	e_{Ny}	Shift of the Z axis in Y direction.
ENZ	e_{Nz}	Shift of the Y axis in Z direction.
NCR_Y	N_{cr}	Elastic critical force for the relevant My buckling mode.
NCR_Z	N_{cr}	Elastic critical force for the relevant Mz buckling mode.
MCR	M_{cr}	Elastic critical moment for lateral-torsional buckling.
LAM_Y	λ_y	Non-dimensional reduced slenderness for bending My.
LAM_Z	λ_z	Non-dimensional reduced slenderness for bending Mz.
LAM_LT	λ_{LT}	Non-dimensional reduced slenderness for lateral-torsional buckling.

7.1.17. Critical Forces and Moments Calculation

The critical forces and moments $N_{cr\,xy}$, $N_{cr\,xz}$ and M_{cr}, are needed for the different types of buckling checks. They are calculated based on the following formulation:

$$N_{cr\,xy} = \frac{AE\pi^2}{\lambda_{xy}^2} = AE\left(\frac{\pi\, i_{xy}}{L_{xy}}\right)^2$$

$$N_{cr\,xz} = \frac{AE\pi^2}{\lambda_{xz}^2} = AE\left(\frac{\pi\, i_{xz}}{L_{xz}}\right)^2$$

where:

$N_{cr\,xy}$	Elastic critical axial force in plane XY.
$N_{cr\,xz}$	Elastic critical axial force in plane XZ.
A	Gross area.
E	Elasticity modulus.
λ_{xy}	Member slenderness in plane XY.
λ_{xz}	Member slenderness in plane XZ.
i_{xy}	Radius of gyration of the member in plane XY.
i_{xz}	Radius of gyration of the member in plane XZ.
L_{xy}	Buckling length of member in plane XY.
L_{xz}	Buckling length of member in plane XZ.

The buckling length in both planes is the length between the ends restrained against lateral movement and it is obtained from the member properties, according to the following expressions:

$$L_{xy} = L \cdot \mathrm{Cfbuckxy}$$

$$L_{xz} = L \cdot \mathrm{Cfbuckxz}$$

where:

Cfbuckxy	Buckling factor in plane XY.
Cfbuckxz	Buckling factor in plane XZ.

For the calculation of the elastic critical moment for lateral-torsional buckling, Mcr, the following equation shall be used. This equation is *only valid for uniform symmetrical cross-sections about the minor axis* (Annex F, ENV 1993-1-1:1992). Eurocode 3 does not provide a method for calculating this moment in nonsymmetrical cross-sections or sections with other symmetry plane (angles, channel section, etc.).

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(kL)^2}\left\{\left[\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z} + \left[C_2 Z_g - C_3 Z_j\right]^2\right]^{1/2} - \left[C_2 Z_g - C_3 Z_j\right]\right\}$$

$$Z_j = Z_s - \frac{0.5}{I_y}\int_A (y^2 + z^2)\ z\,dA$$

where:

M_{cr}	Elastic critical moment for lateral-torsional buckling.
C_1, C_2 y C_3	Factors depending on the loading and end restraint conditions.
k y k_w	Effective length factors.
E	Elasticity modulus.
I_y	Moment of inertia about the principal axis.
I_y	Moment of inertia about the minor axis.
L	Length of the member between end restraints.
G	Shear modulus.
Z_g	$Z_a - Z_s$
Z_a	Coordinate of the point of load application. By default the load is applied at the center of gravity, therefore: $Z_a = 0$.
Z_s	Coordinate of the shear center.
A	Cross-section area.

Factors C and k are read from the properties at structural element level.

The integration of the previous equation is calculated as a summation extending to each plate. This calculation is accomplished for each plate according to its ends coordinates: y_1, z_1 and y_2, z_2 and its thicknesses.

$$\int_A (y^2 + z^2)\,z\,dA = \sum_{i=1}^{n\ plates} S_i{}^{*} \int_{L_i} (y^2 + z^2)\,z\,dl$$

where:

s_i = thickness of plate i

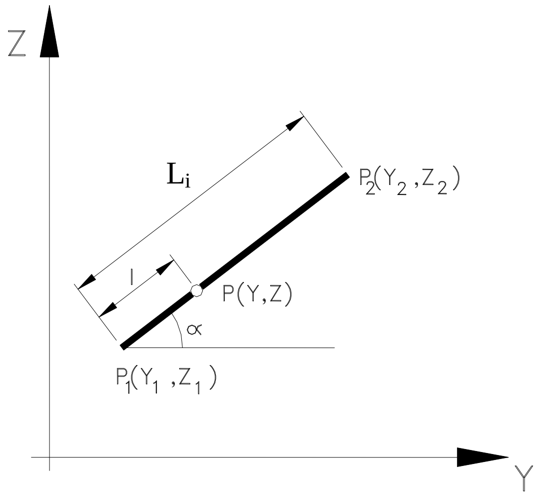
dA = s_i * dl

$$y = y_1 + I^{*}\ \cos\alpha$$

$$z = z_1 + I^{*} \sin\alpha$$

$$\alpha = \arctan\frac{z_2 - z_1}{y_2 - y_1}$$

$$L_i = \sqrt{(y_1 - y_2)^2 + (z_1 - z_2)^2} = \text{plate width}$$



7.2. Steel Structures According to AISC ASD/LRFD 13th Ed.

7.2.1. Material properties

For AISC 13th Edition checking, the following material properties are used:

Description	Property
Steel yield strength	Fy(th)
Ultimate strength	Fu(th)
Elasticity modulus	E
Poisson coefficient	ν
Shear modulus	G

*th =thickness of the plate

7.2.2. Section data

AISC 13th Edition considers the following data set for the section:

- Gross section data
- Net section data
- Effective section data.
- Data belonging to the section and plates class.

Gross section data correspond to the nominal properties of the cross-section. For the net section, only the area is considered. This area is calculated by subtracting the holes for screws, rivets and other holes from the gross section area. (The area of holes is introduced within the structural steel code properties).

The effective section data and the section and plates class data are obtained in the checking process according to chapter B, section B4 of the code. This chapter classifies steel sections into three groups (compact, noncompact and slender), depending upon the width-thickness ratio and other mandatory limits.

The AISC 13TH Edition module utilizes the gross section data in user units and the CivilFEM axis or section axis as initial data. The program calculates the effective section data and the class data, and stores them in CivilFEM's results file, in user units and in CivilFEM or section axis.

The section data used in AISC 13^{TH} Edition are shown in the following tables:

Description	Data
Input data:	
1.- Height	H
2.- Web thickness	Tw
3.- Flanges thickness	Tf
4.- Flanges width	B
5.- Distance between flanges	Hi
6.- Radius of fillet (Rolled shapes)	r_1
7.- Toe radius (Rolled shapes)	r_2
8.- Weld throat thickness (Welded shapes)	a
9.- Web free depth	d
Output data	(None)

Description	Data	Reference axes
Input data:		
1.- Depth in Y	Tky	CivilFEM
2.- Depth in Z	tkz	CivilFEM
3.- Cross-section area	A	
4.- Moments of inertia for torsion	It	CivilFEM
5.- Moments of inertia for bending	Iyy, Izz	CivilFEM
6.- Product of inertia	Izy	CivilFEM
7.- Elastic resistant modulus	Wely, Welz	CivilFEM
8.- Plastic resistant modulus	Wply, Wplz	CivilFEM
9.- Radius of gyration	iy, iz	CivilFEM
10.- Gravity center coordinates	Ycdg, Zcdg	Section
11.- Extreme coordinates of the perimeter	Ymin, Ymax, Zmin, Zmax	Section
12.- Distance between GC and SC in Y and in Z	Yms, Zms	Section
13.- Warping constant	Iw	
14.- Shear resistant areas	Yws, Zws	CivilFEM
15.- Torsional resistant modulus	Xwt	CivilFEM
16.- Moments of inertia for bending about U, V	Iuu, Ivv	Principal
17.- Angle Y->U or Z->V	α	CivilFEM
Output data:	(None)	

Description	Data
Input data: 1.- Gross section area 2.- Area of holes	 Agross Aholes
Output data: 1.- Cross-section area	 Anet

The effective section depends upon the geometry of the section; thus, the effective section is calculated for each element and each of the ends of the element.

Description	Data
Input data:	(None)
Output data: 1.- Reduction factor 2.- Reduction factor 3.- Reduction factor	 Q Qs Qa

7.2.3. Structural steel code properties

For AISC 13th Edition checking, besides the section properties, more data are needed for bucling checks. These data are shown in the following table.

Description	Data
Input data: 1.- Unbraced length of member (global buckling) 2.- Effective length factors Y direction 3.- Effective length factors Z direction 4.- Effective length factors for torsional buckling 5.- Flexural factor relative to bending moment 6.- Length between lateral restraints	 L KY KZ KTOR Cb Lb
Output data: 1.- Compression class 2.- Bending class	 CLS_COMP CLS_FLEX

7.2.4. Check Process

Necessary steps to conduct the different checks in CivilFEM are as follows:

a) Obtain material properties corresponding to the element stored in CivilFEM database and calculate the rest of the properties needed for checking:
Properties obtained from CivilFEM database (materials):

Elasticity modulus	E
Poisson's ratio	ν
Yield strength	Fy (th)
Ultimate strength	Fu (th)
Shear modulus	G
Thickness of corresponding plate	th

b) Obtain the cross-sectional data corresponding to the element.

c) Initiate the values of the plate's reduction factors and the other plate's parameters to determine its class.

d) Perform a check of the section according to the type of external load.

e) Results. In CivilFEM, checking results for each element end are stored in the results file .CRCF

7.2.5. Design requirements

7.2.5.1. Design for Strength Using Load and Resistance Factor Design (LRFD)

Design shall be performed in accordance with:

$$R_u \leq \phi R_n$$

Where:

R_u	Required strength (LRFD).
R_n	Nominal strength.
ϕ	Resistance factor.
ΦR_n	Design strength

7.2.5.2. Design for Strength Using Allowable Strength Design (ASD)

Design shall be performed in accordance with:

$$R_a \leq R_n/\Omega$$

Where:

R_a	Required strength (ASD)
R_n	Nominal strength.
Ω	Safety factor
R_n/Ω	Allowable strength

Section Class and Reduction Factors Calculation.

Steel sections are classified as compact, noncompact or slender-element sections. For a section to qualify as compact its flanges must be continuously connected to the web or webs and the width-thickness ratios of its compression elements must not exceed the limiting width-thickness ratios λ_p (see table B4.1 of AISC 13th Edition). If the width-thickness ratio of one or more compression elements exceeds λ_p but does not exceed λ_r, the section is noncompact. If the width-thickness ratio of any element exceeds λ_r, (see table B4.1 of AISC 13th Edition), the section is referred to as a slender-element compression section.

Therefore, the code suggests different lambda values depending on if the element is subjected to compression, flexure or compression plus flexure.

The section classification is the worst-case scenario of all of its plates. Therefore, the class is calculated for each plate with the exception of pipe sections, which have their own formulation because it cannot be decomposed into plates. This classification will consider the following parameters:

a) Length of elements:

   The program will define the element length (b or h) as the length of the plate (distance between the extreme points), except when otherwise specified.

b) Flange or web distinction:

   To distinguish between flanges or webs, the program follows the criteria below:

   Once the principal axis of bending is defined, the program will examine the plates of the section. Fields Pty and Ptz of the plates indicate if they behave as flanges, webs or undefined, choosing the correct one for the each axis. If undefined, the following criterion will be used to classify the plate as flange or web:

If $|\Delta y| < |\Delta z|$ (increments of end coordinates) and flexure is in the Y axis, it will be considered a web; if not, it will be a flange. The reverse will hold true for flexure in the Z-axis.

- **Hot rolled Steel Shapes**:

  Section I and C:

  The length of the plate h will be taken as the value d for the section dimensions.

  Section Box:

  The length of the plate will be taken as the width length minus three times the thickness.

7.2.5.3. Members subjected to compression

In order to check for compression it is necessary to determine if the element is stiffened or unstiffened.

- For stiffened elements:

$$\lambda_p = 0.0$$

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$

Pipe sections

$$\lambda_r = 0.11\frac{E}{F_y}$$

Box sections

$$\lambda_p = 1.12\sqrt{\frac{E}{F_y}}$$

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$

- Unstiffened elements:

$$\lambda_p = 0.0 \qquad \lambda_r = 0.56\sqrt{\frac{E}{F_y}}$$

Angular sections

$$\lambda_r = 0.45\sqrt{\frac{E}{F_y}}$$

Stem of T sections

$$\lambda_r = 0.75\sqrt{\frac{E}{F_y}}$$

7.2.5.4. Members subjected to bending

The bending check is only applicable to very specific sections. Therefore, the slenderness factor is listed for each section:

- *Section I and C:*

$$P_y = F_y \cdot A_g \; ; \; \Phi_b = 0.90$$

$$k_c = \frac{4}{\sqrt{h/t_w}}$$

F_r = 69 MPa for hot rolled shapes (10 ksi)

114 MPa for welded sections (16.5 ksi)

F_L= minimum of $(F_{yf} - F_r)$ and (F_{yw}) where F_{yf} and F_{yw} are the F_y of flange and web respectively.

Flanges of rolled sections:

$$\lambda_p = 0.38\sqrt{\frac{E}{F_L}} \qquad \lambda_r = 0.83\sqrt{\frac{E}{F_L}}$$

Flanges of welded sections:

$$\lambda_p = 0.38\sqrt{\frac{E}{F_{yf}}} \qquad \lambda_r = 0.95\sqrt{\frac{E}{F_L/k_c}}$$

Flange:

If $P_u/\Phi P_y \leq 0.125$: $\lambda_p = 3.76\sqrt{\frac{E}{F_y}}\left(1 - 2.75\frac{P_u}{\emptyset P_y}\right)$

If ${P_u}/{\Phi P_y} > 0.125$: $\lambda_p = 1.12\sqrt{\frac{E}{F_y}}\left(2.33 - \frac{P_u}{\emptyset P_y}\right) \geq 1.49\sqrt{\frac{E}{F_y}}$

Always: $\lambda_r = 5.70\sqrt{\frac{E}{F_y}}\left(1 - 0.74\frac{P_u}{\Phi P_y}\right)$

P_u is the compression axial force (taken as positive). If in tension, it will be taken as zero.

- *Pipe section:*

  $\lambda_p = 0.07\frac{E}{F_y}$

  $\lambda_r = 0.31\frac{E}{F_y}$

Box section:

Flanges of box section:

$$\lambda_p = 1.12\frac{E}{F_y}$$

$$\lambda_r = 1.40\frac{E}{F_y}$$

Flanges: the program distinguishes between the flange and web upon the principal axis chosen by the user.

If ${P_u}/{\Phi P_y} \leq 0.125$: $\lambda_p = 3.76\sqrt{\frac{E}{F_y}}\left(1 - 2.75\frac{P_u}{\Phi P_y}\right)$

If ${P_u}/{\Phi P_y} > 0.125$: $\lambda_p = 1.12\sqrt{\frac{E}{F_y}}\left(2.33 - \frac{P_u}{\Phi P_y}\right) \geq 1.49\sqrt{\frac{E}{F_y}}$

Always: $\lambda_r = 5.70\sqrt{\frac{E}{F_y}}\left(1 - 0.74\frac{P_u}{\Phi P_y}\right)$

- *T section:*

  $\lambda_p = 0.0$

  Stem: $\lambda_p = 0.75\sqrt{\frac{E}{F_y}}$

Flanges: $\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$

7.2.6. Checking of Members for Tension (Chapter D)

The axial tension force must be taken as positive (if the tension force has a negative value, the element will not be checked)

Design tensile strength $\Phi_t P_n$ and the allowable tensile strength P_n/Ω_t, of tension members, shall be the lower value of :

a) yielding in the gross section:
$P_n = F_y A_g$

$$\Phi_t = 0.90\ (\text{LRFD}) \quad \Omega_t = 1.67\ (\text{ASD})$$

b) rupture in the net section:
$P_n = F_u A_e$

$$\Phi_t = 0.75(\text{LRFD}) \quad \Omega_t = = 2.00\ (\text{ASD})$$

Being:

A_e	Effective net area.
A_g	Gross area.
F_y	Minimum yield stress.
F_u	Minimum tensile strength.

The effective net area will be taken as A_g – A_{HOLES}. The user will need to enter the correct value for A_{HOLES} (the code indicates that the diameter is 1/16th in. (2 mm) greater than the real diameter).

7.2.7. Checking of Members in Axial Compression (Chapter E)

The design compressive strength, $\Phi_c P_n$,and the allowable compressive strength, P_n/Ω_c , are determined as follows:

The nominal compressive strength, P_n, shall be the lowest value obtained according to the limit states of flexural buckling, torsional buckling and flexural-torsional buckling.

$$\emptyset_c = 0.90\ (\text{LRFD}) \quad \Omega_c = 1.67\ (\text{ASD})$$

7.2.8. Compressive Strength for Flexural Buckling

This type of check can be carried out for compact sections as well as for noncompact or slender sections. These three cases adhere to the following steps:

Nominal compressive strength, P_n :

$$P_n = A_g F_{cr} \qquad \text{(E3-1)}$$

$$\lambda_c = \frac{KI}{r\pi}\sqrt{\frac{F_y}{E}}$$

$$Q = Q_s Q_a$$

a) For : $\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{QF_y}}$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

b) for $\frac{KL}{r} > 4.71\sqrt{\frac{E}{QF_y}}$

$$F_{cr} = 0.877F_e$$

Where:

A_g	Gross area of member.
r	Governing radius of gyration about the buckling axis.
K	Effective length factor.
l	Unbraced length.
F_e	Elastic critical buckling stress $F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$

Factor Q for compact and noncompact sections is always 1. Nevertheless, for slender sections, the value of Q has a particular procedure. Such procedure is described below:

Factor Q for slender sections:

For unstiffened plates, Qs must be calculated and for stiffened plates, Qa must be determined. If these cases do not apply (box sections or angular sections, for example), a value of 1 for these factors will be taken.

For circular sections, there is a particular procedure of calculating Q. Such procedure is described below:

- For circular sections, Q is:
  $Q = Q_a = \frac{0.038 \cdot E}{F_y(D/t)} + \frac{2}{3} \qquad 0.11\, ^{E}/_{F_y} \leq D/t < 0.45\, ^{E}/_{F_y}$

Factor Qs:

If there are several plates free, the value of Qs is taken as the biggest value of all of them. The program will check the slenderness of the section in the following order:

- Angular

  If $0.45\sqrt{E/F_y} < \lambda \leq .91\sqrt{E/F_y} \qquad Q_s = 1.340 - 0.76\frac{\lambda}{\sqrt{E/F_y}}$

  If $0.91\sqrt{E/F_y} < \lambda \qquad Q_s = 0.53\frac{E/F_y}{\lambda^2}$

- Stem of T

  If $0.75\sqrt{E/F_y} < \lambda \leq 1.03\sqrt{E/F_y} \qquad Q_s = 1.908 - 1.22\frac{\lambda}{\sqrt{E/F_y}}$

  If $1.03\sqrt{E/F_y} < \lambda \qquad Q_s = 0.69\frac{E/F_y}{\lambda^2}$

- Rolled shapes

  If $0.56\sqrt{E/F_y} < \lambda \leq 1.03\sqrt{E/F_y} \qquad Q_s = 1.415 - 0.74\frac{\lambda}{\sqrt{E/F_y}}$

  If $1.03\sqrt{E/F_y} < \lambda \qquad Q_s = 0.69\frac{E/F_y}{\lambda^2}$

- Other sections

  If $0.64\sqrt{k_c \cdot E/F_y}\,\lambda \leq 1.17\sqrt{k_c \cdot E/F_y} \qquad Q_s = 1.415 - 0.65\frac{\lambda}{\sqrt{k_c \cdot E/F_y}}$

If $1.17\sqrt{k_c \cdot E/F_y} < \lambda$ $\quad Q_s = 0.90 k_c \dfrac{E/F_y}{\lambda^2}$

Where λ is the element slenderness and

$k_c = \dfrac{4}{\sqrt{\lambda}}, 0.35 \leq k_c \leq 0.76$ for I sections

$k_c = 0.76$ for other sections

Factor Qa:

The calculation of factor Qa is an iterative process. Its procedure is the following:

1) An initial value of Q equal to Qs is taken.
2) With this value F_{cr} is calculated.
3) This F_{cr} value is taken to calculate $f (f = P_n/A_{eff})$
4) For elements with stiffened plates, the effective width b_e is calculated.
5) With b_e the effective area is calculated.
6) With the value of the effective area, Qa is calculated, and the process starts again.

$$Q_a = \frac{\text{effective}}{\text{gross area}}$$

- For a box section

  If $\lambda \geq 1.40\sqrt{\dfrac{E}{f}}$ $\quad b_e = 1.92 \cdot t\sqrt{\dfrac{E}{f}}\left[1 - \dfrac{0.38}{\lambda}\sqrt{\dfrac{E}{f}}\right]$

- For other sections

  If $\lambda \geq 1.49\sqrt{\dfrac{E}{f}}$ $\quad b_e = 1.92 \cdot t\sqrt{\dfrac{E}{f}}\left[1 - \dfrac{0.34}{\lambda}\sqrt{\dfrac{E}{f}}\right]$

If it is not within those limits, b_e= b

With the b_e values for each plate, the part that does not contribute [t·(b-b_e)] is subtracted from the area (where t is the plate thickness). Using this procedure, the effective area is calculated.

Finally, with Qs and Qa, Q is calculated, and F_{cr} is obtained.

Output results are written in the CivilFEM results file (.CRCF).

7.2.9. Compressive Strength for Flexural-Torsional Buckling

This type of check can be carried out for compact sections as well as for noncompact or slender sections. The steps for these three cases are as follows:

Nominal compressive strength, P_n:

$$P_n = A_g F_{cr}$$

c) for $\lambda_e\sqrt{Q} \leq 1.5$

$$F_{cr} = Q\left(0.658^{Q\lambda_e^2}\right)F_y$$

(b) for $\lambda_e\sqrt{Q} > 1.5$

$$F_{cr} = \left[\frac{0.877}{\lambda_e^2}\right]F_y$$

Where:

$$\lambda_e = \sqrt{\frac{F_y}{F_e}}$$

$$Q = Q_s Q_a$$

Factor Q for compact and noncompact sections is 1. Nevertheless, for slender sections, the Q factor has a particular procedure of calculation. Such procedure is equal to the one previously described.

The elastic stress for critical torsional buckling or flexural-torsional buckling F_e is calculated as the lowest root of the following third degree equation, in which the axis have been changed to adapt to the CivilFEM normal axis:

$$(F_e - F_{ex})(F_e - F_{ey})(F_e - F_{ez}) - F_e^2(F_e - F_{ez})\left(\frac{y_0}{r_0}\right)^2 - F_e^2(F_e - F_{ey})\left(\frac{z_0}{r_0}\right)^2 = 0$$

Where:

K_x	Effective length factor for torsional buckling.
G	Shear modulus (MPa).

C_w Warping constant (mm^6).

J Torsional constant (mm^4).

I_y, I_z Moments of inertia about the principal axis (mm^4).

X_0, y_o Coordinates of shear center with respect to the center of gravity (mm).

$$\bar{r}_0{}^2 = y_0^2 + Z_0^2 + \frac{I_y + I_z}{A}$$

$$H = 1 - \left(\frac{y_0^2 + Z_0^2}{\bar{r}_0{}^2}\right)$$

$$F_{ey} = \frac{\pi^2 \cdot E}{K_y \cdot I / r_y}$$

$$F_{ez} = \frac{\pi^2 \cdot E}{K_z \cdot I / r_z}$$

$$F_{ex} = \left(\frac{\pi^2 \cdot E \cdot C_w}{(Kx \cdot I)} + G \cdot J\right) \cdot \frac{1}{A \cdot \bar{r}_0{}^2}$$

where:

A Cross-sectional area of member.

l Unbraced length.

K_y, K_z Effective length factor, in the z and y directions.

r_y, r_z Radii of gyration about the principal axes.

$\bar{r}_0{}^2$ Polar radius of gyration about the shear center.

In this formula, CivilFEM principal axes are used. If the CivilFEM axes are the principal axes ±5º sexagesimal degrees, K_y and K_z are calculated with respect to the Y and Z-axes of CivilFEM. If this is not the case (angular shapes, for example) axes U and V will be used as principal axes, with U as the axis with higher inertia.

The torsional inertia (Ixx in CivilFEM, J in AISC 13TH Edition) is calculated for CivilFEM sections, but not for captured sections. Therefore the user will have to introduce this parameter in the mechanical properties of CivilFEM.

Output results are written in the CivilFEM results file (.CRCF).

7.2.10. Compressive Strength for Flexure

Chapter F is only applicable to members subject to simple bending about one principal axis.

The design flexural strength, $\phi_b M_n$, and the allowable flexural strength, M_n/Ω_b , shall be determined as follows:

For all provisions: ϕ_b = 0.90 (LRFD) Ω_b= 1.67 (ASD)

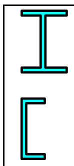
Where M_n is the lowest value of four checks according to sections F2 through F12:

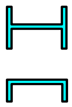
a) Yielding
b) Lateral-torsional buckling
c) Flange local buckling
d) Web local buckling

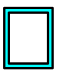
The value of the nominal flexural strength with the following considerations:

- For compact sections, if Lb < Lp only yielding of steel will be checked.
- For T sections, and other compact sections, only yielding and torsional buckling will be checked.
- The case of lateral-torsional buckling does not apply to sections loaded on the minor axis of inertia nor box or square sections.
- The case of lateral-torsional buckling only applies for sections with double symmetry, channel and T sections. Therefore the rest of sections will be checked for torsion plus combined loads and will not be checked under flexure.
- For slender sections, the code contemplates the following cases:

Shape	**Limit State**	**Mr**	**Fcr**	**λ**	**λp**	**λr**
I, C loaded in the axis of higher inertia.	LTB	$F_L S_z$	$\frac{C_b X_1 \sqrt{2}}{\lambda}\sqrt{1+\frac{X_1^2 X_2}{2\lambda^2}}$	$\frac{L_b}{r_z}$	$1.76\sqrt{\frac{E}{F_{yf}}}$	$\frac{X_1}{F_L}\sqrt{1+\sqrt{1+X_2 F_L^2}}$
	FLB	$F_L S_z$	$\frac{0.69E}{\lambda^2}$ rolled $\frac{0.90Ek_c}{\lambda^2}$ welded	$\frac{b}{t}$	Class B4.1	Class B4.1

	WLB	$R_e F_{yf} S_z$	N.A.	$^h/_{t_w}$	Class B4.1	Class B4.1

Shape	Limit State	Mr	Fcr	λ	λp	λr
I, C loaded in the axis of lower inertia. 	LTB	N.A.	N.A.	N.A.	N.A.	N.A.
	FLB	$F_y S_y$	$\frac{0.69E}{\lambda^2}$	$\frac{b}{t}$	Class B4.1	Class B4.1
	WLB	N.A.	N.A.	N.A.	N.A.	N.A.

Shape	Limit State	Mr	Fcr	λ	λp	λr
Box 	LTB	$F_{yt} S_{eff}$	$\frac{2EC_b\sqrt{JA}}{\lambda S_z}$	$\frac{L_b}{r_z}$	$\frac{0.13E\sqrt{JA}}{M_p}$	$\frac{2E\sqrt{JA}}{M_r}$
	FLB	$F_{yt} S_{eff}$	$\frac{S_{eff}}{S} F_y$	$\frac{b}{t}$	Class B4.1	Class B4.1
	WLB	$R_e F_{yf} S_z$	N.A.	$^h/_{t_w}$	Class B4.1	Class B4.1

Shape	Limit State	Mr	Fcr	λ	λp	λr	Notes

Pipe	LTB	NA	NA	NA	NA	NA	Limited by Class B4.1
	FLB	Slender: $F_{cr}S$ Non-compact: $M_n = \left(\frac{0.021E}{D/t} + F_y\right) \cdot S$	$\frac{0.33E}{D/t}$	D/t	Class B4.1	Class B4.1	
	WLB	NA	NA	NA	NA	NA	

Shape	Limit State	Mr	Fcr	λ	λp	λr
T, loaded in web plane	LTB	$M_n = M_{cr} = \frac{\pi\sqrt{EI_zGJ}}{L_b}\left[B + \sqrt{1+B^2}\right]$	N.A.	N.A.	N.A.	N.A.
	FLB	N.A.	N.A.	N.A.	N.A.	N.A.
	WLB	N.A.	N.A.	N.A.	N.A.	N.A.

Where:

$$X_1 = \frac{\pi}{S_z}\sqrt{\frac{EGJA}{2}}$$

$$X_2 = 4\frac{C_w}{I_z}\left(\frac{S_z}{GJ}\right)^2$$

$$B = \underline{+}2.3\frac{d}{L_b}\sqrt{\frac{I_z}{J}}$$

(positive sign if the stem is under tension, negative if it is under compression)

In T sections: $M_n \leq 1.6M_y$ stem in tension; $M_n \leq 1.0M_y$ stem in compression.

For slender webs the nominal flexural strength M_n is the minimum of the following checks:

- tension-flange yield
- compression flange buckling

The first check uses the following formula:

$$M_n = S_{xc} F_y$$

where:

S_{xc} Section modulus referred to tension flange.

F_y Yield strength of tension flange.

The second check uses the following formula:

$$M_n = S_{xc} R_{PG} F_{cr}$$

where:

$$R_{PG} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.70\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

The critical stress depends upon different slenderness parameters such as λ, λ_p, λ_r and C_{pg} in the following way:

For $\lambda \leq \lambda_p$ $\quad F_{cr} = F_{yf}$

For $\lambda_p \leq \lambda \leq \lambda_r$ $\quad F_{cr} = C_b \cdot F_{yf} \cdot \left[1 - 0.3 \cdot \left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right] \leq F_{yf}$

For $\lambda > \lambda_r$ $\quad F_{cr} = \frac{C_{pg}}{\lambda^2}$

The slenderness values have to be calculated for the following limit states:

- Lateral torsional buckling

$$\lambda = \frac{L_b}{r_T}$$

$$\lambda_p = 1.1 \cdot \sqrt{\frac{E}{F_{yf}}}$$

$$\lambda_r = \pi \cdot \sqrt{\frac{E}{0.7F_{yf}}}$$

$$C_{pg} = 1970000 \cdot C_b \text{ (International System units)}$$

r_T is the radius of gyration of compression flange plus one third of the compression portion of the web (mm).

By default, the program takes a conservative value of $C_b = 1$.

- Flange local buckling

$$\lambda = \frac{b_r}{2t_f}$$

$$\lambda_p = 0.38 \cdot \sqrt{\frac{E}{F_{yf}}}$$

$$\lambda_r = 1.35 \cdot \sqrt{\frac{E}{F_{yf}/k_c}}$$

$$C_{pg} = 180650 \cdot k_c \text{ (IS units)}$$

where:

$$C_b = 1$$

$$k_c = 4/\sqrt{h/t_w}$$

and

$$0.35 \leq k_c \leq 0.76$$

Between these two slenderness, the program will choose values the value that produces a lower critical stress.

Output results are written in the CivilFEM results file (.CRCF).

7.2.11. Checking of Members for Shear (Chapter G)

The design shear strength, $\phi_v V_n$, and the allowable shear strength, V_n/Ω_v, shall be determined as follows:

For all provisions: ϕ_v = 0.90 (LRFD) Ω_v= 1.67 (ASD)

According to the limit states of shear yielding and shear buckling, the nominal shear strength, V_n , of unstiffened webs is:

$$V_n = 0.6 F_y A_w C_v$$

For webs of rolled I-shaped members with $h/t_w \leq 2.24\sqrt{E/F_y}$:

$$\phi_v = 1.00 \text{ (LRFD)} \qquad \Omega_v = 1.50 \text{ (ASD)}$$

$$C_v = 1.0 \text{ (web shear coefficient)}$$

For webs of all other doubly symmetric shapes and singly symmetric shapes and channels C_v is determined as follows:

1. For $h/t_w \leq 1.10\sqrt{k_v E/F_y}$

$$C_v = 1.0$$

2. For $1.10\sqrt{k_v E/F_y} < h/t_w \leq 1.37\sqrt{k_v E/F_y}$

$$C_v = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w}$$

3. For $h/t_w \leq 1.37\sqrt{k_v E/F_y}$

$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y}$$

Where A_w is the overall depth times the web thickness.

It is assumed that there are no stiffeners; therefore, the web plate buckling coefficient K_v will be calculated as a constant equal to 5.0.

Output results are written in the CivilFEM results file (.CRCF).

7.2.12. Checking of Members for Combined Flexure and Axial Tension / Compression (Chapter H)

For this check, it is first necessary to determine the value of Mn. This value comes into play in the checking of formulas. The value of M_n, will be calculated in the same way as members subjected to flexure; thus, the nominal flexure strength (M_n) is the minimum of four checks:

1. Yielding
2. Lateral-torsional buckling
3. Flange local buckling
4. Web local buckling

In the case of having bending plus tension or bending plus compression, the interaction between flexure and axial force is limited by the following equations:

(a) For $\frac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rz}}{M_{cz}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad \text{(H1-1a)}$$

(b) For $\frac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rz}}{M_{cz}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad \text{(H1-1b)}$$

If the axial force is tension:

P_r Required tensile strength (N).

P_c Available tensile strength (N):

$\phi_t P_n$(LRFD) or P_n/Ω_t (ASD)

M_r Required flexural strength (N·mm).

M_c Available flexural strength (N·mm):

Design: $\phi_b M_n$ (LRFD) or

Allowable: M_n/Ω_b (ASD)

y Strong axis bending.

z Weak axis bending.

ϕ_t Resistance factor for tension (Sect.D2)

ϕ_b Resistance factor for flexure = 0.90

Ω_t Safety factor for tension (Sect D2)

Ω_b Safety factor for flexure = 1.67

If the axial force is compression:

P_r Required compressive strength (N).

P_c Available compressive strength (N):

Design: $\phi_c P_n$ (LRFD) or

Allowable: P_n/Ω_c (ASD)

M_r Required flexural strength (N·mm).

M_c Available flexural strength (N·mm):

Design: $\phi_b M_n$ (LRFD) or

Allowable: M_n/Ω_b (ASD)

Y Strong axis of bending.

Z Weak axis of bending.

Φ_c Resistance factor for compression =0.90

Φ_b Resistance factor for flexure = 0.90

Ω_c Safety factor for compression =1.67

Ω_b Safety factor for flexure = 1.67

The following checks are carried out by CivilFEM:

- Axial force and flexural buckling
- Bending moment Z direction
- Bending moment Y direction

If one of these checks do not meet the code requirements, it will not be possible to check the member under flexure plus tension / compression.

Output results are written in the CivilFEM results file (.CRCF).

7.2.13. Checking of Members for Combined Torsion, Flexure, Shear and/or Axial Force (Chapter H)

The design torsional strength, $\phi_T T_n$, and the allowable torsional strength, T_n/Ω_T, shall be the lowest value obtained according to the limit states of yielding under normal stress, shear yielding under shear stress or buckling, determined as follows:

$$\phi_T = 0.90 \text{ (LRFD)} \qquad \Omega_T = 1.67 \text{ (ASD)}$$

- For the limit state of yielding, under normal stress:

$$F_n = F_y$$

- For the limit state of yielding, under shear stress:

$$F_n = 0.6F_y$$

- For the limit state of buckling:

$$F_n = F_{cr}$$

- Where F_{cr} is calculated

Output results are written in the CivilFEM results file (.CRCF).

7.3. Steel Structures According to British Standard 5950

The British Standard BS 5950:2000 supersedes BS 5950:1985, which has been withdrawn. BS 5950:2000 is the British Standard for the structural use of steelwork in building, widely in use in regions which experience or have experienced British influence. The purpose of this manual is to define the reach and method of implementing this method within CivilFEM.

The types of analyses considered in this standard have been developed according to the ultimate limit state in agreement with the simple and continuous design methods. Semi-continuous design and experimental verification fall beyond the scope of this specification.

The applicable cross sections for checking procedures include rolled or welded sections subjected to axial forces, shear, and bending in 2D and 3D as well as solid sections subjected to the aforementioned forces.

The calculations made by CivilFEM correspond to the design guidelines of British Standard 5950:2000 Structural use of steelwork in building: Part 1. Code of practice for design – Rolled and welded sections.

7.3.1 Checking Types

With CivilFEM it is possible to accomplish the following checking and analysis types:

Checking of sections subjected to:

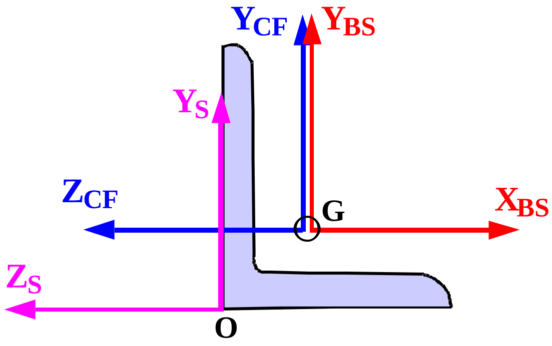
- Bending	British Standard 5950 (2000) apt. 4.2
- Bending and Shear	British Standard 5950 (2000) apt. 4.2
- Lateral Torsional Buckling	British Standard 5950 (2000) apt. 4.3
- Axial Tension	British Standard 5950 (2000) apt. 4.6
- Axial Compression	British Standard 5950 (2000) apt. 4.7
- Axial Tension with Moments	British Standard 5950 (2000) apt. 4.8.2
- Axial Compression with Moments	British Standard 5950 (2000) apt. 4.8.3

7.3.2 Reference Axis

With performing checks according to BS 5950:2000, CivilFEM includes three different coordinate reference systems. All of these systems are right-handed:

1. CivilFEM Reference Axes (X_{CF}, Y_{CF}, Z_{CF}).

2. Cross-Section Reference Axes (X_S, Y_S, Z_S).
3. BS 5950:2000 Reference Axes (Code Axes), (X_{BS}, Y_{BS}, Z_{BS}).



For the BS 5950:2000 axes axes system:

- The origin matches to the CivilFEM axes origin.
- X_{EC3} axis coincides with CivilFEM X-axis.
- Y_{EC3} axis is the relevant axis for bending and its orientation is defined by the user (in steel check process).
- Z_{EC3} axis is perpendicular to the plane defined by X and Y axis, to ensure a right-handed system.

To define this reference system, the user must indicate which direction of the CivilFEM axis (-Z, -Y, +Z or +Y) coincides with the relevant axis for positive bending. The user may define this reference system when checking according to this code. In conclusion, the code reference system coincides with that of CivilFEM, but it is rotated a multiple of 90 degrees, as shown in table below.

7.3.3 Material Properties

BS 5950:2000 uses the following material properties in its checks:

Description	Properties, symbol
Yield strength	Y_S
Tensile strength	U_S
Design strength	p_y (table 9 of BS 5950-1:2000 and table 3 of EN10113-2:1993)

Material strength factor	$\gamma_m = 1$
Modulus of elasticity	E = 205 kN/mm^2
Shear Modulus	$G = E/[2 \cdot (1+\nu)]$
Poisson's ratio	$\nu = 0.3$
Coefficient of linear thermal expansion	$\alpha = 12 \cdot 10^{-6} \circ C^{-1}$
Effective net area coefficient	K_e (BS 5950-1:2000 – Section 3.4.3)
Constant ϵ	$\varepsilon = \left(\frac{275}{p_y}\right)^{1/2}$

The code uses other safety factors (γ_I, γ_p) which depend on the type of loads and which must be used when performing load combinations.

7.3.4 Section Data

BS 5950:2000 considers the following data set for the cross section:

- Gross section data
- Net section data
- Effective section data
- Data concerning the section and element class.

Gross section data correspond to the nominal properties of the cross-section.

From the net section, the net area and the effective net area are considered. The net area is calculated by subtracting the area of holes for screws, rivets and other holes from the gross section area, taking into account the deduction for fastener holes according to section 3.4.4 of the code (see figures 3 and 4 of the code). The area of holes is introduced within the structural steel code properties.

The effective net area is obtained from the net area, multiplying it by a coefficient K_e which depends on type of steel used. This coefficient is calculated by the program and stored together with the material properties.

Effective section data are obtained in the checking process according to the effective width method (Sect. 3.6 of BS 5950:2000). This method discounts the non-resistance zones for local buckling in class 4 cross-sections. For cross-sections of a lower class, this method does not reduce the section because of local buckling.

As an alternative method for slender cross sections calculation, a reduced design strength (ρ_{yr}) may be calculated at which the cross section would be class 3 (section 3.6.5 of the code).

Section and element class data are obtained using tables 11 and 12 of BS 5950:2000 (section 3.5.2). The classification of each element is based on its width to thickness ratio and according to section type (hot-rolled or welded), element type (web or flange) and position (internal or external element). CivilFEM assumes the section class as the largest from all the elements (least favorable).

The initial required data for the BS 5950:2000 module includes the gross section data in user units and the CivilFEM axis or section axis (see the section corresponding to Reference axis in beam sections in Chapter 5 of this Manual). The data are then properly converted from the section's axis into the BS 5950:2000 axis and the results are given in the code axis. The program calculates the effective and net section data and the class data and stores them into CivilFEM's results file in user units and in the CivilFEM coordinate system.

The section data used in BS 5950:2000 is shown in the following tables:

I.- Section Dimensions

Description	Data
Input data:	
1.- Height	H
2.- Web thickness	Tw
3.- Flanges thickness	Tf
4.- Flanges width	B
5.- Distance between flanges	Hi
6.- Radius of fillet (Rolled shapes)	r_1
7.- Toe radius (Rolled shapes)	r_2
8.- Weld throat thickness (Welded shapes)	a
9.- Web free depth	d
Output data	(Nothing)

I.- Gross Section Resistant Properties

Description	Data
Input:	
1.- Area	A
2.- Moments of inertia for torsion	It
3.- Moments of inertia for bending	Ixx, Iyy
4.- Product of inertia	Ixy
5.- Elastic resistant modulus	Wx, Wy
6.- Plastic resistant modulus	Wpx, Wpy
7.- Radius of gyration	ix, iy

8.- Coordinates of the center of gravity	Ymn, Ymx, Xmn, Xmx
9.- Distance between GC and SC in X and in Y	Xm, Ym
10.- Distance CG to shear center along Y axis	Ys
11.- Distance CG to shear center along X axis	Xs
12.- Warping Constant	Iw
13.- Shear resistant areas	Yws, Xws
14.- Torsional resistant modulus	Zwt
Output Data:	
1.- Shear area for major axis (X)	Avx
2.- Sv parameter for major axis (X)	Svx
3.- Shear area for minor axis (Y)	Avy
4.- Sv parameter for minor axis (X)	Svy
5.- Critical shear strength of web panel for major axis	Vcrx
6.- Critical shear strength of web panel for minor axis	Vcry
7.- Y coordinate of plastic center	Yp
8.- X coordinate of plastic center	Xp
* The section properties listed here in are related to the BS coordinate system (X_{BS}, Y_{BS}, Z_{BS})	

III.- Net section data

Description	Data
Input data: AHOLES*	
Output data:	
1.- Net area	Ant
2.- Effective net area	Aneff
A_{nt}= A - AHOLES $A_{neff}= K_e \cdot A_{nt}$ with $A_{neff} \leq$ A (Gross area)	

* Deduction for holes are introduced as a code property

IV.- Effective section data

Description	Data
Input data: None	

Description	Data
Output data:	
1.- Effective Area	Aeff
2.- Moments of inertia for torsion	It
3.- Moments of inertia for Y bending	Iyyeff
4.- Moments of inertia for X bending	Ixxeff
5.- Elastic resistant Y modulus	Wyeff
6.- Elastic resistant X modulus	Wxeff
7.- Plastic resistant Y modulus	Wpyeff
8.- Plastic resistant X modulus	Wpxeff
9.- Section class	Cls
10.- Web class for shear buckling	ClsAlm

V.- Section element data

Description	Data
Input data:	
1.- Number of elements	N
2.- Element type: flange or web (for the relevant axis of bending)	Pltype
3.- Union condition at the ends: free or fixed	Cp1, Cp2
4.- Element thickness	t
5.- Coordinates of the extreme points of the element (using Section axes)	Yp1, Yp2, Zp1, Zp2
Output data:	
6.- Element class	Cl
7.- Reduction factor (for class 4 section- alternative method)	f_r
8.- Web Class	Webclass

7.3.5 Structural steel code properties

For BS 5950:2000 check, besides the section properties, more data are needed for buckling checks. These data are shown in the following table.

Description	Data	Article
Input data:		
1.- Unbraced length of member	L	
2.- Compression buckling factor for X axis	Kcx	Section 4.7.3
3.- Compression buckling factor for Y axis	Kcy	Section 4.7.3
4.- Lateral torsional buckling factor for X axis	KLtx	Section 4.3.5
5.- Lateral torsional buckling factor for Y axis	Klty	Section 4.3.5
6.- Factors by which multiply "L" to found the length between restrictions in planes xz and yz, respectively	Cfbuckx, Cfbucky	
7.- Robertson Constant	CteRob	Appendix C.2
8.- Equivalent uniform moment factor for major axis flexural bending	mx	Section 4.8.3
9.- Equivalent uniform moment factor for minor axis flexural bending	my	Section 4.8.3
10.- Equivalent uniform moment factor for lateral torsional buckling	mlt	Section 4.3.6.6
11.- Depth of the compression flanges lip	DL	Section 4.3.6.7
12.- Intermediate stiffeners depth	d/a	Section 4.4.5
11.- CivilFEM Axis which is the X axis in BS 5950:2000	CHCKAXIS	
0: Not defined		
1: -Z CivilFEM		
2: +Y CivilFEM		
3: +Z CivilFEM		
4: -Y CivilFEM		

7.3.6 Checking Process

The steps for the checking process are the following ones:

4. Read the checking type requested by the user.
5. Read the CivilFEM axis to be considered as the principal axis for bending, so that it coincides with the X-axis of BS5950. In CivilFEM, by default, the principal axis for bending that coincides with the +X axis of BS 5950:2000 is the –Z-axis.
6. The following operations are carried out for each selected element:
   a. Obtain material properties corresponding to the element, stored in CivilFEM database, and calculate the rest of the properties needed for checking:
      Properties obtained from CivilFEM database:
      Elasticity modulus E
      Poisson's ratio ⍰

Yield strength	Y_s
Ultimate strength	U_s
Design strength	ρ_y
Ke parameter	K_e
Safety factor	γ_M

Calculated properties:

Shear Modulus:

$$G = \frac{E}{2^*(1+v)}$$

Epsilon, material coefficient:

$$\varepsilon = \sqrt{275/\rho_y} \quad (\rho_y \text{ in } N/mm^2)$$

b. Obtain the cross-section data corresponding to the element.

c. Determination of section class.

d. There are two calculation procedures for slender cross sections (class 4) that may be may chosen by the user:

1. Initialize reduction factors of section plates and the effective cross section properties calculation.

2. Calculate a reduced design strength that should be used in place of the nominal design strength (section 3.6.5 of the code).

e. Obtain forces acting on the section (FX, F_{vx}, F_{yv}, M_x, M_y).

f. Specific section checking according to the type of external load.

g. Writing of results, which will be stored in the results file .CRCF

7.3.7 Section Class and Reduction Factor Calculation.

According to BS 5950:2000, the sections are made up of different elements, which can be classified according to:

a) The way they work:

Webs and flanges in the X and Y axes, depending on which is the principal bending axis.

b) Their relation to the other elements:

Internal or outside elements

The sections of the shapes included in the program libraries contain this information for each element. CivilFEM classifies elements as either flange or web according to each axis and gives the element union condition at each end. The ends can be classified as fixed or free (i.e. an end is called fixed if it is in contact with another plate and free if it is not).

For checking the structure for safety, BS 5950:2000 classifies cross sections into four different classes to determine whether local buckling influences their capacity (section 3.5.2):

Class 1	Plastic cross sections are those in which a plastic hinge can be developed with sufficient rotation capacity to allow redistribution of moments within the structure.
Class 2	Compact cross sections are those in which the full plastic moment capacity can be developed but local buckling may prevent development of a plastic hinge with sufficient rotation capacity to permit plastic design.
Class 3	Semi-compact sections are those in which the stress at the extreme fibers can reach the design strength but local buckling may prevent the development of the full plastic moment.
Class 4	Slender sections are those which contain slender elements subject to compression due to moment or axial load. Local buckling may prevent the stress in a slender section from reaching the design strength.

The cross-section class is the highest (least favorable) class of its elements: flanges and webs. The class of each element is first determined according to the limits of tables 11 and 12 of BS 5950:2000. According to these tables, the class of an element depends on:

1. The width to thickness ratio. The dimensions of the elements (b, d, t, T) should be taken as shown in Figure 5 of the code.

$$Rd = Width / Thickness$$

2. The limits of this ratio, according to the type of section, element (flange or web) and position (internal or outside). Elements that do not meet the limits for class 3 semi-compact are classified as class 4. The limits are the following (refer to figure 5 of the code for dimensions):

- **Sections other than circular hollow sections (CHS) and rectangular hollow section (RHS)**:

Compression element	**Class 1**	**Class 2**	**Class 3**
Outstand rolled flange Angle, compression due to bending	$9^{*}\varepsilon$	$10^{*}\varepsilon$	$15^{*}\varepsilon$
Angle, axial compression	0	0	$15^{*}\varepsilon$ y $\frac{b+d}{t} \leq 24 \cdot \varepsilon$
Outstand welded flange	$8^{*}\varepsilon$	$9^{*}\varepsilon$	$13^{*}\varepsilon$
Internal flange, compression due to bending	$28^{*}\varepsilon$	$32^{*}\varepsilon$	$40^{*}\varepsilon$
Internal flange, axial compression	0	0	$40^{*}\varepsilon$
Web of an I, H or box section, compression due to bending	$80^{*}\varepsilon/(1+r_1)$ but $\geq 40^{*}\varepsilon$	For $r_1 \leq 0$ $100^{*}\varepsilon/(1+r_1) \geq 40^{*}\varepsilon$ For $r_1 > 0$ $100^{*}\varepsilon/(1+1.5r_1)$ but $\geq 40^{*}\varepsilon$	$120^{*}\varepsilon/(1+2r_2)$ but $\geq 40^{*}\varepsilon$
Web of an I, H or box section, axial compression	0	0	$120^{*}\varepsilon/(1+2r_2)$ but $\geq 40^{*}\varepsilon$
Web of a channel	$40^{*}\varepsilon$	$40^{*}\varepsilon$	$40^{*}\varepsilon$
Stem of a T section, rolled or cut from a rolled I or H section	$8^{*}\varepsilon$	$9^{*}\varepsilon$	$18^{*}\varepsilon$

- **Circular hollow sections (CHS)**:

Circular hollow sections are classified as having only one element and the width to thickness ratio (R_d) is determined as follows:

$R_d = D/t$ D = Diameter.

t = Wall thickness.

	Class 1	Class 2	Class 3
Compression due to bending	$40^*\varepsilon^2$	$50^*\varepsilon^2$	$140^*\varepsilon^2$
Axial compression	0	0	$80^*\varepsilon^2$

- **Rectangular hollow sections hot finished (HF RHS)**:

Compression element	**Class 1**	**Class 2**	**Class 3**
Flange, compression due to bending	$28^*\varepsilon^2$ $\text{but} \leq 80^*\varepsilon - d/t$	$32^*\varepsilon$ $\text{but} \leq 62^*\varepsilon - 0.5d/t$	$40^*\varepsilon$
Flange, axial compression	0	0	$40^*\varepsilon$
Web, compression due to bending	$64^*\varepsilon/(1+0.6r_1)$ $\text{but} \geq 40^*\varepsilon$	$80^*\varepsilon/(1+r_1)$ $\text{but} \geq 40^*\varepsilon$	$120^*\varepsilon/(1+2r_2)$ $\text{but} \geq 40^*\varepsilon$
Web, axial compression	0	0	$120^*\varepsilon/(1+2r_2)$ $\text{but} \geq 40^*\varepsilon$

- **Rectangular hollow sections cold formed (CF RHS)**:

Compression element	**Class 1**	**Class 2**	**Class 3**
Flange, compression due to bending	$26^*\varepsilon$ $\text{but} \leq 72^*\varepsilon - d/t$	$28^*\varepsilon$ $\text{but} \leq 54^*\varepsilon - 0.5d/t$	$35^*\varepsilon$
Flange, axial compression	0	0	$35^*\varepsilon$
Web, compression due to	$56^*\varepsilon/(1+0.6r_1)$	$70^*\varepsilon/(1+r_1)$	$105^*\varepsilon/(1+r_2)$

bending	but $\geq 35^{*}\varepsilon$	but $\geq 35^{*}\varepsilon$	but $\geq 35^{*}\varepsilon$
Web, axial compression	0	0	$105^{*}\varepsilon/(1+2r_2)$ but $\geq 35^{*}\varepsilon$

* The dimensions b and t are defined in figure 5 of the code.

Notes:

1. The classification of the elements according to the way they work (webs or flanges) is included in the program section library. In other cases the user can specify it or, by default, the program will automatically determine it as a function of the angle α with respect to the principal axis of bending, following the below criterion:

   For⍰α⍰⍰⍰Web

   For⍰α⍰⍰⍰Flange

2. Apart from the type of section, type and position of the element, the limits of the width to thickness ratio also depend on the material parameter ⍰ and on the parameters r_1⍰and⍰r_2, which translates into the following relationships⍰

   a) For I or H-sections with equal flanges:

   $$r_1 = \frac{F_c}{d \cdot t \cdot \rho_{yw}} \text{ with } -1 < r_1 \leq 1$$

   $$r_2 = \frac{F_c}{A_g \cdot \rho_{yw}}$$

   b) For I or H-sections with unequal flanges:
   The program deals with this type of sections as generic sections for which the values of r_1 and r_2 are the following:

   $r_1 = 1$
   $r_2 = 1$

   c) Rectangular hollow sections or welded box sections with equal flanges:

   $$r_1 = \frac{F_c}{2 \cdot d \cdot t \cdot \rho_{yw}} \text{ with } -1 < r_1 \leq 1$$

   $$r_2 = \frac{F_c}{A_g \cdot \rho_{yw}}$$

Where:

A_g	Gross section area.
B_c	Width of the compression flange.
B_t	Width of the tension flange.
d	Web depth.
F_c	Axial compression (negative for tension).
f_1	Maximum compressive stress in the web (figure 7 of the code).
f_2	Minimum compressive stress in the web (figure 7 of the code).
ρ_{yf}	Design strength of the flanges.
ρ_{wy}	Design strength of the web (but $\rho_{yw} \leq \rho_{yf}$).
T_c	Thickness of the compression flange.
T_f	Thickness of the tension flange.
t	Web thickness.

3. The webs are also classified for shear buckling resistance according to the following criteria:
   a. For rolled sections with Rd $> 70 * \varepsilon$
   b. For welded sections with Rd $> 62 * \varepsilon$
   In these cases, the shear buckling resistance should be checked according to the section 4.4.5 of the BS 5950:2000.

4. Class 3 semi-compact sections are designed using the effective plastic modulus S_{eff} according to section 3.5.6 and followings of BS 5950:2000.

7.3.7.1 Procedures for Slender Sections (Class 4)

BS 5950:2000 accepts two different procedures for designing slender cross sections.

a) Effective section properties calculation (Sections 3.6.2, 3.6.3, 3.6.4)

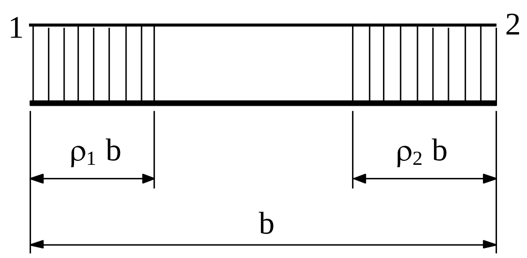
The local buckling resistance of class 4 slender cross sections is performed by adopting effective section properties. The width of the compression elements are reduced in such way that the effective width of a class 4 section will be the same as the maximum width for a class 3 section.

For outstand elements, the reduction is applied to its free end, and for internal elements, the reduction is applied to the non-effective zone, comprised of the central portion of the element with two equal portions of effective zone at the ends.

For each section element, the program calculates two reduction factors ρ_1 and ρ_2 to determine the effective width at each element end. These factors relate the width of the effective zone at each end with the width of the plate.

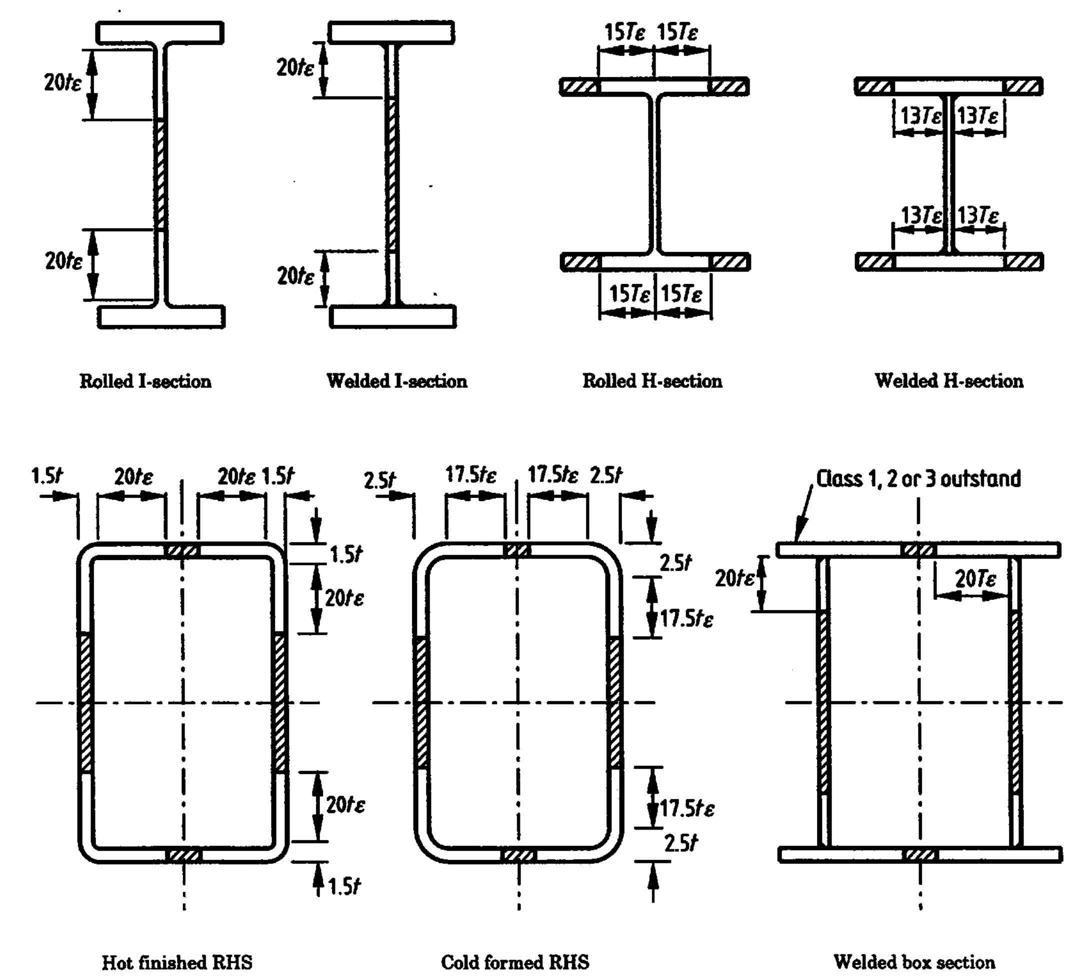
Effective_width_end1 = plate_$\text{width}^*\rho_2$

Effective_ width _end 2 = plate_$\text{width}^*\rho_2$



Effective area calculation (A_{eff})

The effective area is determined from the effective cross section as shown in Figure 8a of the code (section 3.6.2.2).

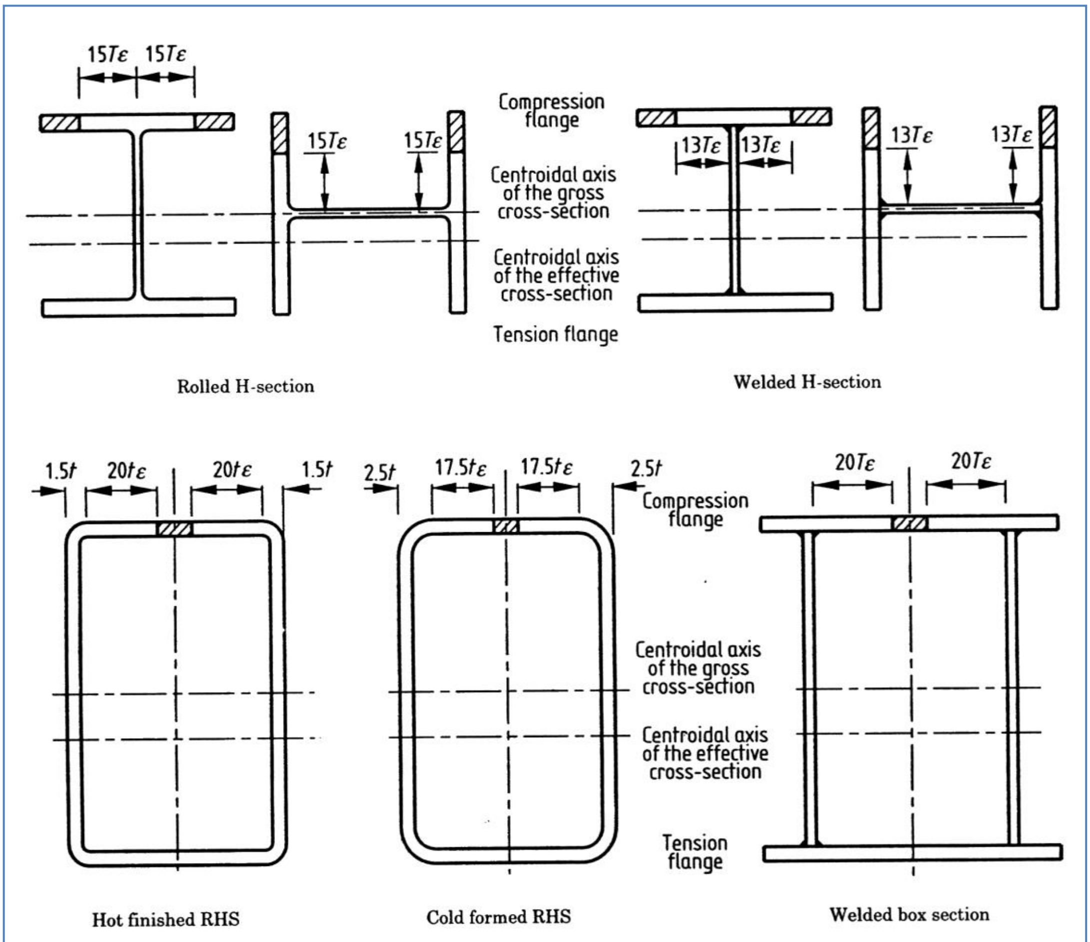


a) Effective cross-sections under pure compression, for determining A_{eff}

Figure 8 — Doubly symmetric slender cross-sections

Effective modulus calculation (Z_{eff})

The effective modulus is determined from the effective cross section as shown in Figure 8b of the code (section 3.6.2.3).



b) Effective cross-sections, webs fully effective under pure moment, for determining Z_{eff}

Figure 8 — Doubly symmetric slender cross-sections (*continued*)

For cross sections with slender webs, the effective modulus is determined from the effective cross section as shown in Figure 9 of the code (section 3.6.2.4).

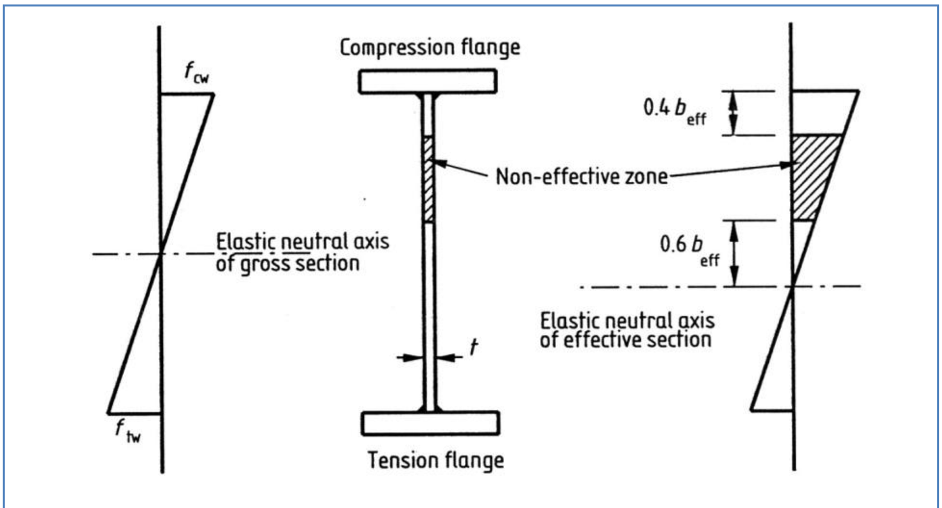


Figure 9 — Effective width for class 4 slender web under pure bending

Circular Hollow Sections

For circular hollow sections, the effective modulus and the effective area is determined according to the section 3.6.6 of BS 5950:2000.

b) Alternative Method (section 3.6.5)

As an alternative to the method previously described, a reduced design strength ρ_{yr} is calculated as if the cross section were a class 3 semi-compact. This reduced design strength is used in place of ρ_y in the checks of section capacity and member buckling resistance. The reduction factor f_r is calculated for each section 4 element according to the below expression:

$$f_r = \left(\frac{\beta_3}{\beta}\right)^2$$

$$\rho_{yr} = f_r \cdot \rho_y$$

Where:

β_3	Limiting value for a class 3 section according to the tables 11 and 12 of the code.
β	Width to thickness ratio for each element.

7.3.8 Checking of Bending Moment and Shear Force (BS Article 4.2)

1. Forces and moments selection
   The forces and moments considered for this checking type are:

F_v = FZ or FY	Design value of the shear force perpendicular to the relevant axis of bending.
M_x = MY or MZ	Design value of the bending moment along the relevant axis of bending.

2. Class determination and calculation either of the effective section properties or the design strength reduction factor for slender sections (depending on the selected method).
3. Criteria calculation

In members subjected to bending moment and shear force, three conditions should be checked:

3.1. Shear checking (Article 4.2.3 of BS 5950:2000)

The first condition to be checked is the shear criteria at each section:

$$F_v \leq P_v \quad \rightarrow \quad Crt\_V = \frac{F_v}{P_v} \leq 1$$

Where:

P_v	Design value of the shear capacity: $P_v = 0.6 \cdot \rho_y \cdot A_v$
ρ_y	Design strength of the material.
A_v	Shear area.

Shear Area Calculation (A_v)

According to section 4.2.3 the shear area is calculated as follows:

Shape	**Shear Area**
Rolled I, H and channel sections, load parallel to web.	$t \cdot D$
Welded I sections, load parallel to web.	$t \cdot d$
Solid bars and plates.	$0 \cdot 9 \cdot A$
Rectangular hollow sections, load parallel to webs.	$\left(\frac{D}{D+B}\right) A$
Welded box sections.	$2 \cdot t \cdot d$
Circular hollow sections.	$0.6 \cdot A$
Any other case.	$0.9A_0$

where:

t	Total web thickness.

B	Breadth.
D	Overall depth.
d	Depth of the web.
A	Area of the section.
A_0	Area of the rectilinear element of the section which has the largest dimension in the direction parallel to the load:

$$\sum\nolimits_{i=\text{web elements}} \text{Breadth}_i \cdot \text{thickness}_i$$

In the case of biaxial bending, it is necessary to consider both shear areas, perpendicular to both the Standard's X- and Y-axis.

3.2. Shear buckling resistance of thin webs (Article 4.4.5)

The shear buckling resistance should be checked if the ratio d/t of the web exceeds 70·ε for a rolled section or 62·ε for welded sections. It should satisfy the following criterion:

$$\text{Crt\_PV} = \frac{F_v}{V_w} \leq 1$$

$$V_w = \sum q_w \cdot d \cdot t$$

Where:

V_w	Shear buckling resistance (summation extended to all section webs).
q_w	Critical shear strength.
d	Depth of the web.
t	Thickness of the web.

The critical shear strength is obtained following the Appendix H.1 of the code where $q_w = \text{Fn}(\rho_y, d/t, d/a)$and a is the distance between stiffeners. The ratio d/a may be introduced by the user. By default, d/a = 1.

If the web of the section is not slender (d/t < 70·ε for rolled sections and d/t < 62·ε for welded sections):

$$Crt\_PV = 0$$

3.3. Bending moment check

Besides the shear checking, the following condition at each section is checked (Article 4.2.5 of BS 5950:2000):

$$M_x \leq M_c \quad \rightarrow \quad Crt\_M = \frac{M_x}{M_c} \leq 1$$

$$M_c = fr \cdot \rho_y \cdot M_{df}$$

Where:

M_c	Moment capacity.
Fr	Stress reduction factor (only for the alternative method for slender sections).
M_{df}	Bending resistant modulus.

The reduction of the bending resistant modulus due to the effect of shear load is only applied if the shear load is above 60% of shear capacity of the section:

$$F_v > 0.6\ P_v$$

The bending resistant modulus is obtained by the following expressions:

1. If $F_v \leq 0.6\ P_v$

a. For plastic or compact sections:

$$M_{df} = S < 1.2 \cdot Z$$

b. For semi-compact sections:

$$M_{df} = S_{eff}$$

c. For slender sections:

$$M_{df} = Z_{eff}$$

2. If $F_v > 0.6\ P_v$

a. For plastic or compact sections:

$$M_{df} = S - S_v \cdot \rho$$

b. For semi-compact sections:

$$M_{df} = S_{eff} - S_v \cdot \rho$$

c. For slender sections:

$$M_{df} = Z_{eff} - \rho \cdot \left(\frac{S_v}{1.5}\right)$$

$$\rho = \left[\frac{2 \cdot F_v}{P_v}\right]^2$$

Where:

Z	Elastic resistant modulus of the section.
Z_{eff}	Effective elastic modulus.
S	Plastic resistant modulus of the section.
S_{eff}	Effective plastic modulus.
S_v	Plastic reduced modulus due to the effect of shear force.

Sv Parameter Calculation

The S_v calculation is done following the expression below:

$$S_v = S - S_f$$

Where:

S	Plastic resistant modulus of the section: S= $\sum_{i = elements} S_i$
S_f	Plastic modulus of the section remaining after deduction of the shear area: $S_r = \sum_{i = webs} S_i$

4. Calculation of the total criterion:

CRT_TOT = Max (Crt_V, Crt_PV, Crt_M)

5. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table:

Result	Concepts	Articles	Description
MX	M_x		Design value of the bending moment
MC	M_c	4.2.5	Moment capacity
FV	F_v		Design value of the shear force
PV	P_v	4.2.3	Design value of the shear capacity
CRT_V	F_v/P_v	4.2.3	Shear criterion
CRT_PV	F_v/V_w	4.4.5	Buckling web criterion
CRT_M	M_x/M_c	4.2.5	Bending criterion
CRT_TOT			BS Global criterion
CLASS		3.5.2	Section class
WEBCLASS		3.5.2	Webs' Class
MDF	M_{df}	4.2.5	Plastic or elastic modulus of the section
VW	V_w	4.4.5	Shear buckling resistance

7.3.9 Checking of Lateral Torsional Buckling Resistance (BS Article 4.3)

1. Forces and moments selection.

The forces and moments considered in this check are:

M_x = MY or MZ   Design value of the bending moment about the relevant axis of bending.

2. Class determination.
3. Criteria calculation.

Resistance to lateral-torsional buckling need not be checked separately for the following cases:

- Bending about the minor axis
- Circular hollow sections (CHS), square RHS or circular or square solid bars

- I, H, Channel or Box sections, if equivalent slenderness (λ_{LT}) does not exceed the limiting equivalent slenderness (λ_{L0})
- RHS, unless the slenderness exceeds the limiting value given in Table 15 of the code for the relevant value D/B.

D/B Depth / Width	Limiting value of λ
1.25	$770^{*}275/\rho_y$
1.33	$670^{*}275/\rho_y$
1.4	$580^{*}275/\rho_y$
1.44	$550^{*}275/\rho_y$
1.5	$515^{*}275/\rho_y$
1.67	$435^{*}275/\rho_y$
1.75	$410^{*}275/\rho_y$
1.8	$395^{*}275/\rho_y$
2	$340^{*}275/\rho_y$
2.5	$275^{*}275/\rho_y$
3	$225^{*}275/\rho_y$
4	$170^{*}275/\rho_y$

When checking for lateral torsional buckling of beams, the criterion shall be taken as:

$$\mathrm{Crt\_TOT} = \frac{m_{LT} \cdot M_x}{M_b} \leq 1$$

Where:

M_b Lateral torsional buckling resistance moment.

m_{LT} Equivalent uniform moment factor for lateral torsional buckling. This can be introduced according to the table 18 of the code (by default, $m_{LT} = 1$)

M_x Maximum major axis bending moment.

3.1 Determination of the buckling resistance moment Mb (Article 4.3.6.4)

The value of M_b may be determined from the following:

- For plastic and compact sections:

$$M_b = \rho_b \cdot S_x$$

- For semi-compact sections:

$$M_b = \rho_b \cdot S_{eff}$$

- For slender sections:

$$M_b = \rho_b \cdot S_{eff}$$

Where ρ_b is the bending strength.

If the equivalent slenderness λ_{LT} is less than or equal to the limiting slenderness λ_{L0} for the relevant design strength given in the tables 16 and 17 of the code, then ρ_b should be taken as equal to ρ_y and no considerations for lateral torsional buckling will be necessary.

$$\text{For } \lambda_{LT} \leq \lambda_{L0} \rightarrow \rho_b = \rho_y$$

Otherwise the bending strength is obtained from the formula given in Appendix B.2.1 of the code:

$$\text{For } \lambda_{LT} > \lambda_{L0} \rightarrow \rho_b = \frac{\rho_E \cdot \rho_y}{\phi_{LT} + \left(\phi_{LT}^2 - \rho_E \cdot \rho_y\right)^{1/2}}$$

$$\phi_{LT} = \frac{\rho_y(\eta_{LT} + 1) \cdot \rho_E}{2}$$

$$\rho_E = \left(\pi^2 \cdot E / \lambda_{LT}^2\right)$$

Where η_{LT} is the Perry coefficient

The Perry coefficient η_{LT} for lateral torsional buckling should be taken as follows:

a) For rolled sections:

$$\eta_{LT} = \alpha_{LT} * (\lambda_{LT} - \lambda_{L0})/1000 \quad \text{con } \eta_{LT} \geq 0$$

b) For welded sections:

If $\lambda_{LT} \leq \lambda_{L0}$ $\eta_{LT} = 0$

If $\lambda_{L0} \leq \lambda_{LT} < 2\lambda_{L0}$ $\eta_{LT} = 2\alpha_{LT}(\lambda_{LT} - \lambda_{L0})/1000$

If $2\lambda_{L0} \leq \lambda_{LT} \leq 3\lambda_{L0}$ $\eta_{LT} = 2\alpha_{LT}(\lambda_{L0})/1000$

If $3\lambda_{L0} \leq \lambda_{LT}$ $\eta_{LT} = (\lambda_{LT} - \lambda_{L0})/1000$

Where:

λ_{L0} Limiting equivalent slenderness: $\lambda_{L0} = 0.4\left(\frac{\pi^2 E}{\rho_y}\right)^{1/2}$

α_{LT} Robertson constant, taken as 0.007.

λ_{LT} Equivalent slenderness.

A. Equivalent Slenderness for I, H and Channel Sections

The equivalent slenderness λ_{LT} is taken as follows:

$$\lambda_{LT} = uv\lambda\sqrt{\beta_w}$$

The ratio β_w depends on the section class:

- For class 1 or 2 sections: $\beta_w = 1.0$
- For class 3 sections: $\beta_w = S_{eff}/S_x$
- For class 4 sections: $\beta_w = Z_{eff}/S_x$

$$\lambda = \max\left(\frac{L_{Ex}}{r_y}, \frac{L_{Ey}}{r_y}\right)$$

$$L_{ex} = L \cdot CfBuckx \cdot Kltx$$

$$L_{ey} = L \cdot CfBuckx \cdot Klty$$

The buckling parameter u and the torsional index x are calculated as follows:

- For I and H sections

$$u = \left(\frac{4S_x^2\gamma}{A^2h_s^2}\right)^{1/4}$$

$$h_s = \left(D - \frac{T_c + T_t}{2}\right)$$

$$x = 0.566 \cdot h_s(A/J)^{0.5}$$

- For Channel sections

$$u = \left(\frac{I_y S_x^2 \gamma}{A^2 H}\right)^{1/4}$$

$$x = 1.132\left(\frac{AH}{I_y J}\right)^{1/2}$$

$$\gamma = \left(1 - \frac{I_y}{I_x}\right)$$

Where:

J	Torsion constant (mechanical property of the section).
T_c	Thickness of the compression flange.
T_t	Thickness of the tension flange.
S_x	Plastic modulus about the major axis.
I_x	Moment of inertia about the major axis (mechanical property of the section).
I_y	Moment of inertia about the minor axis (mechanical property of the section).
A	Cross sectional area (mechanical property of the section).
H	Warping constant (mechanical property of the section).

The slenderness factor (v parameter) is given by:

$$v = \left[\left(4\eta(\eta - 1) + 0.05\left(\frac{\lambda}{x}\right)^2 + \Psi^2\right)^{1/2} + \Psi\right]^{-1/2}$$

$$\eta = \frac{I_{cf}}{I_{cf} + I_{tf}}$$

Where:

I_{cf}	Moment of area of the compression flange about the minor axis of the section.
I_{tf}	Moment of area of the tension flange about the minor axis of the section.

Ψ Monosymmetry index, for I and T sections with lipped flanges.

The monosymmetry index Ψ is calculated as follows:

$$\Psi = 0.8 \cdot (2\eta - 1) \cdot \left(1 + \frac{D_L}{2D}\right) \quad \text{for } \eta > 0.5$$

$$\Psi = 1.0 \cdot (2\eta - 1) \cdot \left(1 + \frac{D_L}{2D}\right) \quad \text{for } \eta < 0.5$$

Where:

D Overall depth of the section (mechanical property of the section).

DL Depth of the lip. By default DL=0.

B. Equivalent slenderness determination for Box Sections including RHS (Appendix B.2.6)

The equivalent slenderness, λ_{LT}, for box sections is taken directly from the expression below:

$$\lambda_{LT} = 2.25 \cdot (\Phi_b \cdot \lambda \cdot \beta_w)^{1/2}$$

$$\Phi_b = \left(\frac{S_x^2 \cdot \gamma'}{A \cdot J}\right)^{1/2}$$

$$\gamma' \left(1 - \frac{I_y}{I_x}\right) \cdot \left(1 - \frac{J}{2.6 \cdot I_x}\right)$$

C. Equivalent slenderness determination for T sections (Appendix B.2.8)

The equivalent slenderness, λ_{LT}, for T sections is obtained from the following:

a) If I_{xx} = I_{yy}: Lateral torsional buckling does not occur and $\lambda_{LT} = 0$

b) If $I_{yy} > I_{xx}$: Lateral torsional buckling occurs about the x-x axis and λ_{LT} is given by:

$$\lambda_{LT} = 2.8 \left(\frac{\beta_W L_E B}{T^2}\right)^{0.5}$$

c) If: $I_{xx} < I_{yy}$ Lateral torsional buckling occurs about the x-x axis and λ_{LT} is given by:

$$\lambda_{LT} = u\nu\lambda\sqrt{\beta_W}$$

$$u = \left(\frac{4S_x^{\,2}\,\gamma}{A^2\left(D - {}^{T}\!/_{2}\right)^2}\right)^{0.25}$$

$$v = \left[\left(w + 0.05\left({}^{\lambda}\!/_{x}\right)^2 + \Psi^2\right)^{1/2} + \Psi\right]^{-1/2}$$

$$w = \left(\frac{4H}{I_y\left(D - {}^{T}\!/_{2}\right)^2}\right)$$

$$x = 0.566(D - T/2)(A/J)^{0.5}$$

$$\gamma = 1 - I_y/I_x$$

$$H = \frac{B^3T^3}{144} + \frac{(D - T/2)^3t^3}{36}$$

D. Equivalent slenderness determination for Equal Angle sections (Appendix B.2.9.1)

The equivalent slenderness, λ_{LT}, for equal angle sections is obtained from the following:

$$\lambda_{LT} = 2.25^*(\phi_a{}^*\lambda_v)^{0.5}$$

$$\phi_a = \left(\frac{Z_u^2\gamma_a}{AJ}\right)^{0.5}$$

$$\gamma_a = \left(1 - \frac{I_v}{I_u}\right)$$

$$\lambda_v = \frac{L_v}{r_v}$$

E. Equivalent slenderness determination for Unequal Angle sections (Appendix B.2.9.2)

The equivalent slenderness, λ_{LT}, for unequal angle sections is obtained from the following:

$$\lambda_{LT} = 2.25^*\nu_a(\phi_a{}^*\lambda_v)^{0.5}$$

$$\nu_a = \left[\left(1 + \left(\frac{4.5\Psi_a}{\lambda_v}\right)^2\right)^{0.5} + \frac{4.5\Psi_a}{\lambda_v}\right]^{-0.5}$$

$$\Psi_a = \left[2\upsilon_0 - \frac{\int \upsilon(u_i^2 + v_i^2)}{I_u}\right]\frac{1}{t}$$

The monosymmetry index Ψ_a for an unequal angle is taken as positive when the short leg is in compression and negative when the long leg is in compression.

ν_0 is the coordinate of the shear center along the v-v axis.

Result	Concepts	Articles	Description
MB	m_b	4.3.6	Buckling resistance moment
UMLT		4.3.6	Equivalent uniform moment
M	m		Equivalent uniform moment factor
LAMBDA	Lambda	B.2	Slenderness
LAMBDALT	LambdaLT	B.2	Equivalent slenderness
LAMBDALO	LambdaLO	B.2	Limiting equivalent slenderness
CRT_TOT	$\frac{m_{LT} \cdot M_x}{M_b}$	4.3.6	Global criterion
CLASS		3.5.2	Section class
WEBCLASS		3.5.2	Web class

7.3.10 Checking of Members in Axial Tension (BS Article 4.6)

1. Forces and moments selection.

   The forces and moments considered for this checking type are:

F = FX — Design value of the axial force (positive if tensile, element not processed if compressive).

2. Class determination.
3. Criteria calculation.
   For members under axial tension, the general criterion Crt_TOT is checked at each section. This criterion coincides with the axial criterion Crt_N:

$$F \leq Pt \quad \rightarrow \quad Crt\_TOT = Crt\_N = \frac{F}{Pt} \leq 1$$

   Where Pt is the tension capacity: $P_t = A_{neff}/\rho_y$

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table:

Result	Concepts	Articles	Description
F	F	4.6.1	Tension Force
PT	P_t	4.6.1	Tension capacity
CRT_TOT	F/P_t	4.6.1	Global criterion

7.3.11 Checking of Members in Axial Compression (BS Article 4.7)

1. Forces and moments selection.
   The forces and moments considered for this checking type are:

F = FX Design value of the axial force (negative if it is compressive). If it is tensile, the element is not processed.

2. Class determination.
3. Criteria calculation.
   For members under axial compression, the general criterion Crt_TOT is checked at each section. This criterion coincides with the axial compression criterion Crt_CB:

$$F \leq Pc \;\rightarrow\; Crt\_TOT = \frac{F}{Pc} \leq 1$$

Where:

F	Axial compression force.
Pc	Compressive strength for buckling.

The compressive strength is determined according to the article 4.7.4 of BS 5950:2000:

- For class 1, 2 or 3 sections:

$$Pc = A_g \cdot \rho_c$$

- For class 4 sections:

$$Pc = A_{eff} \cdot \rho_{cs}$$

Where:

A_g	Gross sectional area.
A_{eff}	Effective cross sectional area.
ρ_c	Compressive strength.

ρ_{cs}	Compressive strength for a reduced slenderness of $\lambda(A_{eff}/A_g)^{0.5}$.

The compressive strength may be obtained from (See Appendix C):

$$\rho_c = \frac{\rho_E \cdot \rho_y}{\phi + \left(\phi^2 - \rho_E \cdot \rho_y\right)^{1/2}}$$

$$\phi = \frac{\rho_y \cdot (\eta + 1) \cdot \rho_E}{2}$$

Where:

ρ_y	Design strength (reduced by 20N/mm^2 for welded I, H or box sections).
ρ_E	Euler strength: $\rho_E = \pi^2 \cdot E/_{\lambda^2}$
E	Material elasticity modulus.
λ	Slenderness: $\lambda = L_E/_{i_g}$
i_g	Radius of gyration about the relevant axis.
L_E	Effective buckling length: $L_E = \max(L \cdot CfBuckx \cdot Kx, L \cdot CfBucky \cdot K_y)$
L	Actual length of the member.
$K_x$and K_y	Correction factors of buckling lengths for planes XZ and YZ.

The Perry coefficient η for flexural buckling under load should be taken as follows (Appendix C.2):

$$\eta = 0.001 \cdot a \cdot (\lambda - \lambda_o)$$

Where λ_o is the limiting slenderness:

$$\lambda_o = 0.2 \cdot \left(\frac{\pi^2 \cdot E}{\rho_y}\right)^{1/2}$$

The constant a (Robertson constant) is determined by the program from the type of section and buckling axis, according to the table 23 of the BS 5950:2000. Therefore, if the user introduces a value for this constant in member properties, the program will give precedence to the user's value.

a= 2.0 for curve (a)

a= 3.5 for curve (b)

a= 5.5 for curve (c)

a= 8.0 for curve (d)

To distinguish between I and H shapes the program follows the criteria below:

I shapes if $i_x/i_y > 2$

H shapes if $i_x/i_y < 2$

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Articles	Description
F	F	4.7	Compression Force
PC	P_c	4.7.4	Compression capacity
RHOC	ρ_c	4.7.5	Compression Resistance
LAMBDA	Lambda	4.7.2	Slenderness
LAMBDA0	Lambda0	C.2	Limiting slenderness
PERRYFCT	NU	C.2	Perry factor
ROBERSTS	a	C.2	Robertson constant
CRT_TOT	F/P_c	4.7	Global criterion
WEBCLASS		3.5.2	Web class
CLASS		3.5.2	Section class

7.3.12 Tension Members with Moments (BS Article 4.8.2)

1. Forces and moments selection.
   The forces and moments considered for this checking type are:
   F = FX Design value of the axial force.

   M_x= MY or MZ Design value of the bending moment along the primary bending axis.

M_y= MZ or MY Design value of the bending moment about the secondary bending axis.

2. Class determination.
3. Criteria calculation.

For members subjected to an axial tension force and bending moments, each section should be checked according the same conditions for members subjected to a shear force and bending moments (see section 9.8.3 of this manual).

Therefore, for this type of checking, the following conditions are checked:

3.1 Shear checking in both directions

$$\mathrm{Crt\_VX} = \frac{\mathrm{Fvx}}{\mathrm{Pvx}} \le 1$$

$$\mathrm{Crt\_VY} = \frac{\mathrm{Fvy}}{\mathrm{Pvy}} \le 1$$

Where Fvx and Fvy are the shear forces about X and Y axis, and Pvx and Pvy the shear capacity about X and Y axis.

3.2 Shear buckling resistance of shear webs

$$\mathrm{Crt\_PVX} = \frac{\mathrm{Fvx}}{\mathrm{Vwx}} \le 1$$

$$\mathrm{Crt\_PVY} = \frac{\mathrm{Fvy}}{\mathrm{Vwy}} \le 1$$

Where Vwx and Vwy are the shear buckling resistance about X and Y axis, respectively.

$$\mathrm{Vwx} = \sum \mathrm{q_{wx}} \cdot \mathrm{d} \cdot \mathrm{t}$$

$$\mathrm{Vwy} = \sum \mathrm{q_{wy}} \cdot \mathrm{d} \cdot \mathrm{t}$$

3.3 Checking of axial force and bending moments

Each section is checked according to the following condition:

$$\frac{F}{A_{neff} \cdot \rho_y} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \leq 1$$

Equivalent to:

$$Crt\_CMP = Crt\_AXL + Crt\_Mx + Crt\_My \leq 1$$

$$Crt\_AXL = \frac{|F|}{P_t}$$

$$Crt\_Mx = \frac{M_x}{M_{cx}}$$

$$Crt\_My = \frac{M_y}{M_{cy}}$$

Where:

F	Axial force.
M_x	Bending moment about major axis.
M_y	Bending moment about minor axis.
A_{neff}	Effective net area of the section.
ρ_y	Design strength of the material.
M_{cx}	Moment capacity about major axis.
M_{cy}	Moment capacity about minor axis.

M_{cx} and M_{cy} are calculated according to the Article 4.2.5 of BS 5950:2000.

For this checking type (moments on both directions), the shear area, the plastic modulus and the S_v parameter are calculated with respect to both directions (X and Y axis).

3.3 Checking of global criterion

CRT_TOT = Max (Crt_CMP, Crt_VX, Crt_VPX, Crt_VY, Crt_VPY)

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Articles	Description
F	F		Axial tension force

Result	Concepts	Articles	Description
MX	M_x	4.2.5	Bending moment about major axis
MY	M_y	4.2.5	Bending moment about minor axis
FVX	F_{vx}		Shear force about major axis
FVY	F_{yv}		Shear force about minor axis
PVX	P_{vx}	4.2.3	Shear capacity about major axis
PVY	P_{vy}	4.2.3	Shear capacity about minor axis
PT	P_t	4.6.1	Axial Tension Capacity
MCX	M_{cx}	4.2.5	Moment capacity about major axis
MCY	M_{cy}	4.2.5	Moment capacity about minor axis
CRT_AXL	$F/(Aneff \cdot \rho_y)$	4.6.1	Axial Criterion
CRT_VX	F_{vx}/P_{vx}	4.2.3	Shear Criterion about major axis
CRT_VY	F_{vy}/P_{vy}	4.2.3	Shear Criterion about minor axis
CRT_MX	M_x/M_{cx}	4.2.5	Bending Criterion about major axis
CRT_MY	M_y/M_{cy}	4.2.5	Bending Criterion about minor axis
CRT_PVX	F_{vx}/V_{wx}	4.4.5	Buckling web Criterion about major axis
CRT_PVY	F_{vy}/V_{wy}	4.4.5	Buckling web Criterion about minor axis
CRT_CMP	Crt_AXL + Crt_MX + Crt_MY	4.8.2	Axial + moments Criterion
SVX	S_{vx}	4.2.6	Sv parameter for major axis
SVY	S_{vy}	4.2.6	Sv parameter for minor axis
CRT_TOT		4.8.2	Global criterion
AVX	A_{vx}	4.2.3	Shear Area for major axis
AVY	A_{vy}	4.2.3	Shear Area for minor axis
VWX	V_{wx}	4.4.5	Shear buckling resistant for major axis

Result	Concepts	Articles	Description
VWY	V_{wy}	4.4.5	Shear buckling resistant for minor axis
MDFX	$S_x, Z_x, S_x - S_{vx} * R_{o1}$	4.2.6	Resistant modulus for major axis
MDFY	$S_y, Z_y, S_y - S_{vy} * R_{o1}$	4.2.6	Resistant modulus for minor axis
ZX	Z_x	4.2.6	Elastic Modulus about major axis
SX	S_x	4.2.6	Plastic Modulus about major axis
ZY	Z_y	4.2.6	Elastic Modulus about minor axis
SY	S_y	4.2.6	Plastic Modulus about minor axis
CLASS		3.5.2	Sections class
WEBCLASS		3.5.2	Web's class

7.3.13 Compression Members with Moments (BS Article 4.8.3)

1. Forces and moments selection.
   The forces and moments considered for this checking type are:

   F = FX — Design value of the axial force.

   F_{vx} = FY or FZ — Design value of the shear force perpendicular to the primary bending axis.

   F_{vy} = FZ or FY — Design value of the shear force perpendicular to the secondary bending axis.

   M_x = MY or MZ — Design value of the bending moment along the primary bending axis.

   M_y = MZ or MY — Design value of the bending moment about the secondary bending axis.

2. Class determination.
3. Criteria calculation.

Compression members are checked for local capacity at the points of greatest bending and axial load. This capacity may be limited by either yielding or local buckling, depending on the section properties. The member is then checked for global buckling. Therefore, for this type of checking, the following conditions are checked:

3.1 **Local Capacity Check**

3.1.1 Axial Criterion

$$Crt\_AX\_L = \frac{F}{F_c} \leq 1$$

Where:

F Axial load

F_c Compression capacity:

For class 1, 2 or 3 sections: $F_c = A_g \cdot \rho_y$

For class 4 sections: $F_c = A_{eff} \cdot \rho_y$

3.1.2 Local criteria as for Tension Members with Moments

Bending criterion (major axis)= Crt_MX_L

Bending criterion (minor axis)= Crt_MY_L

Shear criterion about major axis= Crt_VX

Shear criterion about minor axis = Crt_VY

Buckling web Criterion about major axis = Crt_PVX

Buckling web Criterion about minor axis = Crt_PVY

3.1.3 Component Local Criterion

$$Crt\_CM\_L = Crt\_AX\_L + Crt\_MX\_L + Crt\_MY\_L \leq 1$$

3.2 Overall Buckling Check

3.2.1 Axial Criterion (Buckling)

$$Crt\_AX\_O\_1 = \frac{F}{P_c} \leq 1$$

$$Crt\_AX\_O\_2 = \frac{F}{P_{cy}} \leq 1$$

Where:

F Design value of the axial compressive force.

P_c Compression resistance.

P_{cy} Compresion resistance, considering buckling about the minor axis only:

For class 1, 2 or 3 sections: $P_c = A_g \cdot \rho_c$

For class 4 sections: $P_c = A_{eff} \cdot \rho_c$

A_g Gross sectional area.

ρ_c Compressive strength obtained according article 4.7.5 of the code.

3.2.2 Bending Moment Criterion (primary axis)

$$Crt\_MX\_O\_1 = \frac{m_x \cdot M_x}{\rho_y \cdot Z_x} \leq 1$$

$$Crt\_MX\_O\_2 = \frac{m_{LT} \cdot M_{LT}}{M_b} \leq 1$$

Where:

m_x Equivalent uniform moment factor. By default $m_x = 1$.

m_{LT} Equivalent uniform moment factor for lateral torsional buckling. By default $m_{LT} = 1$.

M_x Bending moment about major axis.

M_b Buckling resistance moment according the article 4.3 of the code.

M_{LT} Maximum major axis moment .

3.2.3 Bending Moment Criterion (secondary axis)

$$Crt\_MY\_O = \frac{m_y \cdot M_y}{\rho_y \cdot Z_y} \leq 1$$

Where:

m_y Equivalent uniform moment factor. By default $m_y = 1$.

M_y Bending moment about minor axis.

Z_y Elastic modulus about the minor axis (for slender class 4 sections Z_{yeff} is taken).

3.2.4 Component Global Criterion

$$Crt\_CM\_O\_1 = Crt\_AX\_O\_1 + Crt\_MX\_O\_1 + Crt\_MY\_O \leq 1$$

$$Crt\_Cmp\_O\_1 = \frac{Fx}{Pc} + \frac{|m_x * M_x|}{\rho_y Z_x} + \frac{|m_y * M_y|}{\rho_y Z_y}$$

$$Crt\_CM\_O\_2 = Crt\_AX\_O\_2 + Crt\_MX\_O\_2 + Crt\_MY\_O \leq 1$$

$$Crt\_Cmp\_O\_2 = \frac{Fx}{Pcy} + \frac{|m_{LT} * M_{LT}|}{Mb} + \frac{|m_y * M_y|}{\rho_y Z_y}$$

$$Crt\_CM\_O = Max(Crt\_CM\_O\_1, Crt\_CM\_O\_2 \leq 1$$

3.3 Total Criterion

Crt_TOT=Max(Crt_CM_L,Crt_CM_O,Crt_VX, Crt_VPX, Crt_VY,Crt_VPY)

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

TABLE	Concepts	Articles	Description
F	F		Design value of the axial compressive force
PC	P_c	4.7.4	Compression resistance
FVX	F_{vx}		Shear force about major axis
MX	M_x		Bending moment about major axis
ZX	Z_x	4.2.5	Elastic Modulus about major axis
SX	S_x	4.2.5	Plastic Modulus about major axis
SVX	S_{vx}	4.2.5	Sv parameter for major axis
AVX	A_{vx}	4.2.3	Shear Area for major axis
VWX	V_{wx}	4.4.5	Shear buckling resistant for major axis
MDFX	$S_x, Z_x, S_x - S_{vx} * R_{o1}$	4.2.5	Resistant modulus for major axis
PVX	P_{vx}	4.2.3	Shear capacity about major axis

TABLE	Concepts	Articles	Description
MCX	M_{cx}	4.2.5	Moment capacity about major axis
FVY	F_{vy}		Shear force about minor axis
MY	M_y		Bending moment about minor axis
ZY	Z_y	4.2.5	Elastic Modulus about minor axis
SY	S_y	4.2.5	Plastic Modulus about minor axis
SVY	S_{vy}	4.2.5	Sv parameter for minor axis
AVY	A_{vy}	4.2.3	Shear Area for minor axis
VWY	V_{wy}	4.4.5	Shear buckling resistant for minor axis
MDFY	$S_y, Z_y, S_y - S_{vy} * R_{o1}$	4.2.5	Resistant modulus for minor axis
PVY	P_{vy}	4.2.3	Shear capacity about minor axis
MCY	M_{cy}	4.2.5	Moment capacity about minor axis
M	M	4.8.3.3	Equivalent uniform moment factor
LAMBDA	Lambda	4.3.7.5	Slenderness
LAMBDA0	Lambda0	C.2	Limiting slenderness
LAMBDALT	LambdaLT	4.3.7.5	Equivalent slenderness
LAMBDAL0	LambdaL0	B.2.4	Limiting equivalent slenderness
PERRYFCT	NU	C.2	Perry Factor
MB	M_b	4.3.7	Buckling resistance moment capacity
CRT_TOT	Max(Crt_CM_L, Crt_CM_O, Crt_VX, Crt_VY, ...)	4.8.3	Total Criterion
CRT_CM_L	Crt_AX_L + Crt_MX_L + Crt_MY_L	4.8.3	Local Axial + moments Criterion
CRT_CM_O	Crt_AX_O + Crt_MX_O + Crt_MY_O	4.8.3	Global Axial + moments Criterion

TABLE	Concepts	Articles	Description
CRT_AX_L	$\frac{F}{F_c}$	4.8.3	Local axial criterion
CRT_MX_L	M_x/M_{cx}	4.2.5	Local bending moment criterion about X axis
CRT_MY_L	M_y/M_{cy}	4.2.5	Local bending moment criterion about Y axis
CRT_AX_O	$F/P_c,/F/P_{cy}$	4.8.3	Global axial criterion
CRT_MX_O	$\frac{m_x \cdot M_x}{\rho_y \cdot Z_x}, \frac{m_{LT} \cdot M_{LT}}{M_b}$	4.8.3	Global bending moment criterion about X axis
CRT_MY_O	$\frac{m_y \cdot M_y}{\rho_y \cdot Z_y}$	4.8.3	Global bending moment criterion about Y axis
CRT_VX	F_{vx}/P_{vx}	4.2.3	Shear criterion about X axis
CRT_PVX	F_{vx}/V_{wx}	4.4.5	Buckling web Criterion about major axis
CRT_VY	F_{vy}/P_{vy}	4.2.3	Shear criterion about Y axis
CRT_PVY	F_{vy}/V_{wy}	4.4.5	Buckling web Criterion about minor axis
CLASS	Class	3.5.2	Section Class
WEBCLASS	Webclass	3.5.2	Web's Class

7.4. Steel Structures According to ASME BPVC III Sub. NF

Steel structures checking according to ASME BPVC III Subsection NF in CivilFEM includes the checking of structures composed of welded or rolled shapes under axial forces, shear forces and bending moments in 3D.

The calculations performed by CivilFEM correspond to the provisions of this code according to the following sections:

1. Allowable Stresses
2. Stability and Slenderness and Width-Thickness Ratios

7.4.1 Checking Types

With CivilFEM it is possible to accomplish the following checking and analysis types:

- Checking of sections (ASME NF-3322.1) subjected to:

Stress in Tension
Stress in Shear
Stress in Compression
Stress in Bending
Axial Compression and Bending
Axial Tension and Bending

- Stability check (ASME NF-3322.2):

Maximum Slenderness Ratios
Width Ratios

7.4.2 Material Properties

The following material properties are used for checking according to ASME BPVC III Subsection NF:

Description	Property
Steel yield strength	S_y (th)
Ultimate strength	S_u (th)
Modulus of Elasticity	E

*th = plate thickness

Furthermore, although austenitic stainless steel is an intrinsic material property, it can be modified by changing the material property.

7.4.3 Section Data

The section data of the element must be included in the CivilFEM database. All geometrical and mechanical properties are automatically obtained when defining the cross section or capturing the solid section. The section data required for checking according to this code are listed below:

Data	Description
A	Area of the cross-section
I_y	Moment of inertia about Y axis
I_z	Moment of inertia about Z axis
I_{yz}	Product of inertia about YZ
Y	Coordinate Y of the considered fiber
Z	Coordinate Z of the considered fiber
i_y	Radius of gyration about Y axis
i_z	Radius of gyration about Z axis
Y_{ws}	Shear area in Y
Z_{ws}	Shear area in Z

From the net section, only the area is considered. This area is calculated by subtracting the holes for screws, rivets and other holes from the gross sectional area. The user should be aware that the code indicates the diameter used to calculate the area of holes is greater than the real diameter. The area of holes is introduced within the structural steel code properties.

In order to determine the effective net area A_e of axially loaded tension members, the reduction coefficient Ct must be set (parameter). By default, Ct=0.75.

7.4.4 Structural Steel Properties

For ASME BPVC III Subsection NF, the data set checked at member level is shown in the following table.

Description	Data	Chapter
1.- Unbraced length of the member	L	3322
2.- Buckling length factor in Y axis	KY	3322
3.- Buckling length factor in Z axis	KZ	3322
4.- Bending coefficient dependent upon moment gradient in Y axis	CBY	3322
5.- Bending coefficient dependent upon moment gradient in Z axis	CBZ	3322
6.- Coefficient applied to bending term in interaction equation and dependent upon column curvature caused by applied moments in Z axis	CMY	3322
7.- Coefficient applied to bending term in interaction equation and dependent upon column curvature caused by applied moments in Y axis	CMZ	3322
8.- Pin-connected members: 0: No (default) 1: Yes	PIN	3322
9.- Member type: 0: Beam (default) 1: Column	COLUMN	3322
9.- Laterally braced in the region of compression: 0: No (default) 1: Yes	BRACED	3322

7.4.5 Checking Process

Necessary steps to conduct the different checks in CivilFEM are as follows:

a) Obtain the cross-sectional data corresponding to the element.

b) Specific section checking according to the type of external load.

c) Results. In CivilFEM, checking results for each element end are stored in the results file .CRCF

The required data for the different types of checking can be found in tables within the corresponding sections in this manual.

7.4.6 Tension Checking

In CivilFEM, elements subjected to tension are checked according to ASME BPVC III Subsection NF code for each end of the selected elements and solid sections of the model with a structural steel cross section. The check for tension adheres to the following steps:

7.4.6.1 Calculation of the Allowable Stress

The allowable stress in tension shall be as given in the equations below:

Except for pin-connected and threaded members, F_t shall be:

$$F_t = 0.6 \cdot S_y \leq 0.5 \cdot S_u^*$$

(*)
S_u on the effective net area

For pin-connected members, using the net area:

$$F_t = 0.45 \cdot S_y$$

7.4.6.2 Calculation of the Stress Criterion

The obtained equivalent stress f_t is divided by the steel design strength F_t in order to obtain a value that is stored as the CRT_STR parameter in the corresponding alternative. This value must be between 0.0 and 1.0 for the element to be valid according to the ASME BPVC III Subsection NF code; consequently, the equivalent stress must be less than the steel design strength.

$$\text{CRT\_STR} = \frac{f_t}{F_t} \leq 1$$

7.4.6.3 Slenderness Ratio

The maximum slenderness ratio l/r for tension members is obtained and stored as SLD_RT. This slenderness ratio is divided by 240 (SLD_RT shall not exceed 240) and stored as the CRT_SLD. Therefore, this value must be between 0.0 and 1.0 for the element to be valid according this code.

$$STR\_RT = \max\left(\frac{I}{r_y}, \frac{I}{r_z}\right)$$

$$CRT\_SLD = \frac{SLD\_RT}{240} \leq 1$$

7.4.6.4 Calculation of the Total Criterion

The Total Criterion is obtained from the maximum value between the stress criterion and the slender criterion; this criterion is stored as the CRT_TOT parameter in the corresponding alternative in CivilFEM's results file for each element end. This value must be between 0.0 and 1.0 for the element to be valid according the ASME BPVC III Subsection NF code.

CRT_TOT=MAX(CRT_STR,CRT_SLD)

7.4.7 Shear Checking

In CivilFEM the elements subjected to a shear force are checked according to ASME BPVC III Subsection NF code is done for each element end of those selected elements or solid sections of the model with a structural steel cross section.

7.4.7.1 Calculation of the Allowable Stress

The allowable stress for shear resistance of the effective section is as follows:

$$F_v = 0.40 \cdot S_y$$

7.4.7.2 Calculation of the Total Criterion

The equivalent stress obtained f_v is divided by the steel design strength F_v in order to obtain a value that is stored as the CRT_TOT parameter in the active alternative in the CivilFEM's results file for each element end. This value must be between 0.0 and 1.0 so that the element will be valid according to the ASME BPVC III Subsection NF code; consequently, the equivalent stress must be less than the steel design strength.

$$CRT\_TOT = \frac{f_v}{F_v} \leq 1$$

This equivalent stress f_v is the maximum value obtained in both directions:

$$F_v = MAX\left(\frac{V_y}{Y_{ws}}, \frac{V_z}{Z_{ws}}\right)$$

7.4.8 Compression Checking

In CivilFEM, elements subjected to compression are checked of according to ASME BPVC III Subsection NF for each element end of the selected elements or solid sections of the model with a structural steel cross section.

7.4.8.1 Calculation of the Allowable Stress

The allowable stress in compression shall be determined as described below:

1- Gross sections of columns, except those fabricated from austenitic stainless steel:

$$FA = \frac{\left[1 - \frac{(KI/r)^2}{2 \cdot Cc^2}\right] \cdot Fy}{\frac{5}{3} + \frac{3(KI/r)}{8 \cdot Cc} - \frac{(KI/r)^3}{8 \cdot Cc^3}} \quad \text{if } kI/r \leq C_c$$

$$FA = \frac{12\pi^2 E}{23 \cdot (KI/r)^2} \qquad \text{if } \frac{kI}{r} > C_c$$

where

$$Cc = \sqrt{\frac{2\pi^2 E}{Sy}}$$

2- Gross sections of columns fabricated from austenitic stainless steel:

$$FA = Sy\left(0.47 - \frac{kI/r}{444}\right) \quad \text{if} \quad kI/r \leq 120$$

$$FA = Sy\left(0.40 - \frac{kI/r}{600}\right) \quad \text{if} \quad kI/r > 120$$

3- Member elements other than columns:

$$FA = 0.60Sy$$

7.4.8.2 Slenderness Ratio

The maximum slenderness ratio l/r for tension members is obtained and stored as SLD_RT. This slenderness ratio is divided by 200 (SLD_RT shall not exceed 200) and stored as the parameter CRT_SLD. Consequently, this value must be between 0.0 and 1.0 for that the element to be valid according this code.

$$STR\_RT = \max\left(\frac{kI \cdot I}{r_y}, \frac{kz \cdot I}{r_z}\right)$$

$$CRT\_SLD = \frac{SLD\_RT}{200} \leq 1$$

7.4.8.3 Stress Reduction Factor

The ASME BPVC III Subsection NF code decreases the efficiency of a section through reduction factors when axially loaded members contain elements subjected to compression and have a width-thickness ratio above the limit below:

7.4.8.4 Unstiffened Compression Elements

Unstiffened compression elements have one free edge parallel to the direction of the compressive stress. Stress on these elements shall be decreased by the reduction factor Qs when the width-thickness ratio exceeds the limits below. The flange width will be the distance from the free edge to the web.

1- For Single Angles,

when $b/t \leq 76/\sqrt{Sy}$

Qs = 1.0

when $76/\sqrt{Sy} < b/t < 155/\sqrt{Sy}$

$$Qs = 1.340 - 0.00447\,(b/t)\sqrt{Sy}$$

when $b/t > 155/\sqrt{Sy}$

$$Qs = 15500/[Sy(b/t)^2]$$

2- For Stems of Tees,

when $b/t < 127/\sqrt{Sy}$

$$Qs = 1.0$$

when $95/\sqrt{Sy} < b/t < 176/\sqrt{Sy}$

$$Qs = 1.908 - 0.00715\,(b/t)\sqrt{Sy}$$

when $b/t > 176/\sqrt{Sy}$

$$Qs = 20000/[Sy(b/t)^2]$$

3- For other Compression Members,

when $b/t < 95/\sqrt{Sy}$

$$Qs = 1.0$$

when $95/\sqrt{Sy} < b/t < 176/\sqrt{Sy}$

$$Qs = 1.415 - 0.00437\,(b/t)\sqrt{Sy}$$

when $b/t > 176/\sqrt{Sy}$

$$Qs = 20000/[Sy(b/t)^2]$$

where Sy is the yield strength, in ksi.

Furthermore, proportions of unstiffened elements of channels and tees that exceed the limits above are checked for the following limits:

Shape	Ratio of Flange Width to Profile Depth	Ratio of Flange Thickness to Web or Stem Thickness
Built-up Channels	≤ 0.25	≤ 3.0
Rolled Channels	≤ 0.50	≤ 2.0
Built-up Tees	≤ 0.50	≥ 1.25

Rolled Tees	≤ 0.50	≥ 1.10

Table NF-3322.2(e)(2)-1

This proportion checking result is defined in the CivilFEM results file (.CRCF) as CTR_W with a value of 0.0 if the proportional limits are fulfilled and 2^{100} if they are not.

7.4.8.5 Stiffened Compression Elements

Stiffened compression elements have lateral support along both edges which are parallel to the direction of the compressive stress. If the width-thickness ratio of these elements exceeds the limit below, a reduced effective width b_e shall be used:

1- For the flanges of square and rectangular sections of uniform thickness:

when $b/t > 238/\sqrt{Sy}$

$$be = \frac{253t}{\sqrt{f}}\left[1 - \frac{50.3}{(b/t)\sqrt{f}}\right] \leq b$$

2- For other uniform compressed elements:

when $b/t > 253/\sqrt{Sy}$

$$be = \frac{253t}{\sqrt{f}}\left[1 - \frac{44.3}{(b/t)\sqrt{f}}\right] \leq b$$

Where f is the axial compressive stress on the member based on the effective area, in ksi.

If unstiffened elements are included in the total cross section, f must be such that the maximum compressive stress in the unstiffened elements does not exceed $F_a Q_s$. Therefore, the calculation of the effective width of stiffened elements adheres to the following iterative process:

a) The axial compressive stress f is obtained.

b) An initial value of the effective width b_e is calculated.

c) A new axial compressive stress f' of the effective area is obtained

d) If f' exceeds F_aQ_s, a new axial compressive stress f'' is obtained by increasing the last axial compressive stress f'.

This process is repeated until the axial compressive stress does not exceed F_aQ_s or until the effective area is equal to the total area.

Using the effective width b_e, the form factor Q_a is then calculated by the ratio of the effective area to the total area.

$$\mathrm{Aef} = \mathrm{A} - \sum (\mathrm{b} - \mathrm{bef}) \cdot \mathrm{t}$$

$$\mathrm{Qa} = \mathrm{Aef} / \mathrm{A}$$

7.4.8.6 Calculation of the Stress Criterion

The allowable stress for axially loaded compression members shall not exceed:

$$\mathrm{FA} \leq \frac{Q_s Q_a \left[1 - \frac{(\mathrm{KI/r})^2}{2 \cdot \mathrm{Cc}^2}\right] \cdot \mathrm{Fy}}{\frac{5}{3} + \frac{3(\mathrm{KI/r})}{8 \cdot \mathrm{Cc}} - \frac{(\mathrm{KI/r})^3}{8 \cdot \mathrm{Cc}^3}} \quad \text{if} \;\; \mathrm{kI/r} \leq C_c$$

After verifying the equation above, the equivalent stress obtained $f_a$is divided by the steel design strength F_a to obtain a value stored as the CRT_STR parameter in the active alternative in CivilFEM's results file for each element end. This value must be between 0.0 and 1.0 so that the element will be valid according to ASME BPVC III Subsection NF; therefore, the equivalent stress must be lower than the steel design strength.

$$\mathrm{CRT\_TOT} = \frac{f_a}{F_a}$$

7.4.8.7 Calculation of the Total Criterion

The Total Criterion is obtained from the maximum value between the stress criterion and the slender criterion and is stored as the CRT_TOT parameter in the active alternative in the CivilFEM's results file for each element end. This value must be between 0.0 and 1.0 for the element to be valid according to the ASME BPVC III Subsection NF.

$$\mathrm{CRT\_TOT} = \mathrm{MAX}\,(\mathrm{CRT\_STR}, \mathrm{CRT\_SLD})$$

7.4.9 Bending Checking

In CivilFEM, elements subjected bending are checked according to ASME BPVC III Subsection NF for each element end of the selected elements or solid sections of the model with a structural steel cross section.

7.4.9.1 Calculation of the Allowable Stress

First, the section is classified as a *compact section*, *member with a high flange width-thickness ratio* or *miscellaneous member*:

(a) Compact sections: For a section to qualify as compact, its flanges must be continuously connected to the web or webs and the width-thickness ratios of its compression elements must not exceed the limiting ratios below:

1- The width-thickness ratio of the compression flanges shall not exceed:
   a. for unstiffened elements $\mathrm{b/t} < 65/\sqrt{\mathrm{Sy}}$
   b. for stiffened elements $\mathrm{b/t} < 190/\sqrt{\mathrm{Sy}}$
2- Depth-thickness ratio of webs

$$\mathrm{d/t} < \left(640/\sqrt{\mathrm{Sy}}\right)[1 - 3.74(\mathrm{fa/Sy})] \quad \text{if} \quad \mathrm{fa/Sy} \le 0.16$$

$$\mathrm{d/t} < 257/\sqrt{\mathrm{Sy}} \quad \text{if} \quad \mathrm{fa/Sy} > 0.16$$

3- Moreover, the compression flanges shall be braced laterally at intervals not exceeding $76\mathrm{b_f}/\sqrt{\mathrm{Sy}}$ nor $20000/(\mathrm{d/A_f})\mathrm{Sy}$. This property is set by the user as a structural steel code property. If the cross section has no compression flanges, the member will be taken into account as braced laterally.

(b) Members with a high flange width-thickness ratio: members shall satisfy the requirements above, except unstiffened flanges shall satisfy:

$$65/\sqrt{Sy} < b/t < 95/\sqrt{Sy}$$

(c) Miscellaneous members: limit ratios above do not apply to these members.

Next, the allowable bending stress is determined by the equations below:

1- I Sections:
   a. Compact sections bent about their minor axis of inertia shall not exceed a bending stress of:
   $$\mathrm{FB} = 0.75 \cdot \mathrm{Sy}$$
   b. Members with a high flange width-thickness ratio bent about their minor axis of inertia shall not exceed a bending stress of:
   $$\mathrm{FB} = \mathrm{Sy}\left[1.075 - 0.005(\mathrm{b_r}/2\mathrm{t_f})\sqrt{\mathrm{Sy}}\right]$$
   c. Compact sections bent about their major axis of inertia shall not exceed:
   $$\mathrm{FB} = 0.66\mathrm{Sy}$$
   d. Members with a high flange width-thickness ratio bent about their major axis of inertia shall not exceed a bending stress of:
   $$\mathrm{FB} = \mathrm{Sy}\left[0.79 - 0.002(\mathrm{b_f}/2\mathrm{t_f})\sqrt{\mathrm{Sy}}\right]$$
   e. Miscellaneous member sections bent about their major axis of inertia shall not exceed the larger value below:
   $$\mathrm{FB1} = \left\{\frac{2}{3}\left[\mathrm{Sy}(\mathrm{I}/\mathrm{r_c})^2\right]/(1530 \cdot 10^3 \cdot \mathrm{C_b})\right\}\mathrm{Sy} \le 0.60 \cdot \mathrm{Sy}$$

   when $[(102 \cdot 10^3 \cdot \mathrm{C_b})/\mathrm{Sy}]^{1/2} \le \mathrm{I}/\mathrm{r_c} \le [(510 \cdot 10^3 \cdot \mathrm{C_b})/\mathrm{Sy}]^{1/2}$

   $$\mathrm{FB1} = (170 \cdot 10^3 \cdot \mathrm{C_b})/(\mathrm{I}/\mathrm{r_c})^2 \le 0.60 \cdot \mathrm{Sy}$$

   when $\mathrm{I}/\mathrm{r_c} \ge [(510 \cdot 10^3 \cdot \mathrm{C_b})/\mathrm{Sy}]^{1/2}$

   where $\mathrm{r_c}$is the radius of the section, comprising of the area of the compression flange plus one-third of the area of the compression web.

   When the area of the compression flange is greater than or equal to the area of the tension flange:

   $$\mathrm{FB2} = (12 \cdot 10^3 \cdot \mathrm{C_C})/(\mathrm{I} \cdot \mathrm{d}/\mathrm{A_f}) \le 0.60 \cdot \mathrm{Sy}$$

f. Members not included above which are braced laterally in the region of the compressive stress shall not exceed a bending stress of:

$$FB = 0.60 \cdot Sy$$

If these members are not braced laterally in the region of the compressive stress, the section will be not checked.

2- Tubular Square Box Sections:

a. Compact sections bent about their minor axis of inertia, but not necessarily braced laterally, shall not exceed a bending stress of:

$$FB = 0.66 \cdot Sy$$

b. Members not included shall not exceed a bending stress of:

$$FB = 0.60 \cdot Sy$$

However, this section strength can be decreased through reduction factors.

3- Pipe Sections:

a. If the diameter-thickness ratio of hollow, circular sections does not exceed $3300/Sy$, the bending stress shall not exceed:

$$FB = 0.66 \cdot Sy$$

If the diameter-thickness ratio is greater than the value above, the section will be not checked.

4- U channel Sections:

a. If the section is bent about its major axis of inertia, the bending stress shall not exceed the larger value below:

$$FB1 = \left\{\frac{2}{3} - [Sy(I/r_c)^2/(1530 \cdot 10^3 \cdot C_b)]\right\} Sy \le 0.60 \cdot Sy$$

when $[(102 \cdot 10^3 \cdot C_b)/Sy]^{1/2} \le {}^{I}/_{r_c} \le [(510 \cdot 10^3 \cdot C_b)/Sy]^{1/2}$

$$FB1 = (170 \cdot 10^3 \cdot C_b)/(I/r_c)^2 \le 0.60 \cdot Sy$$

when $I/r_c \ge [(510 \cdot 10^3 \cdot C_b)/Sy]^{1/2}$

where r_c is the radius of the section, comprising of the area of the compression flange plus one-third of the area of the compression web.

When the area of the compression flange is greater than or equal to the area of the tension flange,

$$FB2 = (12 \cdot 10^3 \cdot c_b)/(I \cdot d/A_f) \leq 0.60 \cdot Sy$$

b. Members not included above which are braced laterally in the region of the compressive stress shall not exceed a bending stress of:

$$FB = 0.60 \cdot Sy$$

If these members are not braced laterally in the region of the compressive stress, the section will be not checked.

5- Tees Sections:

a. Compact sections loaded in the direction of the web which coincides with the minor axis of inertia, shall not exceed a bending stress of:

$$FB = 0.66 \cdot Sy$$

b. Members with a high flange width-thickness ratio which are loaded in the direction of the web coinciding with the minor axis of inertia shall not exceed a bending stress of:

$$FB = Sy\left[0.79 - 0.002(b_f/2t_f)\sqrt{Sy}\right]$$

c. Miscellaneous member sections loaded in the direction of the web coinciding with the minor axis of inertia, shall not exceed the larger bending stress below:

$$FB1 = \left\{\frac{2}{3} - [Sy(I/r_c)^2/(1530 \cdot 10^3 \cdot C_b)]\right\} Sy \leq 0.60 \cdot Sy$$

when $[(102 \cdot 10^3 \cdot c_b)/Sy]^{1/2} \leq I/r_c \leq [(510 \cdot 10^3 \cdot C_b)/]^{1/2}$

$$FB1 = (170 \cdot 10^3 \cdot C_b)/(I/r_c)^2 \leq 0.60 \cdot Sy$$

When $I/r_c \geq [(510 \cdot 10^3 \cdot C_b)/Sy]^{1/2}$

where $r_c$is the radius of a section comprising the area of the compression flange plus one-third of the area the of compression web

When the compression flange area is greater than or equal to the tension flange area:

$$FB2 = (12 \cdot 10^3 \cdot C_b)/(I \cdot d/A_f) \leq 0.60 \cdot Sy$$

d. Members not included above which are braced laterally in the region of the compressive stress shall not exceed a bending stress of:

$$FB = 0.60 \cdot Sy$$

If these members are not braced laterally in the region of the compressive stress, the section will be not checked.

6- All other sections:

a. Members braced laterally in the region of the compressive stress shall not exceed a bending stress of:

$$FB = 0.60 \cdot Sy$$

If these members are not braced laterally in the region of compressive stress, the section will be not checked.

7.4.9.2 Stress Reduction Factor

ASME BPVC III Subsection NF Code decreases the efficiency of a section through reduction factors for flexural members containing elements subject to compression with a width-thickness ratio in excess of the limits below:

7.4.9.3 Unstiffened Compression Elements

Unstiffened compression elements have one free edge parallel to the direction of the compressive stress. When the width-thickness ratio exceeds the limits below, the stress calculation will include a reduction of factor Qs. The width of flanges is taken from distance from the free edge to the weld.

1- For Single Angles:

when $b/t < 76/\sqrt{Sy}$

$$Qs = 1.0$$

when $76/\sqrt{Sy} < b/t < 155/\sqrt{Sy}$

$$Qs = 1.340 - 0.00447(b/t)\sqrt{Sy}$$

when $b/t > 155/\sqrt{Sy}$

$$Qs = 15500/[Sy(b/t)^2]$$

2- For Stems of Tees:

when $b/t < 127/\sqrt{Sy}$

$$Qs = 1.0$$

when $127/\sqrt{Sy} < b/t < 176/\sqrt{Sy}$

$$Qs = 1.908 - 0.00715(b/t)\sqrt{Sy}$$

when $b/t > 176/\sqrt{Sy}$

$$Qs = 20000/[Sy(b/t)^2]$$

3- For Other Compression Members:

when $b/t < 95/\sqrt{Sy}$

$$Qs = 1.0$$

when $95/\sqrt{Sy} < b/t < 176/\sqrt{Sy}$

$$Qs = 1.415 - 0.00437(b/t)\sqrt{Sy}$$

when $b/t > 176/\sqrt{Sy}$

$$Qs = 20000/[Sy(b/t)^2]$$

where Sy is the yield strength, in ksi.

Furthermore, unstiffened elements of channels and tees with proportions that exceed the limits above are checked for the following limits:

Shape	Ratio of Flange Width to Profile Depth	Ratio of Flange Thickness to Web or Stem Thickness
Built-up channels	≤ 0.25	≤ 3.0
Rolled channels	≤ 0.50	≤ 2.0
Built-up tees	≤ 0.50	≥ 1.25

Rolled tees	≤ 0.50	≥ 1.10

Table NF-3322.2(e)(2)-1

This proportion checking result is written in the CivilFEM results file (.CRCF) as CTR_W with a value of 0.0 if the proportion limits are satisfied and 2^{100} if they are not.

7.4.9.4 Stiffened Compression Elements

Stiffened compression elements have lateral support along both edges which are parallel to the direction of the compressive stress. When the width-thickness ratio of these elements exceeds the applicable limit below, a reduced effective width b_e shall be used:

1- For the flanges of square and rectangular sections of uniform thickness,

when $\mathrm{b/t} > 238/\sqrt{\mathrm{Sy}}$

$$\mathrm{be} = \frac{253\mathrm{t}}{\sqrt{\mathrm{f}}}\left[1 - \frac{50.3}{(\mathrm{b/t})\sqrt{\mathrm{f}}}\right] \leq \mathrm{b}$$

2- For other uniform compressed elements,

When $\mathrm{b/t} > 253/\sqrt{\mathrm{Sy}}$

$$\mathrm{be} = \frac{253\mathrm{t}}{\sqrt{\mathrm{f}}}\left[1 - \frac{44.3}{(\mathrm{b/t})\sqrt{\mathrm{f}}}\right] \leq \mathrm{b}$$

Where f is the compressive stress on member based on the effective area, in ksi.

If unstiffened elements are included in the total cross section, f must have a value such that the maximum compressive stress in the unstiffened elements does not exceed $\mathrm{F_b Q_s}$. Therefore, the calculation of the effective width of stiffened elements adheres to the following iterative process:

a) The maximum compressive stress f of the element is obtained

b) An initial value of the effective width $\mathrm{b_e}$ is calculated in all the compressive elements.

c) A new axial compressive stress f' is obtained of the effective area.

d) If f' exceeds $\mathrm{F_b Q_s}$, a new effective width $\mathrm{b_e}$' is obtained by increasing the previous effective width $\mathrm{b_e}$.

This iteration is repeated until the axial compressive stress is less than $F_b Q_s$ or the effective area is equal to the total area.

Using the effective width b_e, the form factor Q_a is then calculated by the ratio of the effective area to the total area.

$$Aef = A - \sum (b - bef) \cdot t$$

$$Qa = Aef/A$$

7.4.9.5 Calculation of the Total Criterion

When reduction factors are required, the maximum allowable bending stress shall not exceed 0.6 $S_y Q_s$ or the F_b value as provided above.

The computed bending stress F_b, obtained from the effective area, is divided by the steel design strength F_b in order to obtain a value that is stored as the CRT_TOT parameter in the active alternative in CivilFEM's results file for each element end. This value must be between 0.0 and 1.0 so that the element will be valid according to ASME BPVC III Subsection NF; therefore, the equivalent stress must be less than the steel design strength.

$$CRT\_TOT = \frac{f_b}{F_b} \leq 1$$

7.4.10 Axial Compression & Bending Checking

In CivilFEM the checking of elements under bending and axial compression forces according to ASME BPVC III Subsection NF code are done for each element end of those selected elements or solid sections of the model whose cross section type is structural steel.

7.4.10.1 Calculation of the Total Criterion

For members subjected to both axial compression and bending, stresses shall satisfy the requirements of the following equations:

$$\frac{f_a}{F_a} + \frac{C_{my} f_{by}}{\left(1 - f_a/F'_{ey}\right) F_{by}} + \frac{C_{mz} f_{bz}}{(1 - f_a/F'_{ez}) F_{bz}} \leq 1.0$$

$$\frac{f_a}{0.60 \cdot S_y} + \frac{f_{by}}{F_{by}} + \frac{f_{bz}}{F_{bz}} \leq 1.0$$

When evaluating both primary and secondary stresses:

$$\frac{f_a}{F_a} + \frac{f_{by}}{F_{by}} + \frac{f_{bz}}{F_{bz}} \leq 1.0$$

When evaluating primary stresses:

$$\frac{f_a}{F_a} + \frac{f_{by}}{F_{by}} + \frac{f_{bz}}{F_{bz}} \leq 1.5$$

The Total Criterion will be the maximum value of the equations below and will be stored as the CRT_TOT parameter in the active alternative in CivilFEM's results file for each element end. This value must be between 0.0 and 1.0 for the element to be valid according to ASME BPVC III Subsection NF; therefore, the equivalent stress must be less than the steel design strength.

7.4.10.2 Axial Tension & Bending Checking

In CivilFEM, elements subjected bending and axial tension forces are checked according to ASME BPVC III Subsection NF for each element end of the selected elements or solid sections of the model with a structural steel cross section.

7.4.10.3 Calculation of the Total Criterion

Members subject to both axial tension and bending stresses shall satisfy the requirements of the following equation:

$$\frac{f_a}{0.60 \cdot S_y} + \frac{f_{by}}{F_{by}} + \frac{f_{bz}}{F_{bz}} \leq 1.0$$

Where f_b is the computed bending tensile stress. However, the computed bending compressive stress, taken alone, shall not exceed the allowable compressive stress F_a.

Therefore, the total criterion will be:

$$\mathrm{CRT\_TOT} = \mathrm{MAX}\left(\frac{f_a}{0.60 \cdot S_y} + \frac{f_{by}}{F_{by}} + \frac{f_{bz}}{F_{bz}}, \frac{f_{bc}}{F_a}\right)$$

Where f_{bc} is the computed bending compressive stress.

The total criterion is stored as the CRT_TOT parameter in the active alternative in CivilFEM's results file for each element end. This value shall be between 0.0 and 1.0 for the element to be valid according to ASME BPVC III Subsection NF; consequently, the equivalent stress must be less than the steel design strength.

7.5. Steel Structures According to GB50017

Steel structures checking according to the Chinese Steel Design Code GB50017 in CivilFEM includes the checking of structures composed of welded or rolled shapes subjected to axial forces, shear forces and bending moments in 3D.
The calculations made by CivilFEM correspond to the provisions of GB50017 from the following sections:

Section 4	Bending element calculations
Section 5	Axially loaded structures and calculation of compression and bending

7.5.1 Checking Types

For checks within CivilFEM according to GB50017, it is possible to accomplish the following checking and analysis types:

Checking of sections subjected to:

- Bending force	GB50017 Art. 4.1.1
- Shear force	GB50017 Art. 4.1.2
- Bending and shear force	GB50017 Art. 4.1.4
- Axial force	GB50017 Art. 5.1.1
- Bending and axial force	GB50017 Art. 5.2.1
- Compression buckling	GB50017 Art. 5.1.2

7.5.2 Material Properties

In GB50017 checking, the following material properties are used:

Description	Property
Steel yield strength	f(th)

Ultimate strength	f_{ce} (th)
Shear strength	f_v (th)
Elasticity modulus	E

*th = plate thickness

7.5.3 Section Data

The section data of the element must be included in the CivilFEM database. All geometrical and mechanical properties are automatically obtained defining the cross section or capturing the solid section. Below, the section data necessary for checking according to GB50017 are listed:

Data	**Description**
A	Area of the cross-section
I_y	Moment of inertia about Y axis
I_z	Moment of inertia about Z axis
I_{yz}	Product of inertia about YZ
Y	Coordinate Y of the considered fiber
Z	Coordinate Z of the considered fiber
i_y	Radius of gyration about Y axis
i_z	Radius of gyration about Z axis
Y_{ws}	Shear area in Y
Z_{ws}	Shear area in Z
X_{wt}	Torsional modulus

From net section, only the area is considered. This area is calculated by subtracting the holes for screws, rivets and other holes from the gross section area. The user should be aware that LRFD indicates the diameter from which to calculate the area of holes is greater than the real diameter. The area of holes is introduced within the structural steel code properties.

7.5.4 Structural steel code properties

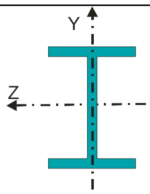
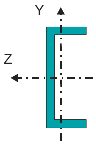
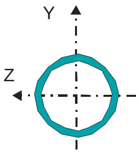
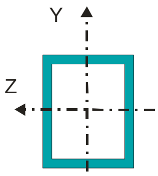
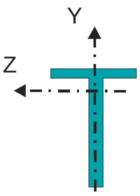
For LRFD, the checked data set used at member level is shown in the following table. All data is stored with the section data in user units and in CivilFEM reference axis.

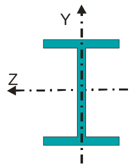
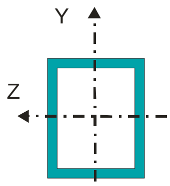
Description	Data	Chapter
Input data:		
1.- Plastic developing coefficient in Y axis	GAMMAy	5.2
0.0: not defined (default)		
2.- Plastic developing coefficient in Z axis	GAMMAz	5.2
0.0: not defined (default)		
3.- Cross section type in Y axis:	TSECy	Table 5.1.2-1 &
0: not defined (default)		Table 5.1.2-2
1: Type a		
2: Type b		
3: Type c		
4: Type d		
4.- Cross section type in Z axis:	TSECz	Table 5.1.2-1 &
0: not defined (default)		Table 5.1.2-2
1: Type a		
2: Type b		
3: Type c		
4: Type d		
5.- Unbraced length of the member	L	5.1.2
6.- Buckling length factor in Y axis	KY	5.1.2
7.- Buckling length factor in Z axis	KZ	5.1.2

7.5.5 Cross Section Type Classification

The cross section type is defined by values introduced in TSECY and TSECZ structural steel code properties. Otherwise they will be computed from the following table 5-3:

CROSS SECTION TYPE			**Y**	**Z**
I Section	Rolled Section	If $b/h \leq 0.8$	b	a

		If $b/h \leq 0.8$	b	b
	Welded Section		b	b
Channel 	Rolled or Welded		b	b
Pipe 	Rolled		a	a
	By dimensions		b	b
L angle	Rolled		b	b
Square Tubing or Box 	Rolled or Welded if $\frac{(b+h)/2}{t} > 20$		c	c
Standard T 	Rolled or Welded		b	b

CROSS SECTION TYPE			Y	Z
I Section 	Rolled	If $\mathrm{t} < 80mm$	c	b
		If $\mathrm{t} \geq 80\mathrm{mm}$	d	c
	Welded (default)		b	b
Square Tubing or Box 	Rolled or Welded if $\frac{(\mathrm{b}+\mathrm{h})/2}{\mathrm{t}} > 20$		b	b
	Rolled or Welded if $\frac{(\mathrm{b}+\mathrm{h})/2}{\mathrm{t}} \leq 20$		c	c

7.5.6 Checking Process

Required steps to conduct the different checks in CivilFEM are as follows:

a) Obtain the cross-section data corresponding to the element.

b) Specific section checking according to the type of external load.

c) Results. Checking results are stored in the results file .CRCF.

In sections corresponding to the different types of checking, the necessary data corresponding to the each type of solicitation is described.

7.5.7 Bending Checking

In CivilFEM the checking of elements under bending according to GB50017 code is done for each element end of those selected elements or solid sections of the model with a cross section type of structural steel. For this check, the program follows the steps below:

7.5.7.1. Calculation of the Maximum Normal Stress

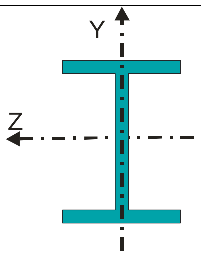
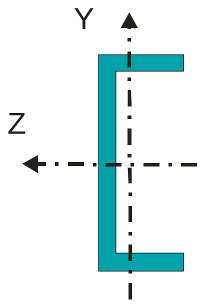
The maximum normal stress is calculated with the general equation for sections subjected to bending moments according to axes, not necessarily principal of inertia:

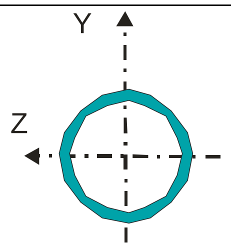
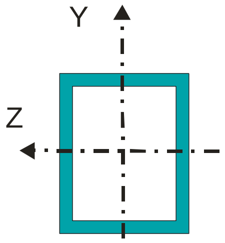
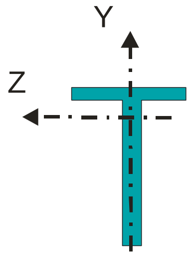
$$\sigma^* = \sigma_n \frac{\frac{M_y}{\gamma_y}(I_z Z - I_{yz} y) - \frac{M_z}{\gamma_z}(I_y y - I_{yz} Z)}{I_y I_z - I_{yz}^2}$$

Where:

M_y	Bending moment in Y direction
M_z	Bending moment in Z direction
I_y	Moment of inertia in Y direction
I_z	Moment of inertia in Z direction
I_{yz}	Product of inertia about YZ

Plastic development coefficients
γ_y, γ_z are obtained from the associated structural steel code properties. Otherwise they should be defined according to 11.5.4.

CROSS SECTION TYPE	γ_y	γ_z
	1.20	1.05
	$1.20 \text{ if } M_y > 0$	1.05
	$1.05 \text{ if } M_y \leq 0$	

	1.15	1.15
	1.05	1.05
	1.20	$1.20 \text{ if } M_z > 0$ $1.05 \text{ if } M_z \leq 0$
Otherwise	1.00	1.00

7.5.7.2. Calculation of GB50017 Criterion

The equivalent stress previously obtained is then divided by the steel design strength σ_u in order to obtain a value that is stored as the CRT_TOT parameter in the active alternative in CivilFEM's results file for each element end. This value shall be between 0.0 and 1.0 so that the element will be valid according to the GB50017 code; therefore, the equivalent stress must be lower than the steel design stress.

$$CRT\_TOT = \frac{\sigma^*}{\sigma_u}$$

7.5.8 Shear Checking

In CivilFEM, the checking of elements under shear force according to the GB50017 code is performed for each element end of those selected elements or solid sections of the model with a cross section type of structural steel.

7.5.8.1. Calculation of the Maximum Tangential Stress

The maximum tangential shear and torsion stresses for each element end are calculated from shear forces and section mechanical properties in the following equation:

$$\sigma^* = \sigma_t = MAX\left(\frac{T_y}{Y_{ws}}, \frac{T_z}{Z_{ws}}\right)$$

Where:

T_y	Shear Force in Y direction
T_z	Shear Force in Z direction
Y_{ws}	Shear area about Y axis.
Z_{ws}	Shear area about Z axis.

7.5.8.2. Calculation of GB50017 Criterion

The equivalent stress obtained is divided by the steel design strength σ_u in order to obtain a value that is stored as the CRT_TOT parameter in the active alternative in CivilFEM's results file for each element end. This value shall be between 0.0 and 1.0 so that the element will be valid according to the GB50017 code; consequently, the equivalent stress must be lower than the steel design stress.

$$CRT\_TOT = \frac{\sigma^*}{\sigma_U} \leq 1$$

7.5.9 Bending & Shear Checking

In CivilFEM the checking of elements under bending and shear forces according to GB50017 code is done for each element end of those selected elements or solid sections of the model with a cross section type of structural steel. The following steps:

7.5.9.1. Calculation of the Maximum Normal Stress

The maximum normal stress is calculated with the general equation for sections subjected to bending moments according to axes, not necessarily the principal axes of inertia:

$$\sigma^* = \sigma_n = \frac{M_y(I_z Z - I_{yz} y) - M_z(I_y y - I_{yz} Z)}{I_y I_z - I_{yz}^2}$$

Where:

M_y Bending moment in Y direction

M_z Bending moment in Z direction

I_y Moment of inertia in Y direction

I_z Moment of inertia in Z direction

I_{yz} Product of inertia about YZ

7.5.9.2. Calculation of the Maximum Tangential Stress

The maximum tangential shear and torsion stresses for each element end are calculated from shear forces and section mechanical properties in the following equation:

$$\sigma^* = \sigma_t = MAX\left(\frac{T_y}{Y_{ws}}, \frac{T_z}{Z_{ws}}\right)$$

Where:

T_y Shear Force in Y direction

T_z Shear Force in Z direction

Y_{ws} Shear area about Y axis.

Z_{ws} Shear area about Z axis.

7.5.9.3. Calculation of the Maximum Equivalent Stress

The maximum equivalent stress in the section σ^* is calculated by using:

$$\sigma^* = \sqrt{{\sigma_n}^2 + 3{\sigma_t}^2}$$

The maximum equivalent stress for each element end is stored in the active alternative in CivilFEM's results file with the parameter named SCEQV.

7.5.9.4. Calculation of GB50017 Criterion

The equivalent stress obtained is divided by the steel design strength σ_u in order to obtain a value that is stored as the CRT_TOT parameter in the active alternative in CivilFEM's results file for each element end. This value shall be between 0.0 and 1.0 so that the element will be valid according to the GB50017 code; thus, the equivalent stress must be lower than the steel design stress.

$$\text{CRT\_TOT} = \frac{\sigma^*}{\beta F} \leq 1$$

Where β is the amplifying factor for the combined design strength. If σ_n and σ_t have different sign β= 1.2, otherwise β= 1.1.

7.5.10 Axial Force Checking

In CivilFEM, the checking of elements under axial forces (without considering buckling) according to the GB50017 code is done for each element end of those selected elements or solid sections of the model with a cross section type of structural steel.

7.5.10.1. Calculation of the Maximum Axial Stress

The maximum tangential shear and torsion stresses for each element end are calculated from shear forces and section mechanical properties in the following equation:

$$\sigma^* = \left(1 - 0.5\frac{n_1}{n}\right)\frac{N}{A_n}$$

Where:

N	Axial force
A_n	Net area of the cross section
n	Number of high-strength frictional bolts
n_t	Number of bolts at the calculated section

In CivilFEM $\frac{n_1}{n}$ coefficient is given by RTB factor which can be modified in the structural steel code properties.

7.5.10.2. Calculation of GB50017 Criterion

The equivalent stress obtained is then divided by the steel design strength σ_u in order to obtain a value that is stored as the CRT_TOT parameter in the active alternative in the CivilFEM's results file for each element end. This value shall be between 0.0 and 1.0 so that the element will be valid according to the GB50017 code; therefore, the equivalent stress must be lower than the steel design stress.

$$\mathrm{CRT\_TOT} = \frac{\sigma^*}{f} \leq 1$$

7.5.11 Bending & Axial Checking

In CivilFEM, checking elements subjected to bending and axial forces according to GB50017 code is conducted for each element end of those selected elements or solid sections of the model with a cross section type of structural steel.

7.5.11.1. Calculation of the Maximum Equivalent Stress

The maximum equivalent stress in the section σ^* is calculated by using:

$$\sigma^* = \frac{N}{A_n} + \frac{\frac{M_y}{\gamma_y}(I_z Z - I_{yz} y) - \frac{M_z}{\gamma_z}(I_y y - I_{yz} Z)}{I_y I_z - I_{yz}^2}$$

Where:

M_y	Bending moment in Y direction
M_z	Bending moment in Z direction
I_y	Moment of inertia in Y direction

I_z	Moment of inertia in Z direction
I_{yz}	Product of inertia about YZ

Plastic development coefficients γ_y, γ_z are obtained from the associated in the structural steel code properties.

7.5.11.2. Calculation of GB50017 Criterion

The equivalent stress obtained is then divided by the steel design strength σ_u in order to obtain a value that is stored as the CRT_TOT parameter in the active alternative in CivilFEM's results file for each element end. This value shall be between 0.0 and 1.0 so that the element will be valid according to the GB50017 code; therefore, the equivalent stress must be lower than the steel design stress.

$$\mathrm{CRT\_TOT} = \frac{\sigma^*}{f} \leq 1$$

7.5.12 Compression Buckling Checking

In CivilFEM the checking of elements considering buckling according to GB50017 code is done for each element end of those selected elements or solid sections of the model with a cross section type of structural steel and subjected to a compressive force.

7.5.12.1. Calculation of the Maximum Equivalent Stress

The maximum equivalent stress in the section σ^* is calculated by using:

$$\sigma^* = \frac{N}{\varphi A}$$

Where φ is the stability coefficient for axially compressed members. The stability coefficient φ is calculated from the slenderness ratio:

$$\lambda_y = L\frac{k_y}{i_y}$$

$$\lambda_z = L\frac{k_z}{i_z}$$

Where:

L	Unbraced length of member
k_y	Buckling length factors in Y axis
k_z	Buckling length factors in Z axis
i_y	Rotational radius to Y axis
i_z	Rotational radius to Z axis

In non symmetric sections, the axes are defined as the directions of principal inertia.

To compute φ:

a) If $\lambda_n = \frac{\lambda}{\pi}\sqrt{f_y/E} \leq 0.215$ then $\varphi = 1 - \alpha_1 \lambda_n^2$

b) Otherwise:

$$\varphi = \frac{1}{2\lambda_n^2}\left[(\alpha_2 + \alpha_3\lambda_n + \lambda_n^2) - \sqrt{(\alpha_2 + \alpha_3\lambda_n + \lambda_n^2)^2 - 4\lambda_n^2}\right]$$

Where $\alpha_1, \alpha_2, \alpha_3$ are chosen according to the following table:

CROSS SECTION		α_1	α_2	α_3
a		0.410	0.986	0.152
b		0.650	0.965	0.300
c	$\lambda_n \leq 1.05$	0.730	0.906	0.595
	$\lambda_n \leq 1.05$		1.216	0.302
d	$\lambda_n \leq 1.05$	1.350	0.868	0.915
	$\lambda_n \leq 1.05$		1.375	0.432

The cross section type is determined from tables of chapter 7.5.5.

7.5.12.2. Calculation of GB50017 Criterion

The equivalent stress obtained is then divided by the steel design strength σ_u in order to obtain a value that is stored as the CRT_TOT parameter in the active alternative in CivilFEM's results file for each element end. This value shall be between 0.0 and 1.0 so that the element will be valid according to the GB50017 code; consequently, the equivalent stress must be lower than the steel design stress.

$$\mathrm{CRT\_TOT} = \frac{\sigma^*}{f} \leq 1$$

7.6. Steel Structures According to IS800-07

Checking steel structures according to Indian Standard IS 800 (2007) is implemented in CivilFEM. It is possible to check structures composed by welded or rolled shapes under axial forces, shear forces and bending moments in 3D.

The calculations made by CivilFEM correspond to the recommendations of Indian Standard General Construction in Steel – Code of Practice (Third Revision).

7.6.1 Checking types

With CivilFEM it is possible to accomplish the following check and analysis types:

- Tension	Section 6.2
- Compression	Section 7.1
- Bending	Section 8.2.1
- Shear force	Section 8.4
- Bending and Shear	Section 9.2
- Lateral Torsional Buckling	Section 8.2.2
- Axial Force with Moments	Section 9.3

7.6.2 Material Properties

For IS 800:2007 checking, the following material properties are used:

Description	Properties, symbol
Yield strength	fy
Ultimate tensile strength	fu
Partial safety factor for material	Resistance, governed by yielding $\gamma_{m0} = 1.1$
	Resistance of member to buckling $\gamma_{m0} = 1.1$
	Resistance, governed by ultimate stress $\gamma_{m1} = 1.25$
Modulus of elasticity	E = 200 kN/mm2
Shear Modulus	$G = \frac{E}{2(1+\upsilon)}$
Poisson's ratio	ν = 0.3

Coefficient of linear thermal expansion	α = 12·10-6 °C-1
Constant ϵ	$\varepsilon = \sqrt{\frac{250}{\text{fy}}}$

7.6.2 Section Data

IS 800:2007 considers the following data set for the section:

- Gross section data
- Net section data
- Effective section data
- Data pertaining to the section and plates class.

Gross section data correspond to the nominal properties of the cross-section.

For the net section, only the area is considered. This area is calculated by subtracting the holes for screws, rivets and other holes from the gross section area.

Effective section data and section and plates class data are obtained in the checking process according to the effective width method. For class 4 cross-sections, this method subtracts the non-resistance zones for local buckling. However, for cross-sections of a lower class, the sections are not reduced for local buckling.

The initial required data for the IS 800:2007 module includes the gross section data in user units and the CivilFEM or section axis. In the following tables, the section data used in IS 800:2007 are shown:

Common data for gross, net and effective sections

Description	Data
Input data:	
1.- Height	H
2.- Web thickness	Tw
3.- Flanges thickness	Tf
4.- Flanges width	B
5.- Distance between flanges	Hi
6.- Radius of fillet (Rolled shapes)	r_1
7.- Toe radius (Rolled shapes)	r_2
8.- Weld throat thickness (Welded shapes)	a
9.- Web free depth	d

Output data	(None)

Gross section data

Description	Data	Reference axis
Input data:		
1.- Depth in Y	Tky	CivilFEM
2.- Depth in Z	tkz	CivilFEM
3.- Cross-section area	A	
4.- Moments of inertia for torsion	It	CivilFEM
5.- Moments of inertia for bending	Iyy, Izz	CivilFEM
6.- Product of inertia	Izy	CivilFEM
7.- Elastic resistant modulus	Wely, Welz	CivilFEM
8.- Plastic resistant modulus	Wply, Wplz	CivilFEM
9.- Radius of gyration	iy, iz	CivilFEM
10.- Gravity center coordinates	Ycdg, Zcdg	Section
11.- Extreme coordinates of the perimeter	Ymin, Ymax, Zmin, Zmax	Section
12.- Distance between GC and SC in Y and in Z	Yms, Zms	Section
13.- Warping constant	Iw	
14.- Shear resistant areas	Yws, Zws	CivilFEM
15.- Torsional resistant modulus	Xwt	CivilFEM
16.- Moments of inertia for bending about U, V	Iuu, Ivv	Principal
17.- Angle Y->U or Z->V	α	CivilFEM
Output data:	(None)	

Net section data

Description	Data
Input data:	
1.- Gross section area	Agross
2.- Area of holes	Aholes
Output data:	
1.- Net Cross-section area	Anet

The effective section depends on the section geometry and on the forces and moments that are applied to it. Consequently, for each element end, the effective section is calculated.

Effective section data

Description	**Data**	**Reference axis**
Imput data:	(None)	
Output data:		
1.- Cross-section area	Aeff	
2.- Moments of inertia for bending	Iyyeff, Izzeff	CivilFEM
3.- Product of inertia	Izyeff	CivilFEM
4.- Elastic resistant modulus	Wyeff, Wzeff	CivilFEM
5.- Gravity center coordinates	Ygeff, Zgeff	Section
6.- Distance between GC and SC in Y and in Z	Ymseff, Zmseff	Section
7.- Warping constant	Iw	
8.- Shear resistant areas	Yws, Zws	CivilFEM

Data referred to the section plates

Description	**Data**
Input data:	
1.- Plates number	N
2.- Plate type: flange or web (for the relevant axis of bending)	Pltype
3.- Union condition at the ends: free or fixed	Cp1, Cp2
4.- Plate thickness	t
5.- Coordinates of the extreme points of the plate (in Section axis)	Yp1, Yp2, Zp1, Zp2
Output data:	
6.- Reduction factors of the plates at each end	Rho1, Rho2
7.- Plates class	Cl

7.6.3 Structural steel code properties

For IS 800:2007 checking, the data set used at member level are shown in the following table. All the data are stored with the section data in user units and in the CivilFEM reference axis. This data is defined as the parameters:

- L, K, KW, C1, C2, C3, CMY, CMZ, CMLT, CFBUCKXY and CFBUCKXZ.

Note that CFBUCKXY and CFBUCKXZ are used for simple buckling calculations and K, KW, C1, C2, C3, CMY, CMZ, CMLT are used for lateral and torsional buckling. This is important for understanding the way CivilFEM obtains buckling length. In simple buckling, buckling effective length (K*L according to 7.2.1) is obtained as CFBUCKXZ*L or CFBUCKXY*L. Lateral buckling effective length (L_{LT} according to 8.3) is obtained using K*L (this K is the one entered in the Member Properties pane).

Member Properties

Description	IS800-07
Input data:	
1.- Unbraced length of member (global buckling). Length between lateral restraints (lateral-torsional buckling)	L
2.- Effective length factors	k, kw
3.- Lateral buckling factors, depending on the load and restraint conditions	C1, C2, C3
4.- Equivalent uniform moment factors for flexural buckling	CMy, CMz
5.- Equivalent uniform moment factors for lateral-torsional buckling	CMLt
6.- Reduction factor for vectorial effects	N/A
7.- Buckling factors for planes XZ and YZ (Effective buckling length for plane XY =L*Cfbuckxy) (Effective buckling length for plane XZ =L*Cfbuckxz)	Cfbuckxy, Cfbuckxz

7.6.4 Check process

The checking process includes the evaluation of the following expression:

$$\frac{N_{Ed}}{N_{c.Rd}} + \frac{M_{y.Ed}}{M_{y.Rd}} + \frac{M_{z.Ed}}{M_{z.Rd}} \leq 1$$

Evaluation steps:

1. Read the checking type requested by the user.
2. Default checking type: Bending, shear and axial force.

3. Read the CivilFEM axis to be considered as the relevant axis for bending so that it coincides with the Y axis of IS 800:2007. In CivilFEM, by default, the principle bending axis that coincides with the +Y axis of IS 800:2007 is the –Z.
4. The following operations are necessary for each selected element:
   a. Obtain material properties of the element stored in CivilFEM database and calculate the rest of the properties needed for checking:
   Properties obtained from CivilFEM database:

   Calculated properties:

   Epsilon, material coefficient:

$$\varepsilon = \sqrt{250/f_y(th)} \quad (f_y \text{ in N/mm}^2)$$

   b. Obtain the cross-section data corresponding to the element.
   c. Initialize values of the effective cross-section.
   d. Initialize reduction factors of section plates and the rest of plate parameters necessary for obtaining the plate class.
   e. If necessary for the type of check (check for buckling), calculate the critical forces and moments of the section for buckling: elastic critical forces for the XY and XZ planes and elastic critical moment for lateral-torsional buckling. (See section: Calculation of critical forces and moments).
   f. Obtain internal forces and moments (T_d, P_d, $V_{y.d}$, $V_{z.d}$, $M_{x.d}$, $M_{y.d}$, $M_{z.d}$ within the section.
   g. Specific section checking according to the type of external load. The specific check includes:
      1. If necessary, selecting the forces and moments considered for the determination of the section class and used for the checking process.
      2. Obtaining the cross-section class and calculating the effective section properties (See Section: General Processing of Sections).
      3. Checking the cross-section according to the external load and its class by calculating the following criteria*: Crt_TOT, Crt_N, Crt_Mx* and *Crt_My*.
   h. Recording the results.

7.6.5 Section Class and Reduction Factors Calculation

Sections, according to IS 800:2007, are made up by plates. These plates can be classified according to:

Plate function: webs and flanges in Y and Z axis, according to the considered relevant axis of bending.

Plate union condition: internal plates or outstand plates.

For sections included in the program libraries, the information above is defined for each plate. CivilFEM classifies plates as flanges or webs according to their axis and provides the plate union condition for each end. Ends can be classified as fixed or free (a fixed end is connected to another plate and free end is not).

For checking the structure for safety, IS 800:2007 classifies sections as one of four possible classes:

Class 1 (Plastic)	Cross-sections, which can develop plastic hinges and have the rotation capacity required for failure of the structure by formation of plastic mechanism.
Class 2 (Compact)	Cross-sections, which can develop plastic moment of resistance, but have inadequate plastic hinge rotation capacity for formation of plastic mechanism, due to local buckling.
Class 3 (Semi-Compact)	Cross-sections, in which the extreme fiber in compression can reach yield stress, but cannot develop the plastic moment of resistance, due to local buckling.
Class 4 (Slender)	Cross-sections in which the elements buckle locally even before reaching yield stress.

The cross-section class is the highest (least favorable) class of all of its elements: flanges and webs (plates). First, the class of each plate is determined according to the limits of IS 800:2007. The plate class depends on the following:

- The geometric width to thickness ratio with the plate width properly corrected according to the plate and shape type.

GeomRat = Corrected_Width / thickness

The width correction consists of subtracting the zone that does not contribute to buckling resistance in the fixed ends. This zone depends on the shape type of the section. Usually, the radii of the fillet in hot rolled shapes or the weld throats in welded shapes determine the deduction zone. The values of the corrected width that CivilFEM uses for each shape type include:

- **Welded Shapes:**

Double T section:

Internal webs or flanges:

Corrected width = d

d Web free depth

Outstand flanges:

Corrected width $\frac{B}{2} - \frac{T_w}{2} - r_1$

Where:

B Flanges width

T_w Web thickness

r_1 Radius of fillet

T section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width = $\frac{B}{2}$

C section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width $B - T_w - r_1$

L section:

Corrected width = $\sqrt{I_1^2 + I_2^2}$

I_1, I_2 Angle flange width

Box section:

Internal webs:

Corrected width = H

H Height

Internal flanges:

Corrected width = $B = 2 \cdot T_W$

T_w Web thickness

Circular hollow section

Corrected width = H

- **Rolled Shapes:**

Double T section:

Internal webs or flanges:

Corrected width = d

d Web free depth

Outstand flanges:

Corrected width = $\frac{B}{2}$

B Flanges width

T Section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width = $\frac{B}{2}$

C Section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width = B

L Section:

Corrected width = $\sqrt{I_1^2 + I_2^2}$

I_1, I_2 Angle flange width

Box section:

Internal webs:

Corrected width = d

Internal flanges:

Corrected width = $B - 3 \cdot T_f$

T_f Flanges thickness

Pipe section:

Corrected width = H

The limit listed below for width to thickness ratio. This limit depends on the material parameter ε and the normal stress distribution in the plate section. The latter value is given by the following parameters: α, ψ, and k_0, and the plate type, internal or outstand; the outstand case depends on if the free end is under tension or compression.

Limit (class) = $f(\varepsilon, \alpha, \Psi, k_0)$

$$\varepsilon = \sqrt{250/f_y} \ \left(f_y \text{ in N/mm}^2\right)$$

where:

α Compressed length / total length

ψ σ_2/σ_1

k_0 Buckling factor

σ_1 The higher stress in the plate ends.

σ_2 The lower stress in the plate ends.

A linear stress distribution on the plate is assumed.

The procedure to determine the section class is as follows:

Obtain stresses at first plate ends from the stresses applied on the section, properly filtered according to the check type requested by the user.

Calculate the parameters: ⍰, ⍰ and k_0
For internal plates:

$0 > \Psi > -1$	$k_0 = 7.81 - 6.29 \cdot \Psi + 9.78 \cdot \Psi^2$
$-1 \geq \Psi > -2$	$k_0 = 5.98 \cdot (1 - \Psi)^2$
$\Psi \leq -2$	k_0 = infinite

For outstand plates with an absolute value of the stress at the free end greater than the corresponding value at the fixed end:

For $1 \geq \Psi \geq -1$

$$k_0 = 0.57 - 0.21 \cdot \Psi + 0.07 \cdot \Psi^2$$

For $-1 > \Psi$

k_0 = infinite

For outstand plates with an absolute value of the stress at the free end lower than the corresponding value at the fixed end:

For $1 \geq \Psi \geq 0$

$$k_0 = \frac{0.578}{\Psi + 0.34}$$

For $0 > \Psi \geq -1$

$$k_0 = 1.7 - 5 \cdot \Psi + 17.1 \cdot \Psi^2$$

For $-1 > \Psi$

k_0 = infinite

Cases in which k_0 = *infinite* are not included in IS 800:2007. With these cases, the plate is considered to be practically in tension and it will not be necessary to determine the class. These cases have been included in the program to avoid errors, and the value k_0=*infinite* has been adopted because the resultant plate class is 1 and the plate reduction factor is $\rho = 1$

(the same values as if the whole plate was in tension). The reduction factor is used later in the effective section calculation.

- Obtain the limiting proportions as functions of: □, □ and k_0 and the plate characteristics (internal, outstand: free end in compression or tension).

Internal plates:

$$\text{Limit}(1) = 396\ \varepsilon/(13\alpha - 1)$$ for □□< 0.5

$$\text{Limit}(1) = 36\ \varepsilon/\alpha$$ for □□< 0.5

$$\text{Limit}(2) = 456\ \varepsilon/(13\alpha - 1)$$ for □□≥ 0.5

$$\text{Limit}(2) = 41.5\ \varepsilon/\alpha$$ for □□<0.5

$$\text{Limit}(3) = 42\ \varepsilon/(0.67 + 0.33\Psi)$$ for □□> -1

$$\text{Limit}(3) = 62\ \varepsilon\ (1 - \Psi)\sqrt{(-\Psi)}$$ for □□□ -1

Outstand plates, free end in compression:

$$\text{Limit}(1) = 9\ \varepsilon/\alpha$$

$$\text{Limit}(2) = 10\ \varepsilon/\alpha$$

$$\text{Limit}(3) = 21\ \varepsilon/\sqrt{k_0}$$

Outstand plates, free end in tension:

$$\text{Limit}(1) = \frac{9\ \varepsilon}{\alpha\sqrt{\alpha}}$$

$$\text{Limit}(2) = \frac{10\ \varepsilon}{\alpha\sqrt{\alpha}}$$

$$\text{Limit}(3) = 21\ \varepsilon/\sqrt{k_0}$$

Above is the general equation used by the program to obtain the limiting proportions for determining plate classes. In addition, plates of IS 800:2007 may be checked according to special cases.

For example:

In sections totally compressed:

□ = 1; □ = 1 for all plates

In sections under pure bending:

□ = 0.5; □= -1 for the web

□ = 1; □ = 1 for compressed flanges

Obtain the plate class:

If		GeomRat	< Limit(1)	Plate Class = 1
If	Limit(1) □	GeomRat	< Limit(2)	Plate Class = 2
If	Limit(2) □	GeomRat	< Limit(3)	Plate Class = 3
If	Limit(3) □	GeomRat		Plate Class = 4

Repeat these steps (1,2,3,4) for each section plate.

Assign of the highest class of the plates to the entire section.
In tubular sections, the section class is directly determined as if it were a unique plate, with GeomRat and the Limits calculated as follows:

GeomRat = outer diameter/ thickness.

$$\text{Limit}(1) = 50\ \varepsilon^2$$

$$\text{Limit}(2) = 70\ \varepsilon^2$$

$$\text{Limit}(3) = 90\ \varepsilon^2$$

For class 4 sections, the section resistance is reduced, using the effective width method.

For each section plate, the effective lengths at both ends of the plate and the reduction factors □$_1$ and □$_2$ are calculated. These factors relate the length of the effective zone at each plate end to its width.

Effective_length_end1 = plate_width*□$_{□}$

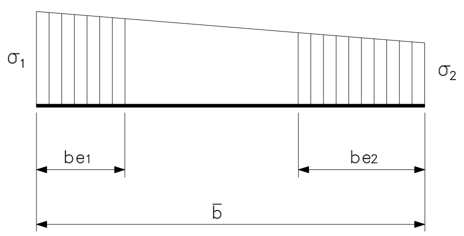
Effective_length_end 2 = plate_width*□$_{□}$

The following formula from IS 800:2007 has been implemented for this process:

$$\Psi = \sigma_2 \sigma_1$$

1. Internal plates:

For $0 \leq \Psi \leq 1$ (Both ends compressed)



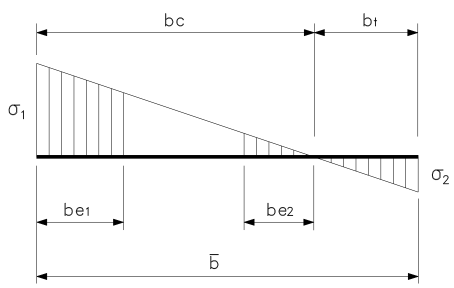
Internal plates

$$b_{eff} = \rho \bar{b}$$
$$b_{e1} = 2b_{eff}/(5 - \Psi)$$
$$b_{e2} = b_{eff} - b_{e1}$$
$$\rho_1 = \frac{b_{e1}}{plate\_width}$$
$$\rho_2 = \frac{b_{e2}}{plate\_width}$$

$\bar{b}$ = corrected plate width

plate_width = real plate width

For $\Psi < 0$ (end 1 in compression and end 2 in tension)



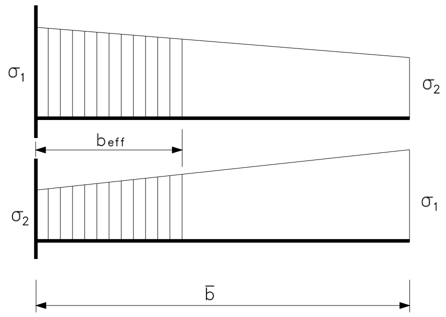
b_{eff}=ρb_c=$\rho\bar{b}/(1-\Psi)$

$$b_{e1} = 0.4\ b_{eff}$$
$$b_{e2} = 0.6\ b_{eff}$$
$$\rho_1 = \frac{b_{e1}}{plate\_width}$$

$$\rho_2 = \frac{b_{e2} + bt}{plate\_width}$$

2. Outstand plates:

For $0 \le \Psi \le 1$ (Both ends in compression: end 1 fixed, end 2 free)

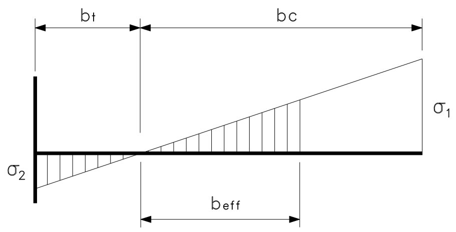


$$b_{eff} = \rho\bar{b}$$

$$\rho_1 = \frac{b_{e1}}{plate\_width}$$

$$\rho_2 = 0$$

For $\Psi < 0$ (end 1 fixed and in tension, end 2 free and in compression)

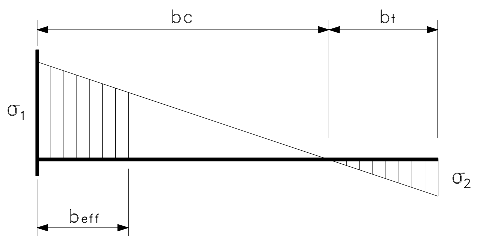


$b_{eff} = \rho b_c = \rho c/(1 - \Psi)$

$$\rho_1 = \frac{b_{eff} + bt}{plate\_width}$$

$$\rho_2 = 0$$

For $\Psi < 0$ (end 1 fixed and in compression, end 2 free and in tension)



çb_{eff} = ρb_c= $\rho c/(1-\Psi)$

$$\rho_1 = \frac{b_{eff}}{plate\_width}$$

$$\rho_2 = \frac{b_t}{plate\_width}$$

If end 2 is the fixed end, the values ρ_1 and ρ_2 are switched.

The global reduction factor ρ is obtained by as follows:

For internal compression elements

For $\bar{\lambda}_p > 0.673$

$$\rho = \frac{\bar{\lambda}_p - 0.055(3+\Psi)}{\bar{\lambda}_p^2}$$

For $\bar{\lambda}_p \leq 0.673$

$$\rho = 1$$

For outstands compression elements:

For $\bar{\lambda}_p > 0.748$

$$\rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2}$$

For $\bar{\lambda}_p \leq 0.748$

$$\rho = 1$$

$\bar{\lambda}_p$ is defined as the plate slenderness given by:

$$\bar{\lambda}_p = \frac{\bar{b}/t}{28.4\varepsilon\sqrt{k_0}}$$

where:

$\bar{b}$ = corrected plate width

t = relevant thickness

⍰⍰⍰ = material parameter

k_0 = buckling factor

To determine effective section properties, three steps are followed:

- *Effective widths of flanges* are calculated from factors ⍰⍰ and ⍰⍰⍰these factors are determined from the gross section properties. As a result, an intermediate section is obtained with reductions taken in the flanges only.

- The resultant section properties are obtained and factors⍰⍰⍰ and ⍰ are calculated again.

- *Effective widths of webs* are calculated so that the finalized effective section is determined. Finally, the section properties are recalculated once more.
The recalculated section properties are included in the effective section data table.
Checking can be accomplished with the gross, net or effective section properties, according to the section class and checking type.

Each checking type follows a specific procedure that will be explained in the following sections.

7.6.6 Checking of Members in Axial Tension

Corresponds to chapter 6 in IS-800-2007.

- Forces and moments selection.
The forces and moments considered for this checking type are:
T = FX Design value of the axial force (positive if tensile, element not processed if compressive).

- Class determination.

- Criteria calculation.
For members under axial tension, the general criterion Crt_TOT is checked at each section.

$$T \leq T_d \rightarrow \text{Crt\_TOT} = \frac{T}{T_d} \leq 1$$

Where T_d is the design strength of the member.

If we only take into account the design strength due to yielding of gross section (article 6.2, IS800:2007):

$$T_d = T_{dg} = A_g f_y / \gamma_{m0}$$

If we take into account the design strength in tension of a plate, ,as governed by rupture of net cross-sectional area:

$$T_d = T_{dn} = 0.9 A_n f_u / \gamma_{m1}$$

A_n is the net cross-sectional area and it will be calculated as $A_g - A_{holes}$

A_{holes} should be included by the user according to the IS 800:2007

$$T_d = \min(T_{dg}, T_{dn})$$

- Output results are written in the CivilFEM results file as an alternative. Checking results: criteria and variables are described in the following table:

Checking of Members in Axial Tension

Result	**Concepts**	**Description**
T	T	Tension Force.
TD	Td	Design strength of the member.
TDG	Tdg	Design strength due to yielding of gross section.
TDN	Tdn	Design strength in tension of a plate, as governed by rupture of net cross-sectional area.
CRT_TOT	T/T_d	Global criterion.

7.6.7 Checking of Members under Bending Moment

Corresponds to chapter 8 in IS-800-2007.

- Forces and moments selection.
The forces and moments considered for this checking type are:

$M_d = My \; or \; Mz$ Design value of the bending moment along the relevant axis for bending. Represented as M_d in IS-800-2007.

- Class definition and effective section properties calculation.
The section class is determined by the general processing of the section with the previously selected forces and moments if the selected option is partial or with all the forces and moments if the selected option is full. The entire calculation process is accomplished with the gross section properties.

- Criteria calculation.
For members subjected to a bending moment in the absence of shear force, the following condition is checked at each section:

$$|M_d| \leq M_{c.Rd} \rightarrow \text{Crt\_TOT} = \text{Crt\_My} = \left|\frac{M_d}{M_{c.Rd}}\right| \leq 1$$

where:

M_d =design value of the bending moment

$M_{c.Rd}$ =design moment resistance of the cross-section

Class 1 or 2 cross-sections:

$$M_{c.Rd} = W_{pl} \cdot f_y / Y_{M0}$$

Class 3 cross sections:

$$M_{c.Rd} = W_{el} \cdot f_y / Y_{M0}$$

Class 4 cross sections:

$$M_{c.Rd} = E_{eff} \cdot f_y / Y_{M0}$$

- Output results are written in the CivilFEM results file as an alternative. Checking results: criteria and variables are described in the following table.

Checking of Members under Bending Moment

Result	**Concepts**	**Description**
MED	M_{Ed}	Design value of the bending moment.
MCRD	$M_{c.Rd}$	Design moment resistance of the cross-section.
CRT_M	$M_d/M_{c.Rd}$	Bending criterion.

CRT_TOT	$M_d/M_{c.Rd}$	IS 800:2007 global criterion.
CLASS		Section Class.
W	W_{el}, W_{pl}, W_{eff}	Used section modulus (Elastic, Plastic or Effective).

7.6.8 Checking of Members under Shear Force

Corresponds to chapter 8.4 in IS-800-2007.

- Forces and moments selection.
The forces and moments considered for this checking type are:

$V_d = Fz \text{ or } Fy$ Design value of the shear force perpendicular to the relevant axis of bending.

- Class definition and effective section properties calculation.
For this checking type, the section class is always 1 and the effective section is the gross section.

- Criteria calculation.
With members under shear force, the following condition is checked at each section:

$$|V_d| \leq V_{Pl.Rd} \rightarrow \text{CRT\_TOT} = \text{Crt\_S} = \left|\frac{V_d}{V_{Pl.Rd}}\right| \leq 1$$

where:

V_d design value of the shear force

$V_{Pl.Rd}$ design plastic shear resistance: $V_{Pl.Rd} = A_v\left(f_y/\sqrt{3}\right)/Y_{M0}$

A_v shear area.

IS800-07 specifies additional cases for the calculation of A_v:

- I and channel sections:

$$A_v = ht_w(\textit{ROLLED SECTIONS})$$

$$A_v = dt_w(\textit{ROLLED SECTIONS})$$

- Rectangular hollow sections of uniform thickness

  Loaded parallel to depth (h) — A h / (b + h)

  Loaded parallel to width (b) — A b / (b + h)

- Circular: 2A/π

- Output results are written in the CivilFEM results file as an alternative. Checking results: criteria and variables are described in the following table.

Checking of Members under Shear Force

Result	Concepts	Description
VED	V_{Ed}	Design value of the shear force.
VPLRD	$V_{pl.Rd}$	Design plastic shear resistance.
CRT_S	$V_d/V_{pl.Rd}$	Shear criterion.
CRT_TOT	$V_d/V_{pl.Rd}$	IS 800:2007 global criterion.
CLASS		Section Class.
S_AREA	A_v	Shear area.

7.6.9 Checking of Members under Bending Moment and Shear Force

Corresponds to chapter 9.2 in IS-800-2007.

- Forces and moments selection.

The forces and moments considered for this checking type are:

$V_d = Fz\ or\ Fy$ Design value of the shear force perpendicular to the relevant axis of bending.

$M_d = My\ or\ Mz$ Design value of the bending moment along the relevant axis of bending.

- Class definition and effective section properties calculation.

The section class is determined by the general processing of the sections with the previously selected forces and moments if the selected option is partial or with all the forces and moments if the selected option is full. The entire calculation is accomplished with gross section properties.

- Criteria calculation.

For members subjected to bending moment and shear force, the following condition is checked at each section:

$$|M_d| \leq M_{V.Rd} \rightarrow \text{Crt\_TOT} = \text{Crt\_BS} = \left|\frac{M_d}{M_{V.Rd}}\right| \leq 1$$

Where:

$M_{V.Rd} =$ design resistance moment of the cross-section, reduced by the presence of shear.

The reduction for shear is applied if the design value of the shear force exceeds 50% of the design plastic shear resistance of the cross-section; written explicitly as:

$$V_d > 0.5\ V_{pl.Rd}$$

The design resistance moment is obtained as follows:

- For double T cross-sections with equal flanges, bending about the major axis:

$$M_{V.Rd}=\left(W_{pl} - \frac{\rho A_v^2}{4t_w}\right) {f_y}/{Y_{M0}}$$

$$\rho = \left(\frac{2V_d}{V_{pl.Rd}} - 1\right)^2$$

$$A_w = h_w t_w$$

- For other cases the yield strength is reduced as follows:

$$f_y= f_y(1 - \rho)$$

Note: This reduction of the yield strength fy is applied to the entire section. IS 800:2007 only requires the reduction to be applied to the shear area, and therefore, it is a conservative simplification.

For both cases, $M_{V.Rd}$ is the smaller value of either $M_{V.Rd}$ or $M_{C.Rd}$.

$M_{C.Rd}$ is the design moment resistance of the cross-section, calculated according to the class.

- Output results are written in the CivilFEM results file as an alternative. Checking results: criteria and variables are described in the following table.

Checking of Members under Bending Moment and Shear Force

Result	**Concepts**	**Description**
MED	M_{Ed}	Design value of the bending moment.
VED	V_{Ed}	Design value of the shear force.
MVRD	$M_{v.Rd}$	Reduced design resistance moment of the cross-section.

CRT_BS	$M_d/M_{v.Rd}$	Bending and Shear criterion.
CRT_TOT	$M_d/M_{v.Rd}$	IS 800:2007 global criterion.
CLASS		Section Class.
S_AREA	A_v	Shear area.
W	$W_{el,}W_{pl,}W_{eff}$	Used section modulus (Elastic, Plastic or Effective).
VPLRD	$V_{pl.Rd}$	Design plastic shear resistance.
RHO	ρ	Reduction factor.

7.6.10 Checking of Members under Bending Moment + Axial Force and Bi-axial Bending + Axial Force

Corresponds to chapter 9.3 in IS-800-2007.

- Forces and moments selection.

The forces and moments considered for this checking type are:

$N_d = FX$ Design value of the axial force..

$M_{y.d} = My \text{ or } Mz$ Design value of the bending moment along the relevant axis of bending.

$M_{z.d} = Mz \text{ or } My$ Design value of the bending moment about the secondary axis of bending.

- Class definition and effective section properties calculation.

The section class is determined by the general processing of the sections with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. These calculations are accomplished with the gross section properties.

- Criteria calculation.

For members subjected to bi-axial bending and in absence of shear force, the following conditions at each section are checked:

Class 1 and 2 sections:

$$\left(\frac{M_{y.d}}{M_{Ny.Rd}}\right)^{\alpha} + \left(\frac{M_{z.d}}{M_{Nz.Rd}}\right)^{\beta} \leq 1$$

This condition is equivalent to:

$$\text{Crt\_My} = (\text{Crt\_My})^{\alpha} + (\text{Crt\_Mz})^{\beta} \leq 1$$

$$\text{Crt\_My} = \left(\frac{M_{y.d}}{M_{Ny.Rd}}\right)$$

$$\text{Crt\_Mz} = \left(\frac{M_{z.d}}{M_{Nz.Rd}}\right)$$

Where $M_{Ny.Rd}$ and $M_{Nz.Rd}$ are the design moment resistance of the cross-section, reduced by the presence of the axial force:

$$M_{Ny.Rd} = My_{pl.Rd}\left[1 - \left({N_d}/{N_{pl.Rd}}\right)^2\right]$$

$$M_{Nz.Rd} = Mz_{pl.Rd}\left[1 - \left({N_d}/{N_{pl.Rd}}\right)^2\right]$$

Where α and β are constants, which may take the following values:

For I and H sections:

$$\alpha = 2 \quad \text{and} \quad \beta = 5n \qquad \beta \geq 1$$

For circular tubes:

$$\alpha = 2 \quad \text{and} \quad \beta = 2$$

For rectangular hollow sections:

$$\alpha = \beta = \frac{1.66}{1-1.13n^2} \qquad \text{but} \qquad \alpha = \beta \leq 6$$

For solid rectangles and plates (the rest of sections):

$$\alpha = \beta = 1.73 + 1.8n^3$$

$$n = \left(\frac{N_d}{N_{pl.Rd}}\right)$$

In absence of $M_{z.d}$, the previous check can be reduced to:

$$\left(\frac{M_{y.d}}{M_{Ny.Rd}}\right) \leq 1$$

Condition equivalent to:

$$Crt\_TOT = Crt\_My = \left(\frac{M_{y.d}}{M_{Ny.Rd}}\right)$$

Class 3 sections (without holes for fasteners):

$$\left(\frac{N_d}{Af_{yd}}\right) + \left(\frac{M_{y.d}}{W_{el.y}f_{yd}}\right) + \left(\frac{M_{z.d}}{W_{el.z}f_{yd}}\right) \leq 1$$

Condition equivalent to:

$$Crt\_TOT = Crt\_N + Crt\_My + Crt\_Mz \leq 1$$

$$Crt\_N = \left(\frac{N_d}{Af_{yd}}\right)$$

$$Crt\_My = \left(\frac{M_{y.d}}{W_{el.y}f_{yd}}\right)$$

$$Crt\_Mz = \left(\frac{M_{z.d}}{M_{el.y}f_{yd}}\right) f_{yd} = f_y/\gamma_{M0}$$

Where $W_{el.y}$ is the elastic resistant modulus about the y axis and $W_{el.z}$ is the elastic resistant modulus about the z axis.

In absence of $M_{z.d}$, the above criterion becomes:

$$\left(\frac{N_d}{Af_{yd}}\right) + \left(\frac{M_{y.d}}{W_{el.y}f_{yd}}\right) \leq 1$$

Which is equivalent to:

$$Crt\_TOT = Crt\_N + Crt\_My \leq 1$$

$$Crt\_N = \left(\frac{N_d}{Af_{yd}}\right)$$

$$Crt\_My = \left(\frac{M_{y.d}}{W_{el.y}f_{yd}}\right)$$

Class 4 sections:

$$\left(\frac{N_d}{A_{eff}f_{yd}}\right) + \left(\frac{M_{y.d} + N_d e_{Ny}}{W_{eff.y}f_{yd}}\right) + \left(\frac{M_{z.d} + N_d e_{Ny}}{W_{eff.z}f_{yd}}\right) \leq 1$$

Condition equivalent to:

$$Crt\_TOT = Crt\_N + Crt\_My + Crt\_Mz \leq 1$$

$$Crt\_N = \left(\frac{N_d}{A_{eff}f_{yd}}\right)$$

$$\text{Crt\_My} = \left(\frac{M_{y.d} + N_d e_{Ny}}{W_{eff.y} f_{yd}}\right)$$

$$\text{Crt\_Mz} = \left(\frac{M_{z.d} + N_d e_{Ny}}{W_{eff.z} f_{yd}}\right)$$

Where:

A_{eff}	effective area of the cross-section
$W_{eff.y}$	effective section modulus of the cross-section when subjected to a moment about the y axis
$W_{eff.z}$	effective section modulus of the cross-section when subjected to a moment about the z axis
e_{Ny}	shift of the center of gravity along the y axis
e_{Nz}	shift of the center of gravity along the z axis

Without $M_{z.d}$, the above criterion becomes:

$$\left(\frac{N_d}{A_{eff} f_{yd}}\right) + \left(\frac{M_{y.d} + N_d e_{Ny}}{W_{eff.y} f_{yd}}\right) + \left(\frac{N_d e_{Ny}}{W_{eff.z} f_{yd}}\right) \leq 1$$

which is equivalent to:

$$\text{Crt\_TOT} = \text{Crt\_N} + \text{Crt\_My} + \text{Crt\_Mz} \leq 1$$

$$\text{Crt\_N} = \left(\frac{N_d}{A_{eff} f_{yd}}\right)$$

$$\text{Crt\_My} = \left(\frac{M_{y.d} + N_d e_{Ny}}{W_{eff.y} f_{yd}}\right)$$

$$\text{Crt\_Mz} = \left(\frac{N_d e_{Ny}}{W_{eff.z} f_{yd}}\right)$$

- Output results are written in the CivilFEM results file as an alternative. Checking results: criteria and variables are described in the following table.

Checking of Members under Bending Moment + Axial Force and Bi-axial Bending + Axial Force

Result	**Concepts**	**Description**

Result	Concepts	Description
NED	N_{Ed}	Design value of the axial force.
MYED	$M_{y.Ed}$	Design value of the bending moment about Y axis.
MZED	$M_{z.Ed}$	Design value of the bending moment about Z axis.
NCRD	$A \cdot f_{yd}$, $A_{eff} \cdot f_{yd}$	Design compression resistance of the cross-section
MNYRD	$M_{Ny \cdot Rd}$, $W_{el.y} \cdot f_{yd}$, $W_{eff \cdot y} \cdot f_{yd}$	Reduced design moment resistance of the cross-section about Y axis
MNZRD	$M_{Nz \cdot Rd}$, $W_{el.z} \cdot f_{yd}$, $W_{eff \cdot Z} \cdot f_{yd}$	Reduced design moment resistance of the cross-section about Z axis
CRT_N	N_d / N_{cRd}	Axial criterion
CRT_MY	M_{yd} / M_{NyRd}	Bending criterion along Y
CRT_MZ	M_{zd} / M_{NzRd}	Bending criterion along Z
ALPHA	α	Alpha constant
BETA	β	Beta constant
CRT_TOT	Crt_tot ≤ 1	IS 800:2007 global criterion
CLASS		Section Class
AREA	A, A_{eff}	Area of the section utilized (Gross or Effective)
WY	$W_{el.y}, W_{pl.y}, W_{eff.y}$	Used section Y modulus (Elastic, Plastic or Effective)
WZ	$W_{el.z}, W_{pl.z}, W_{eff.z}$	Used section Z modulus (Elastic, Plastic or Effective)
SIGXED	$\sigma_{X.Ed}$	Maximum longitudinal stress
ENY	e_{Ny}	Shift of the Z axis in Y direction
ENZ	e_{Nz}	Shift of the Y axis in Z direction
USE_MY	$M_{y.d} + N_d \cdot e_{Ny}$	Modified design value of the bending moment about Y axis
USE_MZ	$M_{z.d} + N_d \cdot e_{Nz}$	Modified design value of the bending moment about Z axis
PARM_N	n	Parameter n

7.6.11 Checking for Buckling of Compression Members

Corresponds to chapter 8 in IS-800-2007.

- Forces and moments selection.

The forces and moments considered in this checking type are:

$N_d = FX$ Design value of the axial force (positive if compressive, otherwise element is not processed). Represented as P_d .

- Class definition and effective section properties calculation.

The section class is determined by the sections general processing with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. The entire calculation is accomplished with the gross section properties.

- Criteria calculation.

When checking the buckling of compression members, the criterion is given by:

$$N_d \leq N_{b.Rd} \rightarrow Crt\_TOT = Crt\_CB = \frac{N_d}{N_{b.Rd}} \leq 1$$

where:

$N_{b.Rd}$ Design buckling resistance. $N_{b.Rd} = \chi\,\beta\,A\,f_y/\gamma_{M1}$

β = 1 for class 1, 2 or 3 sections.

$\beta = A_{eff}/A$ for class 4 sections.

? Reduction factor for the relevant buckling mode, the program does not consider the torsional or the lateral-torsional buckling.

The χ calculation in members of constant cross-section may be determined from:

$$\chi = \frac{1}{\Phi + \left(\Phi^2 - \bar{\lambda}^2\right)^{1/2}} \leq 1$$

$$\Phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

where α is an imperfection factor that depends on the buckling curve. This curve depends on the cross-section type, producing the following values for α:

Imperfection factor α for IS-800-2007

Section type	Limits	Buckling axis	Buckling curve	α
Rolled I	h/b>1.2 and tf≤40mm	y – y	a	0.21
Rolled I	h/b>1.2 and tf≤40mm	z – z	b	0.34
Rolled I	h/b>1.2 and 40mm<t≤100mm	y – y	b	0.34
Rolled I	h/b>1.2 and 40mm<tf≤100mm	z – z	c	0.49
Rolled I	h/b≤1.2 and tf≤100mm	y – y	b	0.34
Rolled I	h/b≤1.2 and tf≤100mm	z – z	c	0.49
Rolled I	tf>100mm	y – y	d	0.76
Rolled I	tf>100mm	z – z	d	0.76
Welded I	tf≤40mm	y – y	b	0.34
Welded I	tf≤40mm	z – z	c	0.49
Welded I	tf >40mm	y – y	c	0.49
Welded I	tf >40mm	z – z	d	0.76
Rolled box and pipe	-	Any	a	0.21
Welded box and pipe (Using fyb)	-	Any	b	0.34
Welded box in other case	-	Any	b	0.34
Welded box	b/tf <30	y – y	c	0.49
Welded box	h/tw <30	z – z	c	0.49
U, L and T	-	Any	c	0.49

$$\bar{\lambda} = \left[\beta_A A f_y / N_{cr}\right]^{1/2}$$

Where N_{cr} is the elastic critical force for the relevant buckling mode. (See section for Critical Forces and Moments Calculation).

The elastic critical axial forces are calculated in the planes XY (Ncr_{xy}) and XZ (Ncr_{xz}) and the corresponding values of χ_{xy} and χ_{xz}, taking the smaller one as the final value for χ.

$$\chi = \min(\chi_{xy}, \chi_{xz})$$

- Output results are written in the CivilFEM results file as an alternative. Checking results: criteria and variables are described in the following table.

Checking for Buckling of Compression Members

Result	Concepts	Description
NED	N_{Ed}	Design value of the compressive force.
NBRD	$N_{b.Rd}$	Design buckling resistance of a compressed member.
CRT_CB	$N_d/N_{b.Rd}$	Compression buckling criterion.
CRT_TOT	$N_d/N_{b.Rd}$	IS 800:2007 global criterion.
CHI	$Min\{\chi_y, \chi_z\}$	Reduction factor for the relevant buckling mode.
BETA_A	β_A	Ratio of the used area to gross area.
AREA	A	Area of the gross section.
CHI_Y	χ_y	Reduction factor for the relevant My buckling mode.
CHI_Z	χ_z	Reduction factor for the relevant Mz buckling mode.
CLASS		Section Class.
PHI_Y	Φ_y	Parameter Phi for bending My.
PHI_Z	Φ_z	Parameter Phi for bending Mz.
LAM_Y	λ_y	Non-dimensional reduced slenderness for bending My.
LAM_Z	λ_z	Non-dimensional reduced slenderness for bending Mz.
NCR_Y	N_{cr}	Elastic critical force for the relevant My buckling mode.
NCR_Z	N_{cr}	Elastic critical force for the relevant Mz buckling mode.
ALP_Y	α_y	Imperfection factor for bending My.

Result	Concepts	Description
ALP_Z	α_z	Imperfection factor for bending Mz.

7.6.12 Checking Lateral-Torsional Buckling of Members Subjected to Combined Bending and Axial Tension

Corresponds to chapter 9.3 in IS-800-2007.

- Forces and moments selection.

The forces and moments considered for this checking type are:

$N_{Sd} = FX$ Design value of the axial force (positive if tensile, otherwise element not processed if compressive).

$M_{Sd} = My\ or\ Mz$ Design value of the bending moment about the relevant bending axis.

- Class definition and effective section properties calculation.

The section class is determined by the general processing of sections with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. The entire calculation is accomplished with the gross section properties.

- Criteria calculation.

With checking lateral-torsional buckling of members subjected to combined bending and axial tension, the value of the axial force is multiplied by a reduction factor Ψ_{vec} in order to consider the axial force and bending moment as a vector magnitude. The value of Ψ_{vec} depends on the country where the code will be applied. That factor is introduced as a property at member level, and typically its value is equal to: Ψ_{vec} = 0.8

The stress in the extreme compression fiber is calculated as follows:

$$\sigma_{com.ed} = \frac{M_{Sd}}{W_{com}} - \frac{\Psi_{vec} N_{t.Sd}}{A}$$

Where W_{com} is the elastic section modulus for the extreme compression fiber and $N_{t.Sd}$ is the design value of the axial tension.

The verification equation is derived to:

$$M_{eff.Sd} < M_{b.Rd} \rightarrow \text{Crt\_TOT} = \text{Crt\_LT} = \frac{M_{eff.Sd}}{M_{b.Rd}} \leq 1$$

Where:

$$M_{eff.Sd} = W_{com}\sigma_{com.ed}$$

- Output results are written in the CivilFEM results file as an alternative. Checking results: criteria and variables are described in the following table.

Checking Lateral-Torsional Buckling of Members Subjected to Combined Bending and Axial Tension

Results	Concepts	Description
NTSD	$N_{t.Sd}$	Design value of the axial tension.
MSD	M_{Sd}	Design value of the bending moment.
MEFFSD	$M_{eff.Sd}$	Effective design internal moment.
MBRD	$M_{b.Rd}$	Buckling resistance moment of a laterally unrestrained beam.
CRT_LT	$M_{eff.Sd}/M_{b.Rd}$	Lateral-torsional buckling criterion.
CRT_TOT	$M_{eff.Sd}/M_{b.Rd}$	IS 800:2007 global criterion.
CLASS		Section Class.
WCOM	W_{com}	Elastic section modulus for the extreme compression fiber.
SCOMED	$\sigma_{Com.Ed}$	Net calculated stress in the extreme compression fiber.
CHI_LT	χ_{LT}	Reduction factor for lateral-torsional buckling.
BETA_W	β_w	Ratio of the used modulus to plastic modulus.
WPL	$W_{pl.y}$	Plastic modulus.
PHI_LT	Φ_{LT}	Parameter Phi for lateral-torsional buckling.
LAM_LT	λ_{LT}	Esbeltez adimensional reducida.
MCR	M_{cr}	Elastic critical moment for lateral-torsional buckling.
ALP_LT	α_{LT}	Non-dimensional reduced slenderness.

7.6.13 Checking for Lateral-Torsional Buckling of Members Subjected to Bending and Axial Compression

Corresponds to chapter 9.3 in IS-800-2007.

- Forces and moments selection.
The forces and moments considered in this checking type are:

$N_d = FX$	Design value of the axial compression (positive if compressive, otherwise element not processed if tensile).
$M_{yd} = My \; or \; Mz$	Design value of the bending moment about the relevant axis of bending.
$M_{zd} = Mz \; or \; My$	Design value of the bending moment about the secondary axis of bending.

- Class definition and effective section properties calculation.
The section class is determined by the general processing of sections with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. The entire calculation is accomplished with the gross section properties.

- Criteria calculation.

When checking the lateral-torsional buckling of members subjected to combined bending and axial compression, the criterion to satisfy is as follows:

$$\left|\frac{N_d}{N_{b.Rd}}\right| + \left|\frac{M_{y.d}}{M_{yb.Rd}}\right| + \left|\frac{M_{z.d}}{M_{zb.Rd}}\right| \leq 1$$

→ Crt_TOT = Crt_N + Crt_My + Crt_Mz ≤ 1

where:

Crt_TOT	IS 800:2007 global criterion.
$Crt\_N = \left|\frac{N_d}{N_{b.Rd}}\right|$	Axial criterion.
$Crt\_My = \left|\frac{M_{y.d}}{M_{yb.Rd}}\right|$	Bending criterion (principal axis).
$Crt\_Mz = \left|\frac{M_{z.d}}{M_{zb.Rd}}\right|$	Bending criterion (secondary axis).
$N_{b.Rd}$	Design buckling resistance for compression.
$M_{yb.Rd}$	Design buckling resistance moment (principal axis)
$M_{zb.Rd}$	Design buckling resistance moment (secondary axis).

The member resistances depend on the cross-section class and on the possibility that the lateral-torsional buckling is a potential failure mode for the structure.

Members with *class 1* and *2* cross-sections shall satisfy:

$$\frac{N_d}{\chi_{min} A f_y/\gamma_{M1}} + \frac{k_y M_{y.d}}{W_{Pl.y} f_y/\gamma_{M1}} + \frac{k_z M_{z.d}}{W_{Pl.z} f_y/\gamma_{M1}} \leq 1$$

where:

$$k_y = 1 - \frac{\mu_y \cdot N_d}{\chi_y \cdot A \cdot f_y} \leq 1.5$$

$$\mu_y = \bar{\lambda}_y\left(2\beta_{My} - 4\right) + \left[\frac{W_{pl.y} - W_{el.y}}{W_{el.y}}\right] \qquad \mu_y \leq 0.90$$

$$k_z = 1 - \frac{\mu_z N_d}{\chi_z A f_y} k_z \leq 1.5$$

$$\mu_z = \bar{\lambda}_z(2\beta_{Mz} - 4) + \left[\frac{W_{pl.z} - W_{elz}}{W_{el.z}}\right] \qquad \mu_z \leq 0.90$$

$$\chi_{min} = \min\left(\chi_y ; \chi_z\right)$$

Where:

χ_y and χ_z are the reduction factors defined at the section corresponding to Checking for Buckling of Compression Members.

β_{My} and β_{Mz} are equivalent uniform moment factors for flexural bending. These factors are entered as properties at member level. (See section Data at Member Level, factors BetaMy and BetaMz).

Members with Class 1 and 2 cross-sections for which lateral-torsional buckling is a potential failure mode shall satisfy:

$$\frac{N_d}{\chi_z A f_y/\gamma_{M1}} + \frac{k_{LT} M_{y.d}}{\chi_{LT} W_{pl.y} f_y/\gamma_{M1}} + \frac{k_z M_{z.d}}{W_{Pl.z} f_y/\gamma_{M1}} \leq 1$$

where:

$$k_{LT} = 1 - \frac{\mu_{LT} \cdot N_d}{\chi_z \cdot A \cdot f_y} \leq 1$$

$$\mu_{LT} = 0.15\,\bar{\lambda}\,C_{M.LT} - 0.15 \qquad \mu_{LT} \leq 0.90$$

where $C_{M.LT}$ is an equivalent uniform moment factor for lateral-torsional buckling. This factor, as the precedent factors CMy and CMz.

Members with *Class 3* cross-sections shall satisfy:

$$\frac{N_d}{\chi_{min}Af_y/\gamma_{M1}}+\frac{k_yM_{y.d}}{W_{el.y}f_y/\gamma_{M1}}+\frac{k_zM_{z.d}}{W_{el.z}F_y/\gamma_{M1}}\leq 1$$

where k_y, k_z and χ_{min} are as for Class 1 and 2 cross-sections.

$$\mu_y=\bar{\lambda}_y(2\beta_{My}-4)\qquad \mu_y\leq 0.90$$
$$\mu_z=\bar{\lambda}_z(2\beta_{Mz}-4)\qquad \mu_z\leq 0.90$$

Members with Class 3 cross-sections for which lateral-torsional buckling is a potential failure mode shall satisfy:

$$\frac{N_d}{\chi_zAf_y/\gamma_{M1}}+\frac{k_yM_{y.d}}{\chi_{LT}W_{el.y}f_y/\gamma_{M1}}+\frac{k_zM_{z.d}}{W_{el.z}F_y/\gamma_{M1}}\leq 1$$

Members with *Class 4* cross-sections shall satisfy:

$$\frac{N_d}{\chi_{min}Af_y/\gamma_{M1}}+\frac{k_y(M_{y.d}+N_de_{Ny})}{W_{eff.y}f_y/\gamma_{M1}}+\frac{k_z(M_{z.d}+N_de_{Nz})}{W_{el.z}F_y/\gamma_{M1}}\leq 1$$

where:

$k_{y,}k_z$ and χ_{min}	are the same as for class 1 and 2 cross-sections, but use the effective area A_{eff}, instead of the gross area A.
μ_y and μ_z	are the same as for class 3 cross-sections, but add the moment $N_d.e_N$that appears by the shift of the center of gravity in the effective cross-section, when determining C_{My} and C_{Mz}.
$A_{eff}, W_{eff.y}, W_{eff.z}, e_{N.y}, e_{Nz}$	are defined in the section corresponding to Checking of members under bending and axial force and bi-axial bending.

Members with Class 4 cross-sections for which lateral-torsional buckling is a potential failure mode shall satisfy:

$$\frac{N_d}{\chi_zAf_y/\gamma_{M1}}+\frac{k_{LT}(M_{y.d}+N_de_{Ny})}{\chi_{LT}W_{el.y}f_y/\gamma_{M1}}+\frac{k_z(M_{z.d}+N_de_{Ny})}{W_{el.z}F_y/\gamma_{M1}}\leq 1$$

where:

k_{LT} is similar to class 1 and 2 cross-sections, but uses the effective area A_{eff}, instead of the gross area A.

μ_{Lt} is similar to class 2 cross-sections, but adds the moment $N_d.e_N$ that appears by the shift of the center of gravity in the effective cross-section, when determining β_{MLT}.

Checking Parameters:

Class	A	W_y	W_z	α_y	α_z	$e_{N,y}$	$e_{N,z}e_{N,z}$
1	A	$W_{pl,y}$	$W_{pl,z}$	0.6	0.6	0	0
2	A	$W_{pl,y}$	$W_{pl,z}$	0.6	0.6	0	0
3	A	$W_{el,y}$	$W_{el,z}$	0.8	1	0	0
4	A_{eff}	$W_{ell,y}$	$W_{eff,z}$	0.8	1	Depending on members and stresses	Depending on members and stresses

Interaction Factors:

Class	Section type	k_y	k_z	k_{yLT}
1 y 2	I, H	$1+(\bar{\lambda}_y-0.2)\frac{N_{Ed}}{\chi_y N_{C,Rd}}$	$1+(\bar{\lambda}_z-0.6)\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	$1-\frac{0.1.\bar{\lambda}_z}{(C_{mLT}-0.25)}.\frac{N_{Ed}}{\chi_z N_{C,Rd}}.0.6+\bar{\lambda}_z$
	RHS		$1+(\bar{\lambda}_y-0.2)\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	
3 y 4	All sections	$1+0.6\bar{\lambda}_y\frac{N_{Ed}}{\chi_y N_{C,Rd}}$	$1+0.6\bar{\lambda}_z\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	$1-\frac{0.05.\bar{\lambda}_z}{(C_{mLT}-0.25)}.\frac{N_{Ed}}{\chi_z N_{C,Rd}}$

where:

λ_y y λ_z Limited slenderness values for y-y and z-z axes, less than 1.

$$N_{C,Rd}=A^*\frac{f_y}{\gamma_{M1}}$$

- Output results are written in the CivilFEM results file as an alternative. Checking results: criteria and variables are described in the following table.

Checking for Lateral-Torsional Buckling of Members Subjected to Bending and Axial Compression

Result	Concepts	Description
NED	N_{Ed}	Design value of the axial compression force.
MYED	$M_{y.Ed}$	Design value of the bending moment about Y axis.
MZED	$M_{z.Ed}$	Design value of the bending moment about Z axis.
NBRD1	$\chi_y \cdot A \cdot f_y/\gamma_{M1}$	Design compression resistance of the cross-section.
MYRD1	$\chi_{LT} \cdot W_y \cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Y axis.
MZRD1	$W_z \cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Z axis.
NBRD2	$\chi_z \cdot Af_y/\gamma_{M1}$	Design compression resistance of the cross-section.
MYRD2	$\chi_{LT} \cdot W_y \cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Y axis.
MZRD2	$W_z \cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Z axis.
K_Y	K_y	Parameter $K_{y.}$
K_Z	K_z	Parameter $K_{z.}$
K_LT	K_{LT}	Parameter $K_{LT.}$

Result	Concepts	Description
CRT_N1	N_{Ed}/N_{cRd1}	Axial criterion.
CRT_MY1	$K_y C_{my}\left(M_{y,Ed} + N_{Ed} \cdot e_{Ny}\right)/M_{yb,Rd1}$	Bending Y criterion.
CRT_MZ1	$\alpha_z \cdot K_z \cdot C_{mz}\ (M_{z,Ed} + N_{ed} e_{Nz})/M_{zb,Rd1}$	Bending Z criterion.
CRT_1	CRT_N1+CRT_MY1 +CRT_MZ1	Criterion 1
CRT_N2	N_{Ed}/N_{cRd2}	Axial criterion.
CRT_MY2	$KC_{my}\ (M_{y,Ed}+N_{Ed} \cdot e_{Ny})/\ M_{yRd2}$	Bending Y criterion. $K=K_{yLT}$ if torsion exists and if not present $K=\alpha_y K_y$
CRT_MZ2	$K_z \cdot C_{mz}\ (M_{z,Ed}+N_{Ed} \cdot e_{Nz})/\ M_{zRd2}$	Bending Z criterion.
CRT_2	CRT_N2+CRT_MY2 +CRT_MZ2	Criterion 2
CRT_TOT	Crt_tot ≤ 1	IS 800:2007 global criterion.
CLASS		Section Class.
CHIMIN	$Min\{\chi_y, \chi_z\}$	Reduction factor for the relevant buckling mode.
CHI_Y	χ_y	Reduction factor for the relevant My buckling mode.
CHI_Z	χ_z	Reduction factor for the relevant Mz buckling mode.
CHI_LT	χ_{LT}	Reduction factor for lateral-torsional buckling.
AREA	A, A_{eff}	Used area of the section (Gross or Effective).
WY	$W_{el.y}, W_{pl.y}, W_{eff.y}$	Used section Y modulus (Elastic, Plastic or Effective).
WZ	$W_{el.z}, W_{pl.z}, W_{eff.z}$	Used section Z modulus (Elastic, Plastic or Effective).
ENY	e_{Ny}	Shift of the Z axis in Y direction.

Result	Concepts	Description
ENZ	e_{Nz}	Shift of the Y axis in Z direction.
NCR_Y	N_{cr}	Elastic critical force for the relevant My buckling mode.
NCR_Z	N_{cr}	Elastic critical force for the relevant Mz buckling mode.
MCR	M_{cr}	Elastic critical moment for lateral-torsional buckling.
LAM_Y	λ_y	Non-dimensional reduced slenderness for bending My.
LAM_Z	λ_z	Non-dimensional reduced slenderness for bending Mz.
LAM_LT	λ_{LT}	Non-dimensional reduced slenderness for lateral-torsional buckling.

7.6.14 Critical Forces and Moments Calculation

The critical forces and moments Ncr_{xy}, Ncr_{xz} and Mcr, are needed for the different types of buckling checks. They are calculated based on the following formulation:

$$Ncr_{xy} = \frac{AE\pi^2}{\lambda_{xy}^2} = AE\left(\frac{\pi\ i_{xy}}{L_{xy}}\right)^2$$

$$Ncr_{xz} = \frac{AE\pi^2}{\lambda_{xz}^2} = AE\left(\frac{\pi\ i_{xz}}{L_{xz}}\right)^2$$

where:

Ncr_{xy}	Elastic critical axial force in plane XY.
Ncr_{xz}	Elastic critical axial force in plane XZ.
A	Gross area.
E	Elasticity modulus.
λ_{xy}	Member slenderness in plane XY.

λ_{xz}	Member slenderness in plane XZ.
i_{xy}	Radius of gyration of the member in plane XY.
i_{xz}	Radius of gyration of the member in plane XZ.
L_{xy}	Buckling length of member in plane XY.
L_{xz}	Buckling length of member in plane XZ.

The buckling length in both planes is the length between the ends restrained against lateral movement and it is obtained from the member properties, according to the following expressions:

$$L_{xy} = L \cdot Cfbuckxy$$

$$L_{xz} = L \cdot Cfbuckxz$$

where:

Cfbuckxy	Buckling factor in plane XY.
Cfbuckxz	Buckling factor in plane XZ.

For the calculation of the elastic critical moment for lateral-torsional buckling, Mcr, the following equation shall be used. This equation is *only valid for uniform symmetrical cross-sections about the minor axis* (Annex E, IS 800:2007). IS 800:2007 does not provide a method for calculating this moment in nonsymmetrical cross-sections or sections with other symmetry plane (angles, channel section, etc.).

$$Mcr = C_1 \frac{\pi^2 EI_z}{(kL)^2}\left\{\left[\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z} + \left[C_2 Z_g - C_3 Z_j\right]^2\right]^{1/2} - \left[C_2 Z_g - C_3 Z_j\right]\right\}$$

$$Z_j = Z_s - \frac{0.5}{I_y}\int_A (y^2 + z^2) z \, dA$$

where:

M_{cr}	Elastic critical moment for lateral-torsional buckling.
C_1, C_2, C_3	Factors depending on the loading and end restraint conditions.
k, k_w	Effective length factors.
E	Elasticity modulus.
I_y	Moment of inertia about the principal axis.

I_z	Moment of inertia about the minor axis.
L	Length of the member between end restraints.
G	Shear modulus.
Z_g	$Z_a - Z_s$
Z_a	Coordinate of the point of load application. ANSYS always considers that the load is applied at the center of gravity, therefore: $Z_a = 0$.
Z_s	Coordinate of the shear center.
A	Cross-section area.

Factors C and k are read from the properties at member level.

The integration of the previous equation is calculated as a summation extending to each plate. This calculation is accomplished for each plate according to its ends coordinates:

y_1,Z_1 and y_2,Z_2 and its thicknesses.

$$\int_A (y^2 + Z^2)\, z \; dA = \sum_{i=1}^{\text{nplates}} S_i{}^* \int_{L_i} (y^2 + z^2) z \; dl$$

where:

S_i = thickness of plate i

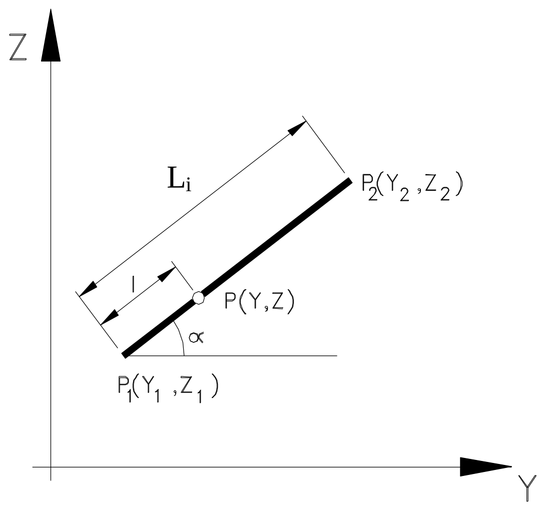
$$dA = S_i{}^* dl$$

$$y = y_1 + I^* \cos\alpha$$

$$z = z_1 + I^* \sin\alpha$$

$$\alpha = \text{arctang}\frac{z_2 - z_1}{y_2 - y_1}$$

$$L_i = \sqrt{(y_1 - y_2)^2 + (z_1 - z_2)^2} = \text{ plate width}$$


Z
Li
P2(Y2 ,Z2)
l
P(Y,Z)
α
P1(Y1 ,Z1)
Y

7.7. Steel Structures According to AASHTO LRFD (2012)

7.7.1. Checking Types

With CivilFEM it is possible to perform the following checking and analysis types:

- Tension	Section 6.8.2
- Flexure	6.12.2.2, A6
- Shear Force	6.10.9
- Flexure and axial force	6.9.2.2, 6.8.2.3
- Bending plus axial force	6.9.2.2, 6.8.2.3
- Compression members	6.9.4.1.2
- Compression members	6.9.4.1.3

7.7.2. Material Properties

For *AASHTO LRFD BRIDGE DESIGN SPECIFICATIONS 2010* checking, the following material properties are used:

Description	**Property**
Steel yield strength	Fy(th)
Ultimate strength	Fu(th)
Elasticity modulus	E
Poisson coefficient	ν
Shear modulus	G

*th =thickness of plate

7.7.3. Check process

Necessary steps to conduct the different checks in CivilFEM are as follows:

a) Obtain material properties corresponding to the element stored in CivilFEM database and calculate the rest of the properties needed for checking:

Properties obtained from CivilFEM database:

Elasticity modulus E

Poisson's ratio: ν

Yield strength: Fy (th)

Ultimate strength Fu (th)

Shear modulus G

Thickness of corresponding plate th

b) Obtain the cross-sectional data corresponding to the element.

c) Initiate the values of the plate's reduction factors and the other plate's parameters to determine its class.

d) Perform a check of the section according to the type of external load.

e) Results. In CivilFEM.

The required data for the different checking types are provided within tables found in their corresponding section of this manual.

7.7.4. Section Class and Reduction Factors Calculation

Steel sections are classified for flexure as compact, noncompact or slender-element sections. For a section to qualify as compact its flanges must be continuously connected to the web or webs and the width-thickness ratios of its compression elements must not exceed the limiting width-thickness ratios λ_p. If the width-thickness ratio of one or more compression elements exceeds λ_p but does not exceed λ_r, the section is noncompact. If the width-thickness ratio of any element exceeds λ_r, the section is referred to as a slender-element compression section. Compression classification is similar but with only one ratio to distinguish slender and non-slender sections

Therefore, the code suggests different lambda values depending on if the element is subjected to compression, flexure or compression plus flexure.

The section classification is the worst-case scenario of all of its plates. Therefore, the class is calculated for each plate with the exception of pipe sections, which have their own formulation because it cannot be decomposed into plates. This classification will consider the following parameters:

a) Length of elements:

The program will define the element length (b or h) as the length of the plate (distance between the extreme points), except when otherwise specified.

b) Flange or web distinction:

To distinguish between flanges or webs, the program follows the criteria below:

Once the principal axis of bending is defined, the program will examine the plates of the section. Fields Pty and Ptz of the plates indicate if they behave as flanges, webs or undefined, choosing the correct one for the each axis. If undefined, the following criterion will be used to classify the plate as flange or web: if $|\Delta y|<|\Delta z|$ (increments of end coordinates) and flexure is in the Y axis, it will be considered a web; if not, it will be a flange. The reverse will hold true for flexure in the Z-axis.

- **Hot rolled Steel Shapes**:

  Section I and C:

  The length of the plate *h* will be taken as the value *d* for the section dimensions.

  Section Box:

  The length of the plate will be taken as the width length minus three times the thickness.

7.7.5. Members Subjected to Compression

In order to check for compression it is necessary to determine if the element is stiffened or unstiffened.

- For stiffened elements:

$$\lambda = 1.49\sqrt{\frac{E}{F_y}}$$

Pipe sections

$$\lambda = 0.11\frac{E}{F_y}$$

Box sections

$$\lambda = 1.40\sqrt{\frac{E}{F_y}}$$

- Unstiffened elements:

$$\lambda = 0.56\sqrt{\frac{E}{F_y}}$$

Angular sections

$$\lambda = 0.45\sqrt{\frac{E}{F_y}}$$

Stem of T sections

$$\lambda = 0.75\sqrt{\frac{E}{F_y}}$$

7.7.6. Members Subjected to Bending

The bending check is only applicable to very specific sections. Therefore, the slenderness factor is listed for each section:

- *Section I:*

  Flanges:

  $$\lambda_p = 0.38\sqrt{\frac{E}{F_{yc}}} \qquad \lambda_r = 0.95\sqrt{\frac{EK_c}{F_{yr}}}$$

  For hot rolled shapes $K_c = 0.76$

  For welded sections $0.35 \le K_c \le 0.76$, $k_c = \frac{4}{\sqrt{D/t_w}}$

  F_{yr} = minimum of $0.7F_{yc}$, $R_h F_{yw} S_{xt}/S_{xc}$ y F_{yw} but no less than $0.5F_{yc}$.

  Web:

  $$\lambda_p = \frac{\sqrt{\frac{E}{F_{yc}}}}{\left(0.54\frac{M_P}{R_h M_y} - 0.09\right)^2}$$

$$\lambda_r = 5.70\sqrt{\frac{E}{F_y}}$$

- *Section C*

  Flanges:

  $\lambda_p = 3.76\sqrt{\frac{E}{F_y}}$ $\lambda_r = 0.95\sqrt{\frac{EK_c}{F_{yr}}}$

  For hot rolled shapes $K_c = 0.76$

  For welded sections $0.35 \leq K_c \leq 0.76$, $k_c = \frac{4}{\sqrt{D/t_w}}$

  F_{yr} = minimum of $0.7F_{yc}, R_h F_{yw} S_{xt}/S_{xc}$ y F_{yw} but no less than $0.5F_{yc}$.

  Web:

  $$\lambda_p = 3.76\sqrt{\frac{E}{F_y}}$$

  $$\lambda_r = 5.70\sqrt{\frac{E}{F_y}}$$

- *Pipe section:*

  $$\lambda_p = 0.07\frac{E}{F_y}$$

  $$\lambda_r = 0.31\frac{E}{F_y}$$

- *Box section:*

  Flanges of box section:

  $$\lambda_p = 1.12\sqrt{\frac{E}{F_y}}$$

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$

Webs: the program distinguishes between the flange and web upon the principal axis chosen by the user.

$$\lambda_p = 2.42\sqrt{\frac{E}{F_y}}$$

$$\lambda_r = 5.70\sqrt{\frac{E}{F_y}}$$

- *T section:*

Flange: $\lambda_r = 0.38\sqrt{\frac{E}{F_y}}$

$$\lambda_p = 1.0\sqrt{\frac{E}{F_y}}$$

Web: No limits are included for flexure classification, so class section is only checked for flange limit.

7.7.7. Members Subjected to Tension

The axial tension force must be taken as positive (if the tension force has a negative value, the element will not be checked)

The factored tensile resistance, P_r, shall be taken as the lesser of :

a) yielding in the gross section:

$$P_r = \Phi_y F_y A_g$$

$$\Phi_y = 0.95$$

b) rupture in the net section:

$$P_r = \Phi_u F_u A_n R_p U$$

$$\Phi_u = 0.80$$

Being:

A_n Effective net area.

A_g Gross area.

F_y Minimum yield stress.

F_u Minimum tensile strength.

Values of R_p and U must be introduced by the user according article 6.8.2.1.

The effective net area will be taken as A_g – AHOLES. The user will need to enter the correct value for AHOLES (the code indicates that the diameter is 1/16th in. (2 mm) greater than the real diameter).

7.7.8. Members Subjected to Axial Compression

Axial compression check by la *AASHTO LRFD BRIDGE DESIGN SPECIFICATIONS 2010* of the design compressive strength, $\Phi_c P_n$, are determined as follows:

$$\Phi_c = 0.90$$

Compressive Strength for Flexural Buckling

Compressive Strength for Flexural Buckling

$$Q = Q_s Q_a$$

(a) for $\frac{P_e}{P_0} \geq 0.44$

$$P_n = \left(0.658^{\frac{P_0}{P_e}}\right) P_0$$

(b) for $\frac{P_e}{P_0} < 0.44$

$$P_n = 0.877 P_e$$

Being:

$$P_0 = Q F_y A_g$$

$$P_e = \frac{\pi^2}{\left(\frac{Kl}{r_s}\right)^2} EA_g$$

Where:

A_g	Gross area of member.
Q	Slender element reduction factor.
r_s	Governing radius of gyration about the buckling axis.
K	Effective length factor.
l	Unbraced length.

Factor Q for compact and noncompact sections is always 1. Nevertheless, for slender sections (exceed ratio given in 10-G.6.1.1) , the value of $Q = Q_s Q_a$ has a particular procedure. Such procedure is described below:

Factor Q for slender sections*:*

For unstiffened plates, Qs must be calculated and for stiffened plates, Qa must be determined. If these cases do not apply (box sections or angular sections, for example), a value of 1 for these factors will be taken.

For circular sections, there is a particular procedure of calculating Q. Such procedure is described below:

- For circular sections, Q is:

$$Q = Q_a = \frac{0.038 \cdot E}{F_y (D/t)} + \frac{2}{3} \qquad 0.11\, E/F_y \le D/t < 0.45\, E/F_y$$

Factor Qs:

If there are several plates free, the value of Qs is taken as the biggest value of all of them. The program will check the slenderness of the section in the following order:

- Angular

If $0.45\sqrt{E/F_y} < \lambda \le 0.91\sqrt{E/F_y}$ $\quad Q_s = 1.340 - 0.76\frac{\lambda}{\sqrt{E/F_y}}$

If $0.91\sqrt{E/F_y} < \lambda$ $\quad Q_s = 0.53\frac{E/F_y}{\lambda^2}$

- Stem of T

If $0.75\sqrt{E/F_y} < \lambda \leq 1.03\sqrt{E/F_y}$ $\quad Q_s = 1.908 - 1.22\frac{\lambda}{\sqrt{E/F_y}}$

If $1.03\sqrt{E/F_y} < \lambda$ $\quad Q_s = 0.69\frac{E/F_y}{\lambda^2}$

- Rolled shapes

If $0.56\sqrt{E/F_y} < \lambda \leq 1.03\sqrt{E/F_y}$ $\quad Q_s = 1.415 - 0.74\frac{\lambda}{\sqrt{E/F_y}}$

If $1.03\sqrt{E/F_y} < \lambda$ $\quad Q_s = 0.69\frac{E/F_y}{\lambda^2}$

- Other sections

If $0.64\sqrt{k_c \cdot E/F_y} < \lambda \leq 1.17\sqrt{k_c \cdot E/F_y}$ $\quad Q_s = 1.415 - 0.65\frac{\lambda}{\sqrt{k_c \cdot E/F_y}}$

If $1.17\sqrt{k_c \cdot E/F_y} < \lambda$ $\quad Q_s = 0.90\frac{E/F_y}{\lambda^2}$

Where λ is the element slenderness and

$k_c = \frac{4}{\sqrt{\lambda}}, 0.35 \leq k_c \leq 0.76$ For hot rolled I sections

$k_c = 0.76$ for other sections

Factor Qa:

The calculation of factor Qa is an iterative process. Its procedure is the following:

7) An initial value of Q equal to Qs calculated before is taken.
8) With this value $f = Q_sF_y$ is calculated.
9) For elements with stiffened plates, the effective width b_e is calculated.
10) With b_e the effective area is calculated.
11) With the value of the effective area, Qa is calculated.

$$Q_a = \frac{\text{effctive area}}{\text{gross area}}$$

- For a box section

If $\lambda \geq 1.40\sqrt{\frac{E}{f}}$ $\quad b_e = 1.92 \cdot t\sqrt{\frac{E}{f}}\left[1 - \frac{0.38}{\lambda}\sqrt{\frac{E}{f}}\right]$

- For other sections

If $\lambda \geq 1.49\sqrt{\frac{E}{f}}$ $\quad b_e = 1.92 \cdot t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{\lambda}\sqrt{\frac{E}{f}}\right]$

If it is not within those limits, $b_e = b$

With the b_e values for each plate, the part that does not contribute [t·(b-b_e)] is subtracted from the area (where t is the plate thickness). Using this procedure, the effective area is calculated.

Finally, with Qs and Qa, Q is calculated.

Output results are written in the CivilFEM results file.

Compressive Strength for Flexural-Torsional Buckling

This type of check can be carried out for compact sections as well as for noncompact or slender sections. The steps for these three cases are as follows:

Nominal compressive strength, P_n :

(a) for $\frac{P_e}{P_o} \geq 0.44$

$$P_n = \left(0.658^{\frac{P_0}{P_e}}\right) P_0$$

(b) for $\frac{P_e}{P_o} < 0.44$

$$P_n = 0.877P_e$$

Where:

$$P_0 = QF_yA_g$$

$$Q = Q_sQ_a$$

Factor Q for compact and noncompact sections is 1. Nevertheless, for slender sections, the Q factor has a particular procedure of calculation. Such procedure is equal to the one previously described.

The elastic stress for critical torsional buckling or flexural-torsional buckling F_e is calculated as the lowest root of the following third degree equation, in which the axis have been changed to adapt to the CivilFEM normal axis:

$$(P_e - P_{ex})(P_e - P_{ey})(P_e - P_{ez}) - P_e^2(P_e - P_{ez})\left(\frac{y_0}{r_0}\right)^2 - P_e^2(P_e - P_{ey})\left(\frac{Z_0}{r_0}\right)^2 = 0 \quad (6.9.4.1.3\text{-}7)$$

Where:

K_x Effective length factor for torsional buckling.

G Shear modulus (MPa).

C_w Warping constant (mm^6).

J Torsional constant (mm^4).

I_y, I_z Moments of inertia about the principal axis (mm^4).

X_0, y_0 Coordinates of shear center with respect to the center of gravity (mm).

$$\bar{r}_0^2 = y_0^2 + z_0^2 + \frac{I_y + I_z}{A}$$

$$H = 1 - \left(\frac{y_0^2 + Z_0^2}{\bar{r}_0^2}\right)$$

$$P_{ey} = \frac{\pi^2 \cdot E}{(K_y \cdot I/r_y)^2} A$$

$$P_{ez} = \frac{\pi^2 \cdot E}{(K_z \cdot I/r_z)^2} A$$

$$P_{ex} = \left(\frac{\pi^2 \cdot E \cdot C_w}{(K_x \cdot I)^2}\right)$$

where:

A Cross-sectional area of member.

l Unbraced length.

K_y, K_z Effective length factor, in the z and y directions.

$r_y r_z$ Radii of gyration about the principal axes.

$\bar{r}_0^2$ Polar radius of gyration about the shear center.

In this formula, CivilFEM principal axes are used. If the CivilFEM axes are the principal axes ±5º sexagesimal degrees, Ky and Kz are calculated with respect to the Y and Z-axes of CivilFEM. If this is not the case (angular shapes, for example) axes U and V will be used as principal axes, with U as the axis with higher inertia.

Output results are written in the CivilFEM results file.

7.7.9. Members Subjected to Flexure

Summary of the checks done by CivilFEM:

SECTION TYPE	YIELDING	LTB	FLB	WLB	Conditions
BOX	X (6.12.2.2.2)		X (6.12.2.2.2)	X (6.12.2.2.2)	Non-slender web
PIPE	X (6.12.2.2.3)		X(local buckling) (6.12.2.2.3)		Compact, non-compact and slender under the limit for flexure check.
T SECTION	X (6.12.2.2.4)	X (6.12.2.2.4)	X (6.12.2.2.4)	X (6.12.2.2.4)	Non-slender flange
I SECTION (FLEXUREABOUT STRONG AXIS)	X	X (A.6.3.3)	X (A.6.3.2)		Non-slender web and Fy<70 ksi
DOUBLE T (FLEXURE ABOUT WEAK AXIS)	X		X (6.12.2.2.1)		Non-slender flanges
SECTION C (FLEXURE ABOUT STRONG AXIS)	X (6.12.2.2.5)	X (6.12.2.2.5)			Compact members
SECTION C (FLEXURE ABOUT WEAK	X		X (6.12.2.2.5)		Non-slender flanges

AXIS)	

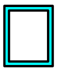
The design flexural strength, $\phi_f M_n$, shall be determined as follows:

$$\phi_f = 1.00$$

Where M_n is the lowest value of four checks:

e) Yielding (Y)
f) Lateral-torsional buckling (LTB)
g) Flange local buckling (FLB)
h) Web local buckling (WLB)

The checks done depends on the section:

- Box  (non-slender webs)
- Yielding

$$M_n = M_p$$

- FLB

$$\text{If } \lambda_p \leq \lambda \leq \lambda_r$$

$$M_n = M_p = \left(M_p - F_y\, S\right)\left(3.57\frac{b_f}{t_f}\sqrt{\frac{F_y}{E}} - 4.0\right) \leq M_p$$

If $\lambda > \lambda_r$

$$M_n = F_y\, S_{eff}$$

- WLB

If $\lambda > \lambda_p$

$$M_n = M_p - \left(M_p - F_y S\right)\left(0.305\frac{D}{t_w}\sqrt{\frac{F_y}{E}} - 0.738\right) \leq M_p$$

- Circular tubes  (compact, non-compact and slender under the ratio limit

$\lambda = 0.45\frac{E}{F_y})$

1. Yielding

$$M_n = M_p$$

2. Local buckling

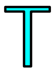
If $\lambda_p \leq \lambda \leq \lambda_r$

$$M_n = \left[\frac{0.021E}{\lambda} + F_y\right]S$$

If $\lambda > \lambda_r$

$$M_n = F_{cr}S$$

$$F_{cr} = \frac{0.33E}{\lambda}$$

- T shape 

1. Yielding $M_n = M_p$

If stem is in tension, the limit on M_n is $1.6M_y$
If stem is in compression M_n is limited to M_y

2. LTB

$$M_n = \frac{\pi\sqrt{EI_yGJ}}{L_b}\left[B + \sqrt{1 + B^2}\right] \leq M_p$$

$$B = \pm 2.3\frac{d}{L_b}\sqrt{\frac{I_y}{J}}$$

(The plus sign for B shall apply when the stem is in tension and the minus sign shall apply when the stem is in compression)

3. FLB

If $\lambda_p \leq \lambda \leq \lambda_r$

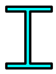
$$M_n = M_p - \left(M_p - 0.7F_yS_{xc}\right)\left(\frac{\lambda-\lambda_p}{\lambda_r-\lambda_p}\right) \leq 1.6M_y$$

S_{xc} : Elastic section modulus with respect to the compression flange

$\lambda > \lambda_r$ is not provided because the limiting slenderness value is larger than 12 (Eq. 6.10.2.2-1)

4. Local buckling of the stem

$$M_n = 0.424\frac{EJ}{d} \leq M_y$$

- I shape loaded on the strong axis  (non-slender web)

1. Yielding

$$M_n = M_yR_{pc}$$

2. LTB

$$L_p = 1.0r_t\sqrt{\frac{E}{F_y}}$$

$$L_r = 1.95r_t\frac{E}{F_{yr}}\sqrt{\frac{J}{S_xh}}\sqrt{1+\sqrt{1+6.76\left(\frac{F_{yr}S_xh}{EJ}\right)^2}}$$

Where:

$$r_t = \text{radius of gyration} = \frac{b_f}{\sqrt{12\left(1+\frac{D_cf_w}{3b_ft_f}\right)}}$$

If $L_b \leq L_p$

$$M_n = M_yR_{pc}$$

If $L_p < L_b \leq L_r$

$$M_n = C_b\left[1-(1-\frac{F_{yr}S_x}{M_yR_{pc}})(\frac{L_b-L_p}{L_r-L_p})\right]M_yR_{pc} \le M_yR_{pc}$$

If $L_b > L_r$

$$M_n = F_{cr}S_x$$

$$F_{cr}=\frac{C_b\pi^2E}{\left(\frac{L_b}{r_t}\right)^2}\sqrt{1+0.078\frac{J}{S_xh}\left(\frac{L_b}{r_t}\right)^2}$$

3. FLB

If $\lambda \le \lambda_p$

$$M_n = M_yR_{pc}$$

If $\lambda \le \lambda_p$

$$M_n = \left[1-\left(1-\frac{F_{yr}S_x}{R_{pc}M_y}\right)\left(\frac{\lambda-\lambda_p}{\lambda_r-\lambda_p}\right)\right]R_{pc}M_y$$

R_{pc} is the web plastification factor for the compression flange determined as specified in Article A6.2.1 or Article A6.2.2:

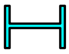
If is compact web $\quad \lambda \le \frac{\sqrt{\frac{E}{F_y}}}{\left(0.54\frac{M_p}{R_hM_y}-0.09\right)^2} \le 5.7\sqrt{\frac{E}{F_y}}$

$$R_{pc} = \frac{M_p}{M_y}$$

If is non-compact web, $\lambda \le 5.7\sqrt{\frac{E}{F_y}}$

$$R_{pc} = \left[1-\left(1-\frac{R_hM_y}{M_p}\right)\left(\frac{\lambda-\lambda_p}{\lambda_r-\lambda_p}\right)\right]\frac{M_p}{M_y} \le \frac{M_p}{M_y}$$

R_h is the hybrid factor and for sections that are checked in CivilFEM takes a value of 1.

- T shape loaded on weak axis  (flanges compact or non-compact)

    1. Yielding:
       $M_n = M_p$
    2. FLB

$$\text{If } \lambda \leq \lambda_p$$

$$M_n = M_p$$

If $\lambda_p \leq \lambda \leq \lambda_r$

$$M_n = \left[1 - \left(1 - \frac{S_y}{Z_y}\right)\left(\frac{\lambda - \lambda_p}{0.45\sqrt{\frac{E}{F_{fy}}}}\right)\right] F_{yf} Z_y$$

- C shape loaded on the strong axis  (web and flanges compact)
    1. Yielding
       $M_n = M_p$
    2. LTB

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}}$$

$$L_r = 1.95\, r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_0}} \sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7F_y S_x h_0}{EJc}\right)^2}}$$

If $L_p < L_b < L_r$

$$M_n = C_b \left[M_p - \left(M_p - 0.7F_y S_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p$$

If $L_b > L_r$

$$M_n = F_{cr} S_x$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_0} \left(\frac{L_b}{r_{ts}}\right)^2}$$

$$C = \frac{h_0}{2} \sqrt{\frac{I_y}{C_w}}$$

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x}$$

Where:

r_y= radius of gyration about the weak axis(in)

J = Torsional constant St. Venant (in^4)

S_x= Elastic section modulus about strong axis(in^3)

h_0= distance between centroids of the flanges(in)

C_w=warping constant(in^6)

C_b=Moment gradient modifier. Must be introduced by the user.

- C shape loaded on the weak axis  (flanges compact or non-compact)
  1. Yielding
     M_n = min (M_p, $1.6F_yS_y$)

  2. FLB

If $\lambda \leq \lambda_p$

$M_n = M_p$

If$\lambda_p \leq \lambda \leq \lambda_r$

$$M_n = \left[1 - \left(1 - \frac{S_y}{Z_y}\right)\left(\frac{\lambda - \lambda_p}{0.45\sqrt{\frac{E}{F_{fy}}}}\right)\right] F_{yf} Z_y$$

Output results are written in the CivilFEM results file.

7.7.10. Members Subjected to Shear

The design shear strength, Φ_v, V_n, shall be determined as follows:

For all provisions: $\Phi_v = 1.0$

To calculate the nominal shear strength CivilFEM follows the provisions of the article 6.10.9.2 except for box-shaped (6.12.1.2.3b) and circular tubes (6.12.1.2.3c)

$V_n = C^* V_p$

$V_p = 0.58^* F_{yw} * D * t_w$, where D is total depth of the web.

C is the ratio of the shear-buckling resistance to the shear yield strength determined as:

a. For $\frac{D}{t_w} \leq 1.12\sqrt{\frac{k\ E}{F_y}}$ $\qquad C_v = 1.0$ (AASHTO 6.10.9.3.2-4)

b. For $1.12\sqrt{\frac{Ek}{F_y}} < \frac{D}{t_w} \leq 1.40\sqrt{\frac{Ek}{F_y}}$ $\qquad C_v = \frac{1.12}{\frac{h}{t_w}}\sqrt{\frac{Ek}{t_w}}$ (AASHTO 6.10.9.3.2-5)

c. For $\frac{D}{t_w} \leq 1.40\sqrt{\frac{Ek}{F_y}}$ $\qquad C_y = \frac{1.57}{\left(\frac{h}{t_w}\right)^2}\left(\frac{Ek}{F_y}\right)$ (AASHTO 6.10.9.3.2-6)

The web plate buckling coefficient, Kv, will be calculated as a constant equal to 5.0.

For shape-box sections D is the clear distance between flanges less inside corner radius on each side. Both webs area shall be considered effective in resisting the shear.

For circular tubes the nominal shear strength will be taken as:

$$V_n = 0.5F_{cr}A$$

F_{cr} , shear buckling resistance (ksi)taken as the larger of either:

$$F_{cr1} = \frac{1.60E}{\sqrt{\frac{L_V}{D}}\left(\frac{D}{t}\right)^{\frac{5}{4}}} \leq 0.58F_y$$

$$F_{cr2} = \frac{0.78E}{\left(\frac{D}{t}\right)^{\frac{3}{2}}} \leq 0.58F_y$$

Output results are written in the CivilFEM results file.

7.7.11. Members Subjected to Combined Forces

Checking of Members Subject to Flexure and Axial Tension / Compression

For this check, it is first necessary to determine the value of Mn. This value comes into play in the checking of formulas. The value of Mn, will be calculated in the same way as members subjected to flexure; thus, the nominal flexure strength (M_n) is the minimum of four checks:

1. Yielding
2. Lateral-torsional buckling
3. Flange local buckling
4. Web local buckling

In the case of having bending plus tension or bending plus compression, the interaction between flexure and axial force is limited by the following equations:

(c) For $\frac{P_r}{P_c} \geq 0.2$

$$\frac{P_u}{P_r} + \frac{8}{9}\left(\frac{M_{uz}}{M_{rz}} + \frac{M_{uy}}{M_{ry}}\right) \leq 1.0 \quad \text{(6.8.2.3-2, 6.9.2.2-2)}$$

(d) For $\frac{P_r}{P_c} < 0.2$

$$\frac{P_u}{2P_r} + \left(\frac{M_{uz}}{M_{rz}} + \frac{M_{uy}}{M_{ry}}\right) \leq 1.0 \quad \text{(6.8.2.3-1, 6.9.2.2-1)}$$

Where:

P_u	Axial force resulting from factored loads.
P_r	Factored resistance.
M_u	Moment resulting from factored loads.
M_r	Factored flexural resistance .

y Strong axis bending.

z Weak axis bending.

The following checks are carried out by CivilFEM:

- Axial force and flexural buckling
- Bending moment Z direction
- Bending moment Y direction

If one of these checks do not meet the code requirements, it will not be possible to check the member under flexure plus tension / compression.

Output results are written in the CivilFEM results file

7.8. Steel Structures According to AISC ASD/LRFD 14th Ed.

7.8.1. Material properties

For AISC 13th Edition checking, the following material properties are used:

Description	Property
Steel yield strength	Fy(th)
Ultimate strength	Fu(th)
Elasticity modulus	E
Poisson coefficient	ν
Shear modulus	G

*th =thickness of the plate

7.8.2. Section data

AISC 14th Edition considers the following data set for the section:

- Gross section data
- Net section data
- Effective section data.
- Data belonging to the section and plates class.

Gross section data correspond to the nominal properties of the cross-section. For the net section, only the area is considered. This area is calculated by subtracting the holes for screws, rivets and other holes from the gross section area. (The area of holes is introduced within the structural steel code properties).

The effective section data and the section and plates class data are obtained in the checking process according to chapter B, section B4 of the code. This chapter classifies steel sections into three groups (compact, noncompact and slender), depending upon the width-thickness ratio and other mandatory limits.

The AISC 14TH Edition module utilizes the gross section data in user units and the CivilFEM axis or section axis as initial data. The program calculates the effective section data and the class data, and stores them in CivilFEM's results file, in user units and in CivilFEM or section axis.

The section data used in AISC 14TH Edition are shown in the following tables:

Description	Data
Input data:	
1.- Height	H
2.- Web thickness	Tw
3.- Flanges thickness	Tf
4.- Flanges width	B
5.- Distance between flanges	Hi
6.- Radius of fillet (Rolled shapes)	r_1
7.- Toe radius (Rolled shapes)	r_2
8.- Weld throat thickness (Welded shapes)	a
9.- Web free depth	d
Output data	(None)

Description	Data	Reference axes
Input data:		
1.- Depth in Y	Tky	CivilFEM
2.- Depth in Z	tkz	CivilFEM
3.- Cross-section area	A	
4.- Moments of inertia for torsion	It	CivilFEM
5.- Moments of inertia for bending	Iyy, Izz	CivilFEM
6.- Product of inertia	Izy	CivilFEM
7.- Elastic resistant modulus	Wely, Welz	CivilFEM
8.- Plastic resistant modulus	Wply, Wplz	CivilFEM
9.- Radius of gyration	iy, iz	CivilFEM
10.- Gravity center coordinates	Ycdg, Zcdg	Section
11.- Extreme coordinates of the perimeter	Ymin, Ymax, Zmin, Zmax	Section
12.- Distance between GC and SC in Y and in Z	Yms, Zms	Section
13.- Warping constant	Iw	
14.- Shear resistant areas	Yws, Zws	CivilFEM
15.- Torsional resistant modulus	Xwt	CivilFEM
16.- Moments of inertia for bending about U, V	Iuu, Ivv	Principal
17.- Angle Y->U or Z->V	α	CivilFEM
Output data:	(None)	

Description	Data
Input data:	
1.- Gross section area	Agross
2.- Area of holes	Aholes
Output data:	
1.- Cross-section area	Anet

The effective section depends upon the geometry of the section; thus, the effective section is calculated for each element and each of the ends of the element.

Description	Data
Input data:	(None)
Output data:	
1.- Reduction factor	Q
2.- Reduction factor	Qs
3.- Reduction factor	Qa

7.8.3. Structural steel code properties

For AISC 14th Edition checking, besides the section properties, more data are needed for bucling checks. These data are shown in the following table.

Description	Data
Input data:	
1.- Unbraced length of member (global buckling)	L
2.- Effective length factors Y direction	KY
3.- Effective length factors Z direction	KZ
4.- Effective length factors for torsional buckling	KTOR
5.- Flexural factor relative to bending moment	Cb
6.- Length between lateral restraints	Lb
Output data:	
1.- Compression class	CLS_COMP
2.- Bending class	CLS_FLEX

7.8.4. Check Process

Necessary steps to conduct the different checks in CivilFEM are as follows:

f) Obtain material properties corresponding to the element stored in CivilFEM database and calculate the rest of the properties needed for checking:
Properties obtained from CivilFEM database (materials):

Elasticity modulus	E
Poisson's ratio	ν
Yield strength	Fy (th)
Ultimate strength	Fu (th)
Shear modulus	G
Thickness of corresponding plate	th

g) Obtain the cross-sectional data corresponding to the element.

h) Initiate the values of the plate's reduction factors and the other plate's parameters to determine its class.

i) Perform a check of the section according to the type of external load.

j) Results. In CivilFEM, checking results for each element end are stored in the results file .CRCF

7.8.5. Design requirements

7.8.5.1. Design for Strength Using Load and Resistance Factor Design (LRFD)

Design shall be performed in accordance with:

$$R_u \leq \phi R_n$$

Where:

R_u Required strength (LRFD).

R_n Nominal strength.

ϕ Resistance factor.

ΦR_n Design strength

7.8.5.2. Design for Strength Using Allowable Strength Design (ASD)

Design shall be performed in accordance with:

$$R_a \leq R_n/\Omega$$

Where:

R_a	Required strength (ASD)
R_n	Nominal strength.
Ω	Safety factor
R_n/Ω	Allowable strength

Section Class and Reduction Factors Calculation.

Steel sections are classified as compact, noncompact or slender-element sections for bending sections and slender or non slender for compression sections. For a section to qualify as compact its flanges must be continuously connected to the web or webs and the width-thickness ratios of its compression elements must not exceed the limiting width-thickness ratios λ_p (see table B4.1 of AISC 14th Edition). If the width-thickness ratio of one or more compression elements exceeds λ_p but does not exceed λ_r, the section is noncompact. If the width-thickness ratio of any element exceeds λ_r, (see table B4.1 of AISC 14th Edition), the section is referred to as a slender-element compression section.

Therefore, the code suggests different lambda values depending on if the element is subjected to compression, flexure or compression plus flexure.

The section classification is the worst-case scenario of all of its plates. Therefore, the class is calculated for each plate with the exception of pipe sections, which have their own formulation because it cannot be decomposed into plates. This classification will consider the following parameters:

a) Length of elements:

   The program will define the element length (b or h) as the length of the plate (distance between the extreme points), except when otherwise specified.

b) Flange or web distinction:

   To distinguish between flanges or webs, the program follows the criteria below:

   Once the principal axis of bending is defined, the program will examine the plates of the section. Fields Pty and Ptz of the plates indicate if they behave as flanges, webs or undefined, choosing the correct one for the each axis. If undefined, the following criterion will be used to classify the plate as flange or web:

If $|\Delta y| < |\Delta z|$ (increments of end coordinates) and flexure is in the Y axis, it will be considered a web; if not, it will be a flange. The reverse will hold true for flexure in the Z-axis.

- **Hot rolled Steel Shapes**:
  Section I and C:

  The length of the plate h will be taken as the value d for the section dimensions.

  Section Box:

  The length of the plate will be taken as the width length minus three times the thickness.

7.8.5.3. Members subjected to compression

In order to check for compression it is necessary to determine if the element is stiffened or unstiffened.

- For stiffened elements:

$$\lambda_p = 0.0$$

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$

Pipe sections

$$\lambda_r = 0.11\frac{E}{F_y}$$

Box sections

$$\lambda_p = 1.12\sqrt{\frac{E}{F_y}}$$

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$

- Unstiffened elements:

$\lambda_p = 0.0 \qquad \lambda_r = 0.56\sqrt{\frac{E}{F_y}}$

Angular sections

$\lambda_r = 0.45\sqrt{\frac{E}{F_y}}$

Stem of T sections

$\lambda_r = 0.75\sqrt{\frac{E}{F_y}}$

7.8.5.4. *Members subjected to bending*

The bending check is only applicable to very specific sections. Therefore, the slenderness factor is listed for each section:

- *Section I and C:*

  $P_y = F_y \cdot A_g \; ; \; \Phi_b = 0.90$

  $$k_c = \frac{4}{\sqrt{h/t_w}}$$

  $F_r =$ 69 MPa for hot rolled shapes (10 ksi)

  114 MPa for welded sections (16.5 ksi)

  F_L= minimum of ($F_{yf} - F_r$) and (F_{yw}) where F_{yf} and F_{yw} are the F_y of flange and web respectively.

  Flanges of rolled sections:

  $\lambda_p = 0.38\sqrt{\frac{E}{F_L}} \qquad \lambda_r = 0.83\sqrt{\frac{E}{F_L}}$

  Flanges of welded sections:

  $\lambda_p = 0.38\sqrt{\frac{E}{F_{yf}}} \qquad \lambda_r = 0.95\sqrt{\frac{E}{F_L/k_c}}$

  Flange:

  If $P_u/\Phi P_y \leq 0.125$: $\quad \lambda_p = 3.76\sqrt{\frac{E}{F_y}}\left(1 - 2.75\frac{P_u}{\emptyset P_y}\right)$

If $P_u/\Phi P_y > 0.125$: $\lambda_p = 1.12\sqrt{\frac{E}{F_y}}\left(2.33 - \frac{P_u}{\emptyset P_y}\right) \geq 1.49\sqrt{\frac{E}{F_y}}$

Always: $\lambda_r = 5.70\sqrt{\frac{E}{F_y}}\left(1 - 0.74\frac{P_u}{\Phi P_y}\right)$

P_u is the compression axial force (taken as positive). If in tension, it will be taken as zero.

- *Pipe section:*

  $\lambda_p = 0.07\frac{E}{F_y}$

  $\lambda_r = 0.31\frac{E}{F_y}$

Box section:

Flanges of box section:

$$\lambda_p = 1.12\frac{E}{F_y}$$

$$\lambda_r = 1.40\frac{E}{F_y}$$

Flanges: the program distinguishes between the flange and web upon the principal axis chosen by the user.

If $P_u/\Phi P_y \leq 0.125$: $\lambda_p = 3.76\sqrt{\frac{E}{F_y}}\left(1 - 2.75\frac{P_u}{\Phi P_y}\right)$

If $P_u/\Phi P_y > 0.125$: $\lambda_p = 1.12\sqrt{\frac{E}{F_y}}\left(2.33 - \frac{P_u}{\Phi P_y}\right) \geq 1.49\sqrt{\frac{E}{F_y}}$

Always: $\lambda_r = 5.70\sqrt{\frac{E}{F_y}}\left(1 - 0.74\frac{P_u}{\Phi P_y}\right)$

- *T section:*

  $\lambda_p = 0.0$

  Stem: $\lambda_p = 0.75\sqrt{\frac{E}{F_y}}$

Flanges: $\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$

7.8.6. Checking of Members for Tension (Chapter D)

The axial tension force must be taken as positive (if the tension force has a negative value, the element will not be checked)

Design tensile strength $\Phi_t P_n$ and the allowable tensile strength P_n/Ω_t, of tension members, shall be the lower value of :

c) yielding in the gross section:
$P_n = F_y A_g$

$$\Phi_t = 0.90 \text{ (LRFD)} \quad \Omega_t = 1.67 \text{ (ASD)}$$

d) rupture in the net section:
$P_n = F_u A_e$

$$\Phi_t = 0.75 \text{(LRFD)} \quad \Omega_t = = 2.00 \text{ (ASD)}$$

Being:

A_e	Effective net area.
A_g	Gross area.
F_y	Minimum yield stress.
F_u	Minimum tensile strength.

The effective net area will be taken as A_g – A_{HOLES}. The user will need to enter the correct value for A_{HOLES} (the code indicates that the diameter is 1/16th in. (2 mm) greater than the real diameter).

7.8.7. Checking of Members in Axial Compression (Chapter E)

The design compressive strength, $\Phi_c P_n$,and the allowable compressive strength, P_n/Ω_c , are determined as follows:

The nominal compressive strength, P_n, shall be the lowest value obtained according to the limit states of flexural buckling, torsional buckling and flexural-torsional buckling.

$$\emptyset_c = 0.90\ (\text{LRFD}) \quad \Omega_c = 1.67\ (\text{ASD})$$

7.8.8. Compressive Strength for Flexural Buckling

This type of check can be carried out for compact sections as well as for noncompact or slender sections. These three cases adhere to the following steps:

Nominal compressive strength, P_n :

$$P_n = A_g F_{cr} \qquad \text{(E3-1)}$$

$$\lambda_c = \frac{KI}{r\pi}\sqrt{\frac{F_y}{E}}$$

$$Q = Q_s Q_a$$

d) For : $\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{QF_y}}$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

e) for $\frac{KL}{r} > 4.71\sqrt{\frac{E}{QF_y}}$

$$F_{cr} = 0.877F_e$$

Where:

A_g Gross area of member.

r Governing radius of gyration about the buckling axis.

K Effective length factor.

l Unbraced length.

F_e Elastic critical buckling stress $F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$

Factor Q for compact and noncompact sections is always 1. Nevertheless, for slender sections, the value of Q has a particular procedure. Such procedure is described below:

Factor Q for slender sections:

For unstiffened plates, Qs must be calculated and for stiffened plates, Qa must be determined. If these cases do not apply (box sections or angular sections, for example), a value of 1 for these factors will be taken.

For circular sections, there is a particular procedure of calculating Q. Such procedure is described below:

- For circular sections, Q is:
  $$Q = Q_a = \frac{0.038 \cdot E}{F_y (D/t)} + \frac{2}{3} \qquad 0.11\, {}^{E}/_{F_y} \leq D/t < 0.45\, {}^{E}/_{F_y}$$

Factor Qs:

If there are several plates free, the value of Qs is taken as the biggest value of all of them. The program will check the slenderness of the section in the following order:

- Angular

  If $0.45\sqrt{E/F_y} < \lambda \leq .91\sqrt{E/F_y}$ $\qquad Q_s = 1.340 - 0.76\dfrac{\lambda}{\sqrt{E/F_y}}$

  If $0.91\sqrt{E/F_y} < \lambda$ $\qquad Q_s = 0.53\dfrac{E/F_y}{\lambda^2}$

- Stem of T

  If $0.75\sqrt{E/F_y} < \lambda \leq 1.03\sqrt{E/F_y}$ $\qquad Q_s = 1.908 - 1.22\dfrac{\lambda}{\sqrt{E/F_y}}$

  If $1.03\sqrt{E/F_y} < \lambda$ $\qquad Q_s = 0.69\dfrac{E/F_y}{\lambda^2}$

- Rolled shapes

  If $0.56\sqrt{E/F_y} < \lambda \leq 1.03\sqrt{E/F_y}$ $\qquad Q_s = 1.415 - 0.74\dfrac{\lambda}{\sqrt{E/F_y}}$

  If $1.03\sqrt{E/F_y} < \lambda$ $\qquad Q_s = 0.69\dfrac{E/F_y}{\lambda^2}$

- Other sections

  If $0.64\sqrt{k_c \cdot E/F_y}\, \lambda \leq 1.17\sqrt{k_c \cdot E/F_y}$ $\qquad Q_s = 1.415 - 0.65\dfrac{\lambda}{\sqrt{k_c \cdot E/F_y}}$

If $1.17\sqrt{k_c \cdot E/F_y} < \lambda$ $\quad Q_s = 0.90 k_c \frac{E/F_y}{\lambda^2}$

Where λ is the element slenderness and

$k_c = \frac{4}{\sqrt{\lambda}}, 0.35 \leq k_c \leq 0.76$ for I sections

$k_c = 0.76$ for other sections

Factor Qa:

The calculation of factor Qa is an iterative process. Its procedure is the following:

12) An initial value of Q equal to Qs is taken.
13) With this value F_{cr} is calculated.
14) This F_{cr} value is taken to calculate $f(f = P_n/A_{eff})$
15) For elements with stiffened plates, the effective width b_e is calculated.
16) With b_e the effective area is calculated.
17) With the value of the effective area, Qa is calculated, and the process starts again.

$$Q_a = \frac{\text{effective}}{\text{gross area}}$$

- For a box section

  If $\lambda \geq 1.40\sqrt{\frac{E}{f}}$ $\quad b_e = 1.92 \cdot t\sqrt{\frac{E}{f}}\left[1 - \frac{0.38}{\lambda}\sqrt{\frac{E}{f}}\right]$

- For other sections

  If $\lambda \geq 1.49\sqrt{\frac{E}{f}}$ $\quad b_e = 1.92 \cdot t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{\lambda}\sqrt{\frac{E}{f}}\right]$

If it is not within those limits, b_e= b

With the b_e values for each plate, the part that does not contribute [t·(b-b_e)] is subtracted from the area (where t is the plate thickness). Using this procedure, the effective area is calculated.

Finally, with Qs and Qa, Q is calculated, and F_{cr} is obtained.

Output results are written in the CivilFEM results file (.CRCF).

7.8.9. Compressive Strength for Flexural-Torsional Buckling

This type of check can be carried out for compact sections as well as for noncompact or slender sections. The steps for these three cases are as follows:

Nominal compressive strength,P_n:

$$P_n = A_g F_{cr}$$

f) for $\lambda_e\sqrt{Q} \leq 1.5$

$$F_{cr} = Q\left(0.658^{Q\lambda_e^2}\right)F_y$$

(b) for $\lambda_e\sqrt{Q} > 1.5$

$$F_{cr} = \left[\frac{0.877}{\lambda_e^2}\right]F_y$$

Where:

$$\lambda_e = \sqrt{\frac{F_y}{F_e}}$$

$$Q = Q_s Q_a$$

Factor Q for compact and noncompact sections is 1. Nevertheless, for slender sections, the Q factor has a particular procedure of calculation. Such procedure is equal to the one previously described.

The elastic stress for critical torsional buckling or flexural-torsional buckling F_e is calculated as the lowest root of the following third degree equation, in which the axis have been changed to adapt to the CivilFEM normal axis:

$$(F_e - F_{ex})(F_e - F_{ey})(F_e - F_{ez}) - F_e^2(F_e - F_{ez})\left(\frac{y_0}{r_0}\right)^2 - F_e^2(F_e - F_{ey})\left(\frac{z_0}{r_0}\right)^2 = 0$$

Where:

K_x	Effective length factor for torsional buckling.
G	Shear modulus (MPa).

C_w Warping constant (mm^6).

J Torsional constant (mm^4).

I_y, I_z Moments of inertia about the principal axis (mm^4).

X_0, y_o Coordinates of shear center with respect to the center of gravity (mm).

$$\bar{r}_0{}^2 = y_0^2 + Z_0^2 + \frac{I_y + I_z}{A}$$

$$H = 1 - \left(\frac{y_0^2 + Z_0^2}{\bar{r}_0{}^2}\right)$$

$$F_{ey} = \frac{\pi^2 \cdot E}{K_y \cdot I/r_y}$$

$$F_{ez} = \frac{\pi^2 \cdot E}{K_z \cdot I/r_z}$$

$$F_{ex} = \left(\frac{\pi^2 \cdot E \cdot C_w}{(Kx \cdot I)} + G \cdot J\right) \cdot \frac{1}{A \cdot \bar{r}_0{}^2}$$

where:

A Cross-sectional area of member.

l Unbraced length.

K_y, K_z Effective length factor, in the z and y directions.

r_y, r_z Radii of gyration about the principal axes.

$\bar{r}_0{}^2$ Polar radius of gyration about the shear center.

In this formula, CivilFEM principal axes are used. If the CivilFEM axes are the principal axes ±5º sexagesimal degrees, K_y and K_z are calculated with respect to the Y and Z-axes of CivilFEM. If this is not the case (angular shapes, for example) axes U and V will be used as principal axes, with U as the axis with higher inertia.

The torsional inertia (Ixx in CivilFEM, J in AISC 13TH Edition) is calculated for CivilFEM sections, but not for captured sections. Therefore the user will have to introduce this parameter in the mechanical properties of CivilFEM.

Output results are written in the CivilFEM results file (.CRCF).

7.8.10. Compressive Strength for Flexure

Chapter F is only applicable to members subject to simple bending about one principal axis.

The design flexural strength, $\phi_b M_n$, and the allowable flexural strength, M_n/Ω_b , shall be determined as follows:

For all provisions: ϕ_b = 0.90 (LRFD) Ω_b= 1.67 (ASD)

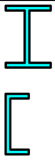
Where M_n is the lowest value of four checks according to sections F2 through F12:

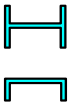
i) Yielding
j) Lateral-torsional buckling
k) Flange local buckling
l) Web local buckling

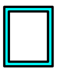
The value of the nominal flexural strength with the following considerations:

- For compact sections, if Lb < Lp only yielding of steel will be checked.
- For T sections, and other compact sections, only yielding and torsional buckling will be checked.
- The case of lateral-torsional buckling does not apply to sections loaded on the minor axis of inertia nor box or square sections.
- The case of lateral-torsional buckling only applies for sections with double symmetry, channel and T sections. Therefore the rest of sections will be checked for torsion plus combined loads and will not be checked under flexure.
- For slender sections, the code contemplates the following cases:

Shape	**Limit State**	**Mr**	**Fcr**	**λ**	**λp**	**λr**
I, C loaded in the axis of higher inertia.	LTB	$F_L S_z$	$\frac{C_b X_1 \sqrt{2}}{\lambda} \sqrt{1 + \frac{X_1^2 X_2}{2\lambda^2}}$	$\frac{L_b}{r_z}$	$1.76 \sqrt{\frac{E}{F_{yf}}}$	$\frac{X_1}{F_L} \sqrt{1 + \sqrt{1 + X_2 F_L^2}}$
	FLB	$F_L S_z$	$\frac{0.69E}{\lambda^2}$ rolled $\frac{0.90Ek_c}{\lambda^2}$ welded	$\frac{b}{t}$	Class B4.1	Class B4.1

	WLB	$R_eF_{yf}S_z$	N.A.	$^h/_{t_w}$	Class B4.1	Class B4.1

Shape	Limit State	Mr	Fcr	λ	λp	λr
I, C loaded in the axis of lower inertia. 	LTB	N.A.	N.A.	N.A.	N.A.	N.A.
	FLB	F_yS_y	$\frac{0.69E}{\lambda^2}$	$\frac{b}{t}$	Class B4.1	Class B4.1
	WLB	N.A.	N.A.	N.A.	N.A.	N.A.

Shape	Limit State	Mr	Fcr	λ	λp	λr
Box 	LTB	$F_{yt}S_{eff}$	$\frac{2EC_b\sqrt{JA}}{\lambda S_z}$	$\frac{L_b}{r_z}$	$\frac{0.13E\sqrt{JA}}{M_p}$	$\frac{2E\sqrt{JA}}{M_r}$
	FLB	$F_{yt}S_{eff}$	$\frac{S_{eff}}{S}F_y$	$\frac{b}{t}$	Class B4.1	Class B4.1
	WLB	$R_eF_{yf}S_z$	N.A.	$^h/_{t_w}$	Class B4.1	Class B4.1

Shape	Limit State	Mr	Fcr	λ	λp	λr	Notes

Pipe	LTB	NA	NA	NA	NA	NA	Limited by Class B4.1
	FLB	Slender: $F_{cr}S$ Non-compact: $M_n = \left(\frac{0.021E}{D/t} + F_y\right) \cdot S$	$\frac{0.33E}{D/t}$	D/t	Class B4.1	Class B4.1	
	WLB	NA	NA	NA	NA	NA	

Shape	Limit State	Mr	Fcr	λ	λp	λr
T, loaded in web plane	LTB	$M_n = M_{cr} = \frac{\pi\sqrt{EI_zGJ}}{L_b}\left[B + \sqrt{1 + B^2}\right]$	N.A.	N.A.	N.A.	N.A.
	FLB	N.A.	N.A.	N.A.	N.A.	N.A.
	WLB	N.A.	N.A.	N.A.	N.A.	N.A.

Where:

$$X_1 = \frac{\pi}{S_z}\sqrt{\frac{EGJA}{2}}$$

$$X_2 = 4\frac{C_w}{I_z}\left(\frac{S_z}{GJ}\right)^2$$

$$B = \underline{+}2.3\frac{d}{L_b}\sqrt{\frac{I_z}{J}}$$

(positive sign if the stem is under tension, negative if it is under compression)

In T sections: $M_n \leq 1.6M_y$ stem in tension; $M_n \leq 1.0M_y$ stem in compression.

For slender webs the nominal flexural strength M_n is the minimum of the following checks:

- tension-flange yield
- compression flange buckling

The first check uses the following formula:

$$M_n = S_{xc}F_y$$

where:

S_{xc} Section modulus referred to tension flange.

F_y Yield strength of tension flange.

The second check uses the following formula:

$$M_n = S_{xc}R_{PG}F_{cr}$$

where:

$$R_{PG} = 1 - \frac{a_w}{1200 + 300a_w}\left(\frac{h_c}{t_w} - 5.70\sqrt{\frac{E}{F_y}}\right) \leq 1.0$$

The critical stress depends upon different slenderness parameters such as λ, λ_p, λ_r and C_{pg} in the following way:

For $\lambda \leq \lambda_p$ $\quad F_{cr} = F_{yf}$

For $\lambda_p \leq \lambda \leq \lambda_r$ $\quad F_{cr} = C_b \cdot F_{yf} \cdot \left[1 - 0.3 \cdot \left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right] \leq F_{yf}$

For $\lambda > \lambda_r$ $\quad F_{cr} = \frac{C_{pg}}{\lambda^2}$

The slenderness values have to be calculated for the following limit states:

- Lateral torsional buckling

$$\lambda = \frac{L_b}{r_T}$$

$$\lambda_p = 1.1 \cdot \sqrt{\frac{E}{F_{yf}}}$$

$$\lambda_r = \pi \cdot \sqrt{\frac{E}{0.7F_{yf}}}$$

$$C_{pg} = 1970000 \cdot C_b \text{ (International System units)}$$

r_T is the radius of gyration of compression flange plus one third of the compression portion of the web (mm).

By default, the program takes a conservative value of $C_b = 1$.

- Flange local buckling

$$\lambda = \frac{b_r}{2t_f}$$

$$\lambda_p = 0.38 \cdot \sqrt{\frac{E}{F_{yf}}}$$

$$\lambda_r = 1.35 \cdot \sqrt{\frac{E}{F_{yf}/k_c}}$$

$$C_{pg} = 180650 \cdot k_c \text{ (IS units)}$$

where:

$$C_b = 1$$

$$k_c = 4/\sqrt{h/t_w}$$

and

$$0.35 \leq k_c \leq 0.76$$

Between these two slenderness, the program will choose values the value that produces a lower critical stress.

Output results are written in the CivilFEM results file (.CRCF).

7.8.11. Checking of Members for Shear (Chapter G)

The design shear strength, $\phi_v V_n$, and the allowable shear strength, V_n/Ω_v, shall be determined as follows:

For all provisions: ϕ_v = 0.90 (LRFD) Ω_v= 1.67 (ASD)

According to the limit states of shear yielding and shear buckling, the nominal shear strength, V_n , of unstiffened webs is:

$$V_n = 0.6 F_y A_w C_v$$

For webs of rolled I-shaped members with $h/t_w \leq 2.24\sqrt{E/F_y}$:

$$\phi_v = 1.00 \text{ (LRFD)} \qquad \Omega_v = 1.50 \text{ (ASD)}$$

$$C_v = 1.0 \text{ (web shear coefficient)}$$

For webs of all other doubly symmetric shapes and singly symmetric shapes and channels C_v is determined as follows:

4. For $h/t_w \leq 1.10\sqrt{k_v E/F_y}$

$$C_v = 1.0$$

5. For $1.10\sqrt{k_v E/F_y} < h/t_w \leq 1.37\sqrt{k_v E/F_y}$

$$C_v = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w}$$

6. For $h/t_w \leq 1.37\sqrt{k_v E/F_y}$

$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y}$$

Where A_w is the overall depth times the web thickness.

It is assumed that there are no stiffeners; therefore, the web plate buckling coefficient K_v will be calculated as a constant equal to 5.0.

Output results are written in the CivilFEM results file (.CRCF).

7.8.12. Checking of Members for Combined Flexure and Axial Tension / Compression (Chapter H)

For this check, it is first necessary to determine the value of Mn. This value comes into play in the checking of formulas. The value of M_n, will be calculated in the same way as members subjected to flexure; thus, the nominal flexure strength (M_n) is the minimum of four checks:

5. Yielding
6. Lateral-torsional buckling
7. Flange local buckling
8. Web local buckling

In the case of having bending plus tension or bending plus compression, the interaction between flexure and axial force is limited by the following equations:

(e) For $\frac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rz}}{M_{cz}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad \text{(H1-1a)}$$

(f) For $\frac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rz}}{M_{cz}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1b)}$$

If the axial force is tension:

P_r	Required tensile strength (N).
P_c	Available tensile strength (N):
	$\phi_t P_n$(LRFD) or P_n/Ω_t (ASD)
M_r	Required flexural strength (N·mm).
M_c	Available flexural strength (N·mm):
	Design: $\phi_b M_n$ (LRFD) or
	Allowable: M_n/Ω_b (ASD)
y	Strong axis bending.
z	Weak axis bending.
ϕ_t	Resistance factor for tension (Sect.D2)
ϕ_b	Resistance factor for flexure = 0.90
Ω_t	Safety factor for tension (Sect D2)
Ω_b	Safety factor for flexure = 1.67

If the axial force is compression:

P_r	Required compressive strength (N).
P_c	Available compressive strength (N):
	Design: $\phi_c P_n$ (LRFD) or
	Allowable: P_n/Ω_c (ASD)
M_r	Required flexural strength (N·mm).

M_c Available flexural strength (N·mm):

Design: $\phi_b M_n$ (LRFD) or

Allowable: M_n/Ω_b (ASD)

Y Strong axis of bending.

Z Weak axis of bending.

Φ_c Resistance factor for compression =0.90

Φ_b Resistance factor for flexure = 0.90

Ω_c Safety factor for compression =1.67

Ω_b Safety factor for flexure = 1.67

The following checks are carried out by CivilFEM:

- Axial force and flexural buckling
- Bending moment Z direction
- Bending moment Y direction

If one of these checks do not meet the code requirements, it will not be possible to check the member under flexure plus tension / compression.

Output results are written in the CivilFEM results file (.CRCF).

7.8.13. Checking of Members for Combined Torsion, Flexure, Shear and/or Axial Force (Chapter H)

The design torsional strength, $\phi_T T_n$, and the allowable torsional strength, T_n/Ω_T , shall be the lowest value obtained according to the limit states of yielding under normal stress, shear yielding under shear stress or buckling, determined as follows:

$$\phi_T = 0.90 \text{ (LRFD)} \qquad \Omega_T = 1.67 \text{ (ASD)}$$

- For the limit state of yielding, under normal stress:

$$F_n = F_y$$

- For the limit state of yielding, under shear stress:

$$F_n = 0.6F_y$$

- For the limit state of buckling:

$$F_n = F_{cr}$$

- Where F_{cr} is calculated

  Output results are written in the CivilFEM results file (.CRCF).

7.9. Steel Structures According to Structural Code (Spanish code)

For checking steel structures according to Structural code (Annex 22) in CivilFEM, it is possible to check structures composed by welded or rolled shapes under axial forces, shear forces and bending moments in 3D.

With CivilFEM it is possible to accomplish the following check and analysis types:

Check steel sections subjected to	
- Tension	Art. 6.2.3
- Compression	Art. 6.2.4
- Bending	Art. 6.2.5
- Shear force	Art. 6.2.6
- Bending and Shear	Art. 6.2.8
- Bending and axial force	Art. 6.2.9
- Bending, shear and axial force	Art. 6.2.10
Check for buckling	
- Compression members with constant cross-section	Art. 6.3.1
- Lateral-torsional buckling of beams	Art. 6.3.2
- Members subjected to bending and axial tension	N/A
- Members subjected to bending and axial compression	Art. 6.3.3

Valid cross-sections supported by CivilFEM for checks according to Structural code are the following:

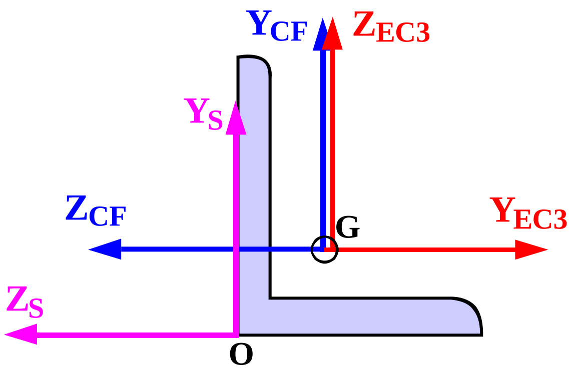
- All rolled shapes included in the program libraries (see the hot rolled shapes library).
- The following welded beams: double T shapes, U or channel shapes, T shapes, box, equal and unequal legs angles and pipes.
- Structural steel sections defined by plates.

CivilFEM considers the above sections as sections composed of plates; for example, an I-section is composed by five plates: four flanges and one web. These cross sections are therefore adapted to the method of analysis of Structural code. Obviously circular sections cannot be decomposed into plates, so these sections are analyzed separately.

7.1.1. Reference axis

With checks according to Structural code (Annex 22), CivilFEM includes three different coordinate reference systems. All of these systems are right-handed:

1. CivilFEM Reference Axis. (X_{CF}, Y_{CF}, Z_{CF}).
2. Cross-Section Reference Axis. (X_S, Y_S, Z_S).
3. Structural code Reference Axis. (Code axis). (X_{code}, Y_{code}, Z_{code}).



For the Structural code axes system:

- The origin matches to the CivilFEM axes origin.
- X_{code} axis coincides with CivilFEM X-axis.
- Y_{code} axis is the relevant axis for bending and its orientation is defined by the user (in steel check process).
- Z_{code} axis is perpendicular to the plane defined by X and Y axis, to ensure a right-handed system.

To define this reference system, the user must indicate which direction of the CivilFEM axis (-Z, -Y, +Z or +Y) coincides with the relevant axis for positive bending. The user may define this reference system when checking according to this code. In conclusion, the code reference system coincides with that of CivilFEM, but it is rotated a multiple of 90 degrees, as shown in table below.

Relevant Axis for Bending in CivilFEM Reference System	Angle of Rotation (clockwise) of Structural Code Reference System respect to the CivilFEM Reference System

- Z_{CF}	90 º (Default value)
- Y_{CF}	180 º
+ Z_{CF}	270 º
+ Y_{CF}	0 º

7.1.2. Material properties

For Structural code checking, the following material properties are used:

Description	**Property**
Steel yield strength	Fy(th)
Ultimate strength	Fu(th)
Partial safety factors	γ_{M0} γ_{M1} γ_{M2}
Elasticity modulus	E
Poisson coefficient	ν
Shear modulus	G

*th =thickness of the plate

7.1.3. Section data

Structural code considers the following data set for the section:

- Gross section data
- Net section data
- Effective section data
- Data belonging to the section and plates class.

Gross section data correspond to the nominal properties of the cross-section. For the net section, only the area is considered. This area is calculated by subtracting the holes for screws, rivets and other holes from the gross section area. The area of holes is introduced within the structural steel code properties.

Effective section data and section and plates class data are obtained in the checking process according to the effective width method. For class 4 cross-sections, this method subtracts the non-resistance zones for local buckling. However, for cross-sections of a lower class, the sections are not reduced for local buckling.

In the following tables, the section data used in Structural code are shown:

Description	Data
Input data:	
1.- Height	H
2.- Web thickness	Tw
3.- Flanges thickness	Tf
4.- Flanges width	B
5.- Distance between flanges	Hi
6.- Radius of fillet (Rolled shapes)	r_1
7.- Toe radius (Rolled shapes)	r_2
8.- Weld throat thickness (Welded shapes)	a
9.- Web free depth	d
Output data	(None)

Description	Data	Reference axis
Input data:		
1.- Depth in Y	Tky	CivilFEM
2.- Depth in Z	tkz	CivilFEM
3.- Cross-section area	A	
4.- Moments of inertia for torsion	It	CivilFEM
5.- Moments of inertia for bending	Iyy, Izz	CivilFEM
6.- Product of inertia	Izy	CivilFEM
7.- Elastic resistant modulus	Wely, Welz	CivilFEM
8.- Plastic resistant modulus	Wply, Wplz	CivilFEM
9.- Radius of gyration	iy, iz	CivilFEM
10.- Gravity center coordinates	Ycdg, Zcdg	Section
11.- Extreme coordinates of the perimeter	Ymin, Ymax, Zmin, Zmax	Section
12.- Distance between GC and SC in Y and in Z	Yms, Zms	Section
13.- Warping constant	Iw	
14.- Shear resistant areas	Yws, Zws	CivilFEM
15.- Torsional resistant modulus	Xwt	CivilFEM
16.- Moments of inertia for bending about U, V	Iuu, Ivv	Principal
17.- Angle Y->U or Z->V	α	CivilFEM

Description	Data	Reference axis
Output data:	(None)	

The effective section depends on the section geometry and on the forces and moments that are applied to it. Consequently, for each element end, the effective section is calculated.

Description	Data	Reference axis
Imput data:	(None)	
Output data:		
1.- Cross-section area	Aeff	
2.- Moments of inertia for bending	Iyyeff, Izzeff	CivilFEM
3.- Product of inertia	Izyeff	CivilFEM
4.- Elastic resistant modulus	Wyeff, Wzeff	CivilFEM
5.- Gravity center coordinates	Ygeff, Zgeff	Section
6.- Distance between GC and SC in Y and in Z	Ymseff, Zmseff	Section
7.- Warping constant	Iw	
8.- Shear resistant areas	Yws, Zws	CivilFEM

7.1.4. Structural steel code properties

For Structural code checking, besides the section properties, more data are needed for buckling checks. These data are shown in the following table.

Description	Structural code
Input data:	
1.- Unbraced length of member (global buckling). Length between lateral restraints (lateral-torsional buckling).	L
2.- Buckling effective length factors in XY, XZ planes YZ (Effective buckling length for plane XY =L*K XY) (Effective buckling length for plane XZ =L*K XZ).	K XY, K XZ
3.- Lateral buckling factors, depending on the load and restraint conditions.	C1, C2, C3
4.- Equivalent uniform moment factors for flexural buckling.	CMy, CMz

Description	Structural code
5.- Equivalent uniform moment factors for lateral-torsional buckling.	CMLt
6.- Effective length factor regarding the boundar conditions.	K
7.- Warping effective factor.	KW

7.1.5. Check Process

The checking process includes the evaluation of the following expression:

$$\frac{N_{Ed}}{N_{c.Rd}} + \frac{M_{y.Ed}}{M_{y.Rd}} + \frac{M_{z.Ed}}{M_{z.Rd}} \leq 1$$

Evaluation steps:

1. Read the loading check requested by the user.
2. Read the CivilFEM axis to be considered as the relevant axis for bending so that it coincides with the Y axis of Structural code. In CivilFEM, by default, the principle bending axis that coincides with the +Y axis of Structural code is the –Z.
3. The following operations are necessary for each selected element:
    a. Obtain material properties of the element stored in CivilFEM database and calculate the rest of the properties needed for checking:
    Properties obtained from CivilFEM database:

Calculated properties:

Epsilon, material coefficient:

$$\varepsilon = \sqrt{235/f_y(th)} \qquad (f_y \text{ in } N/mm^2)$$

    b. Obtain the cross-section data corresponding to the element.
    c. Initialize values of the effective cross-section.
    d. Initialize reduction factors of section plates and the rest of plate parameters necessary for obtaining the plate class.
    e. If necessary for the type of check (check for buckling), calculate the critical forces and moments of the section for buckling: elastic critical forces for the XY and XZ planes and elastic critical moment for lateral-torsional buckling. (See section: Calculation of critical forces and moments).

f. Obtain internal forces and moments: N_{Ed}, $V_{y.Ed}$, $V_{z.Ed}$, $M_{x.Ed}$, $M_{y.Ed}$, $M_{z.Ed}$ within the section.

g. Specific section checking according to the type of external load. The specific check includes:

1. If necessary, selecting the forces and moments considered for the determination of the section class and used for the checking process.
2. Obtaining the cross-section class and calculating the effective section properties.
3. Checking the cross-section according to the external load and its class by calculating the check criterion.

h. Store the results.

7.1.6. Section Class and Reduction Factors Calculation

Sections, according to Structural code, are made up by plates. These plates can be classified according to:

4. Plate function: webs and flanges in Y and Z axis, according to the considered relevant axis of bending.
5. Plate union condition: internal plates or outstand plates.

For sections included in the program libraries, the information above is defined for each plate. CivilFEM classifies plates as flanges or webs according to their axis and provides the plate union condition for each end. Ends can be classified as fixed or free (a fixed end is connected to another plate and free end is not).

For checking the structure for safety, Structural code classifies sections as one of four possible classes:

Class 1	Cross-sections which can form a plastic hinge with the rotation capacity required for plastic analysis.
Class 2	Cross-sections which can reach their plastic moment resistance, but have limited rotation capacity.
Class 3	Cross-sections for which the stress in the extreme compression fiber of the steel member can reach the yield strength, but local buckling is liable to prevent the development of the plastic moment resistance.
Class 4	Cross-sections for which it is necessary to make explicit allowances for the

effects of local buckling when determining their moment resistance or compression resistance.

The cross-section class is the highest (least favorable) class of all of its elements: flanges and webs (plates). First, the class of each plate is determined according to the limits of Structural code. The plate class depends on the following:

1. The geometric width to thickness ratio with the plate width properly corrected according to the plate and shape type.

GeomRat = Corrected_Width / thickness

The width correction consists of subtracting the zone that does not contribute to buckling resistance in the fixed ends. This zone depends on the shape type of the section. Usually, the radii of the fillet in hot rolled shapes or the weld throats in welded shapes determine the deduction zone. The values of the corrected width that CivilFEM uses for each shape type include:

- **Welded Shapes:**

Double T section:

Internal webs or flanges:

Corrected width = d

d	Web free depth

Outstand flanges:

Corrected width $\frac{B}{2}-\frac{T_w}{2}-r_1$

Where:

B	Flanges width
T_w	Web thickness
r_1	Radius of fillet

T section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width = B/d

C section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width	$B - T_w - r_1$

L section:

Corrected width = $\sqrt{I_1^2 + I_2^2}$

l_1, l_2 Angle flange width

Box section:

Internal webs:

Corrected width = H

H Height

Internal flanges:

Corrected width $= B - 2 \cdot T_w$

T_w Web thickness

Circular hollow section

Corrected width = H

- **Rolled Shapes:**

  Double T section:

  Internal webs or flanges:

Corrected width = d

d Web free depth

Outstand flanges:

Corrected width = B/2

B Flanges width

T Section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width = B/2

C Section:

Internal webs or flanges:

Corrected width = d

Outstand flanges:

Corrected width = B

L Section:

Corrected width = $= \sqrt{l_1^2 + l_2^2}$

l_1, l_2 Angle flange width

Box section:

Internal webs:

Corrected width = d

Internal flanges:

Corrected width $= B - 3 \cdot T_f$

T_f Flanges thickness

Pipe section:

Corrected width = H

2. The limit listed below for width to thickness ratio. This limit depends on the material parameter ε and the normal stress distribution in the plate section. The latter value is given by the following parameters: α, Ψ and k_0, and the plate type, internal or outstand; the outstand case depends on if the free end is under tension or compression.

$$\text{Limit (class)} = f(\varepsilon, \alpha, \Psi, k_0)$$

$$\varepsilon = \sqrt{235/f_y}\ (f_y \text{ in } N/mm^2)$$

where:

α	Compressed length / total length
ψ	σ_2/σ_1
k_0	Buckling factor
σ_2	The higher stress in the plate ends.
σ_1	The lower stress in the plate ends.

A linear stress distribution on the plate is assumed.

The procedure to determine the section class is as follows:

1. Obtain stresses at first plate ends from the stresses applied on the section, properly filtered according to the check type requested by the user.
2. Calculate the parameters: α, Ψ and k_0
For internal plates:

$1 \geq \Psi \leq 0$	$k_0 \dfrac{8.2}{1.05 + \Psi}$
$0 > \Psi > -1$	$k_0 = 7.81 - 6.29 \cdot \Psi + 9.78 \cdot \Psi^2$
$-1 \geq \Psi - 2$	$k_0 = 5.98 \cdot (1 - \Psi)^2$
$\Psi \leq -2$	k_0 = infinite

For outstand plates with an absolute value of the stress at the free end greater than the corresponding value at the fixed end:

For $1 \geq \Psi \geq -1$

$$k_0 = 0.57 - 0.21 \cdot \Psi + 0.07 \cdot \Psi^2$$

For $-1 > \Psi$

$$k_0 = \text{infinite}$$

For outstand plates with an absolute value of the stress at the free end lower than the corresponding value at the fixed end:

For $1 \geq \Psi \geq 0$

$$k_0 = \frac{0.578}{\Psi + 0.34}$$

For $0 > \Psi \geq -1$

$$k_0 = 1.7 - 5 \cdot \Psi + 17.1 \cdot \Psi^2$$

For $-1 > \Psi$

$$k_0 = \text{infinite}$$

Cases in which $k_0 = infinite$ are not included in Structural code. With these cases, the plate is considered to be practically in tension and it will not be necessary to determine the class. These cases have been included in the program to avoid errors, and the value $k_0 = infinite$ has been adopted because the resultant plate class is 1 and the plate reduction factor is ρ = 1 (the same values as if the whole plate was in tension). The reduction factor is used later in the effective section calculation.

3. Obtain the limiting proportions as functions of: α, Ψ and k_0 and the plate characteristics (internal, outstand: free end in compression or tension).

Internal plates:

$\text{Limit}(1) = 396\ \varepsilon\ /\ (13\ \alpha - 1)$ for $\alpha < 0.5$

$\text{Limit}(1) = 36\ \varepsilon\ /\ \alpha$ for $\alpha < 0.5$

$\text{Limit}(2) = 456\ \varepsilon\ /\ (13\ \alpha - 1)$ for $\alpha \geq 0.5$

$\text{Limit}(2) = 41.5\ \varepsilon\ /\ \alpha$ for $\alpha < 0.5$

$\text{Limit}(3) = 42\ \varepsilon\ /\ (0.67 + 0.33\ \Psi)$ for $\Psi > -1$

$\text{Limit}(3) = 62\ \varepsilon\ (1 - \Psi)\sqrt{(-\Psi)}$ for $\Psi \leq -1$

Outstand plates, free end in compression:

$$\text{Limit}(1) = 9\ \varepsilon\ /\ \alpha$$

$$\text{Limit}(2) = 10\ \varepsilon\ /\ \alpha$$

$$\text{Limit}(3) = 21\ \varepsilon\sqrt{K_0}$$

Outstand plates, free end in tension:

$$\text{Limit}(1) = \frac{9\ \varepsilon}{\alpha\sqrt{\alpha}}$$

$$\text{Limit}(2) = \frac{10\ \varepsilon}{\alpha\sqrt{\alpha}}$$

$$\text{Limit}(3) = 21\ \varepsilon\sqrt{K_0}$$

Above is the general equation used by the program to obtain the limiting proportions for determining plate classes. In addition, plates of Structural code may be checked according to special cases.

For example:

In sections totally compressed:

α= 1; Ψ = 1 for all plates

In sections under pure bending:

α = 0.5; Ψ = -1 for the web

α = 1; Ψ = 1 for compressed flanges

4. Obtain the plate class:

If		GeomRat	< Limit(1)	Plate Class = 1
If	Limit(1) ≤	GeomRat	< Limit(2)	Plate Class = 2
If	Limit(2) ≤	GeomRat	< Limit(3)	Plate Class = 3
If	Limit(3) ≤	GeomRat		Plate Class = 4

Repeat these steps (1,2,3,4) for each section plate.

5. Assign of the highest class of the plates to the entire section.
   In tubular sections, the section class is directly determined as if it were a unique plate, with GeomRat and the Limits calculated as follows:

   6. GeomRat = outer diameter/ thickness.

      $\text{Limit}(1) = 50\ \varepsilon^2$

      $\text{Limit}(2) = 70\ \varepsilon^2$

      $\text{Limit}(3) = 90\ \varepsilon^2$

For class 4 sections, the section resistance is reduced, using the effective width method.

For each section plate, the effective lengths at both ends of the plate and the reduction factors ρ_1 and ρ_2 are calculated. These factors relate the length of the effective zone at each plate end to its width.

Effective_length_end 1 = $\text{plate\_width}^{*}\rho_1$

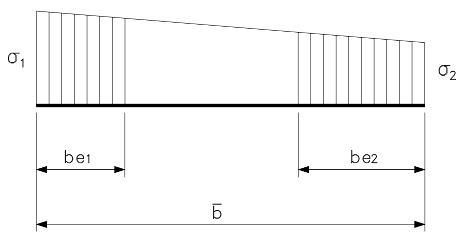
Effective_length_end 2 = $\text{plate\_width}^{*}\rho_2$

The following formula from Structural code has been implemented for this process:

$$\Psi = \sigma_2/\sigma_1$$

1. Internal plates:

   For $0 \le \Psi \le 1$ (Both ends compressed)



$$b_{eff} = \rho\ \bar{b}$$

$$b_{e1} = 2\ b_{eff}/\ (5 - \Psi)$$

$$b_{e2} = b_{eff} - b_{e1}$$

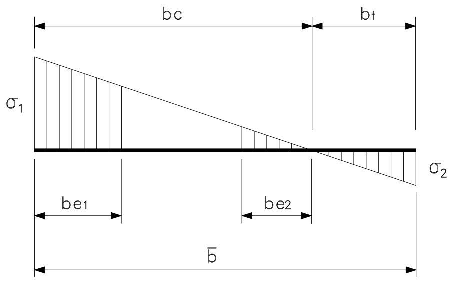
$$\rho_1 = \frac{b_{e1}}{plate\_width}$$

$$\rho_2 = \frac{b_{e2}}{plate\_width}$$

$$\bar{b} = \text{ corrected plate width}$$

plate_width = real plate width

For $\Psi < 0$ (end 1 in compression and end 2 in tension)



$$b_{eff} = \rho\, b_c = \rho\, \bar{b}/(1-\Psi)$$

$$b_{e1} = 0.4\, b_{eff}$$

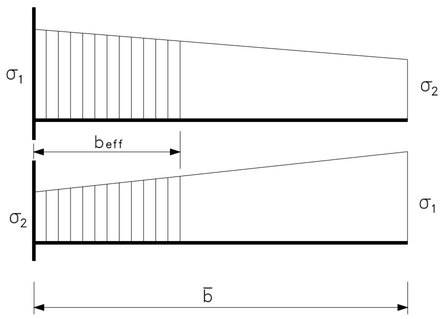
$$b_{e2} = 0.6\, b_{eff}$$

$$\rho_1 = \frac{b_{e1}}{plate\_width}$$

$$\rho_2 = \frac{b_{e2} + bt}{plate\_width}$$

2. Outstand plates:

For $0 \leq \Psi \leq 1$ (Both ends in compression: end 1 fixed, end 2 free)

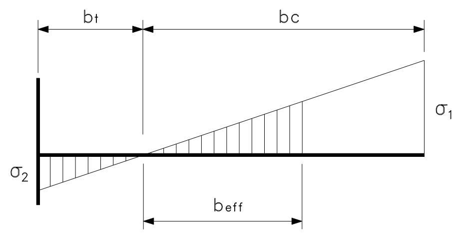


$$b_{eff} = \rho\, \bar{b}$$

$$\rho_1 = \frac{b_{eff}}{plate\_width}$$

$$\rho_2 = 0$$

For $\Psi < 0$(end 1 fixed and in tension, end 2 free and in compression)

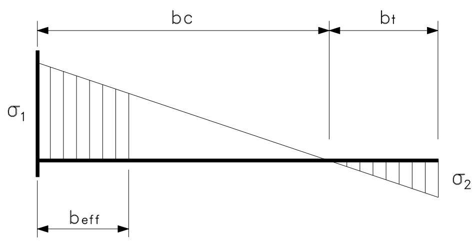


$$b_{eff} = \rho b_c = \rho c/(1-\Psi)$$

$$\rho_1 = \frac{b_{eff} + bt}{plate\_width}$$

$$\rho_2 = 0$$

For $\Psi < 0$ (end 1 fixed and in compression, end 2 free and in tension)



$$b_{eff} = \rho b_c = \rho c/(1-\Psi)$$

$$\rho_1 = \frac{b_{eff}}{plate\_width}$$

$$\rho_2 = \frac{b_t}{plate\_width}$$

If end 2 is the fixed end, the values ρ_1 and ρ_2 are switched.

The global reduction factor ρ is obtained by as follows:

For internal compression elements

For $\bar{\lambda}_p > 0.673$

$$\rho = \frac{\bar{\lambda}_p - 0.055(3+\Psi)}{\bar{\lambda}_p{}^2}$$

For $\bar{\lambda}_p \leq 0.673$

$$\rho = 1$$

For outstands compression elements:

For $\bar{\lambda}_p > 0.748$

$$\rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p{}^2}$$

For $\bar{\lambda}_p \leq 0.748$

$$\rho = 1$$

The plate slendernesss given by:

$$\bar{\lambda}_p = \frac{\bar{b}/t}{28.4\varepsilon\sqrt{k_0}}$$

where:

$\bar{b}$ = corrected plate width

t = relevant thickness

ε = material parameter

k_0 = buckling factor

To determine effective section properties, three steps are followed:

1. *Effective widths of flanges* are calculated from factors α and Ψ these factors are determined from the gross section properties. As a result, an intermediate section is obtained with reductions taken in the flanges only.
2. The resultant section properties are obtained and factors α and Ψ are calculated again.

3. *Effective widths of webs* are calculated so that the finalized effective section is determined. Finally, the section properties are recalculated once more.

The recalculated section properties are included in the effective section data table. Checking can be accomplished with the gross, net or effective section properties, according to the section class and checking type.

Each checking type follows a specific procedure that will be explained in the following sections.

7.1.7. Checking of Members in Axial Tension

Corresponds to chapter 6.2.3 in Structural Code (Annex 22).

1. Forces and moments selection.
   The forces and moments considered for this checking type are:

N_{Ed}= FX Design value of the axial force (positive if tensile, element not processed if compressive).

2. Class definition and effective section properties calculation.
   For this checking type, the section class is always 1 and the considered section is either the gross or net section.
3. Criteria calculation.
   For members under axial tension, the general criterion *Crt_TOT* is checked at each section. This criterion coincides with the axial criterion *Crt_N*.

$$N_{Ed} \leq N_{t.Rd} \rightarrow \quad Crt\_TOT = Crt\_N = \frac{N_{Ed}}{N_{t.Rd}} \leq 1$$

where $N_{t.Rd}$ is the design tension resistance of the cross-section, taken as the smaller value of:

$$N_{Pl,Rd} = Af_y/Y_{M\,0}$$ plastic design strength of the gross cross-section

$$N_{u.Rd} = 0.9A_{net}\,f_u/Y_{M2}$$ ultimate design strength of the net cross-section

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table:

Result	**Concepts**	**Description**

NED	N_{Ed}	Design value of the tensile force.
NTRD	$N_{t.Rd}$	Design tensile strength of the cross-section.
CRT_N	$N_{Ed}/N_{t.Rd}$	Axial criterion.
CRT_TOT	$N_{Ed}/N_{t.Rd}$	Structural Code global criterion.
NPLRD	$N_{pl.Rd}$	Design plastic strength of the gross cross-section.
NURD	$N_{u.Rd}$	Ultimate design strength

7.1.8. Checking of Members in Axial Compression

Corresponds to chapter 6.2.4 in Structural Code (Annex 22).

1. Forces and moments selection.
   The forces and moments considered for this checking type are:

N_{Ed}= FX Design value of the axial force (positive if compressive, element not processed if tensile).

2. Class definition and effective section properties calculation.
   For this check type, the section class is always 1 and the considered section is the gross or net section.
3. Criteria calculation.
   For members in axial compression, the general criterion *Crt_TOT* is checked at each section. This criterion coincides with the axial criterion *Crt_N:*

$$N_{Ed} \leq N_{c.Rd} \rightarrow \text{Crt\_TOT} = \text{Crt\_N} = \frac{N_{Ed}}{N_{c.Rd}} \leq 1$$

where $N_{c.Rd}$ is the design compression resistance of the cross-section

Class 1,2 or 3 cross-sections:

$N_{c.Rd} = Af_y/\Upsilon_{M\,0}$ design plastic resistance of the gross section

Class 4 cross sections:

$$N_{c.Rd} = Af_{eff}f_y/\Upsilon_{M\,0}$$

4. Output results written in the CivilFEM results file (.CRCF) . Checking results: criteria and variables are described at the following table.

Result	**Concepts**	**Description**

NED	N_{Ed}	Design axial force.
NCRD	$N_{c.Rd}$	Design compression strength of the cross-section.
CRT_N	$N_d/N_{c.Rd}$	Axial criterion.
CRT_TOT	$N_d/N_{c.Rd}$	Structural global criterion.
CLASS		Section Class.
AREA	A, A_{eff}	Area of the section (Gross or Effective).

7.1.9. Checking of Members under Bending Moment

Corresponds to chapter 6.2.5 in Structural Code (Annex 22).

1. Forces and moments selection.
   The forces and moments considered for this checking type are:

$M_{Ed} = MY \text{ or } MZ$ Design value of the bending moment along the relevant axis for bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of the section with the previously selected forces and moments if the selected option is partial or with all the forces and moments if the selected option is full. The entire calculation process is accomplished with the gross section properties.

3. Criteria calculation.
   For members subjected to a bending moment in the absence of shear force, the following condition is checked at each section:
   where:

$$|M_{Ed}| \leq M_{c.Rd} \rightarrow Crt\_TOT = Crt\_My = \left|\frac{M_{Ed}}{M_{c.Rd}}\right| \leq 1$$

M_{Ed} = design value of the bending moment

$M_{c.Rd}$ = design moment resistance of the cross-section

Class 1 or 2 cross-sections:

$$M_{c.Rd} = W_{pl} \cdot f_y / \Upsilon_{M0}$$

Class 3 cross sections:

$$M_{c.Rd} = W_{el} \cdot f_y \ / \ Y_{M\,0}$$

Class 4 cross sections:

$$M_{c.Rd} = W_{eff} \cdot f_y \ / \ Y_{M\,0}$$

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
MED	M_{Ed}	Design value of the bending moment.
MCRD	$M_{c.Rd}$	Design moment resistance of the cross-section.
CRT_M	$M_d/M_{c.Rd}$	Bending criterion.
CRT_TOT	$M_d/M_{c.Rd}$	Structural Code global criterion.
CLASS		Section Class.
W	$W_{el}, W_{pl} W_{eff}$	Used section modulus (Elastic, Plastic or Effective).

7.1.10. Checking of Members under Shear Force

Corresponds to chapter 6.2.6 in Structural Code (Annex 22)..

1. Forces and moments selection.
   The forces and moments considered for this checking type are:

$V_{Ed} = \ FZ \text{ or } FY$ Design value of the shear force perpendicular to the relevant axis of bending.

2. Class definition and effective section properties calculation.
   For this checking type, the section class is always 1 and the effective section is the gross section.
3. Criteria calculation.
   With members under shear force, the following condition is checked at each section:

$$|V_{Ed}| \leq V_{Pl.Rd} \rightarrow \ Crt\_TOT = Crt\_S = \left|\frac{V_{Ed}}{V_{Pl.Rd}}\right| \leq 1$$

where:

V_{Ed} design value of the shear force

$V_{Pl.Rd}$ design plastic shear resistance: $V_{Pl.Rd} = A_v \left(f_y/\sqrt{3}\right)/Y_{M0}$

A_v shear area, obtained subtracting from the gross area the summation of the flanges areas: $A_v = A - \sum \text{Flanges\_Area}$

Modifications to the previous computation of A_v are as follows:

- Rolled I and H sections, load parallel to web:
  $A_v = A_v + (t_w + 2r)t_f$
- Rolled channel sections, load parallel to web:
  $A_v = A_v + (t_w + r)t_f$

- Rolled I and H sections with load parallel to web:

  $A_v = A_v + (t_w + 2r)t_f$ but not less than η $h_w t_w$

- Rolled T shaped sections with load parallel to web:
  $A_v = 0.9 \cdot (A - b \cdot t_r)$

Where:

η η = 1.2 for steels with fy = 460 MPa

η= 1.0 for steels with fy > 460 MPa

h_w Web depth

t_w Web thickness

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
VED	V_{Ed}	Design value of the shear force.
VPLRD	$V_{pl.Rd}$	Design plastic shear resistance.
CRT_S	$V_d/V_{pl.Rd}$	Shear criterion.
CRT_TOT	$V_d/V_{pl.Rd}$	Structural Code global criterion.
CLASS		Section Class.
S_AREA	Av	Shear area.

7.1.11. Checking of Members under Bending Moment and Shear Force

Corresponds to chapter 6.2.8 in Structural Code (Annex 22).

1. Forces and moments selection.
   The forces and moments considered for this checking type are:

$V_{Ed} = $ FZ or FY — Design value of the shear force perpendicular to the relevant axis of bending.

$M_{Ed} = $ MY or MZ — Design value of the bending moment along the relevant axis of bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of the sections with the previously selected forces and moments if the selected option is partial or with all the forces and moments if the selected option is full. The entire calculation is accomplished with gross section properties.
3. Criteria calculation.
   For members subjected to bending moment and shear force, the following condition is checked at each section:

$$|M_{Ed}| \leq M_{V.Rd} \rightarrow \ \text{Crt\_TOT} = \text{Crt\_BS} = \left|\frac{M_d}{M_{V.Rd}}\right| \leq 1$$

Where:

$M_{V.Rd} = $ design resistance moment of the cross-section, reduced by the presence of shear.

The reduction for shear is applied if the design value of the shear force exceeds 50% of the design plastic shear resistance of the cross-section; written explicitly as:

$$V_{Ed} > 0.5\ V_{pl.Rd}$$

The design resistance moment is obtained as follows:

a. For double T cross-sections with equal flanges, bending about the major axis:

$$M_{V.Rd} = \left(W_{pl} - \frac{\rho A_v^2}{4t_w}\right) f_y / \gamma_{M0}$$

$$\rho = \left(\frac{2V_{Ed}}{V_{pl.Rd}} - 1\right)^2$$

$$A_w = h_w t_w$$

b. For other cases the yield strength is reduced as follows:

$$f_y = f_y(1 - \rho)$$

Note: This reduction of the yield strength fy is applied to the entire section. Structural code only requires the reduction to be applied to the shear area, and therefore, it is a conservative simplification.

For both cases, $M_{V.Rd}$ is the smaller value of either $M_{V.Rd}$ or $M_{C.Rd}$.

$M_{C.Rd}$ is the design moment resistance of the cross-section, calculated according to the class.

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
MED	M_{Ed}	Design value of the bending moment.
VED	V_{Ed}	Design value of the shear force.
MVRD	$M_{v.Rd}$	Reduced design resistance moment of the cross-section.
CRT_BS	$M_d/M_{v.Rd}$	Bending and Shear criterion.
CRT_TOT	$M_d/M_{v.Rd}$	Structural code global criterion.
CLASS		Section Class.
S_AREA	A_v	Shear area.
W	W_{el}, W_{pl}, W_{eff}	Used section modulus (Elastic, Plastic or Effective).
VPLRD	$V_{pl.Rd}$	Design plastic shear resistance.
RHO	ρ	Reduction factor.

7.1.12. Checking of Members under Bending Moment and Axial Force

Corresponds to chapter 6.2.9 in Structural Code (Annex 22).

1. Forces and moments selection.
   The forces and moments considered for this checking type are:

$N_{Ed} = FX$ Design value of the axial force.

$M_{y.Ed} = MY \text{ or } MZ$ Design value of the bending moment along the relevant axis of bending.

$M_{z.Ed} = MZ \text{ or } MY$ Design value of the bending moment about the secondary axis of bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of the sections with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. These calculations are accomplished with the gross section properties.

3. Criteria calculation.
   For members subjected to bi-axial bending and in absence of shear force, the following conditions at each section are checked:

Class 1 and 2 sections:

$$\left(\frac{M_{y.Ed}}{M_{Ny.Rd}}\right)^{\alpha} + \left(\frac{M_{z.Ed}}{M_{Nz.Rd}}\right)^{\beta} \leq 1$$

This condition is equivalent to:

$$Crt\_TOT = (Crt\_My)^{\alpha} + (Crt\_Mz)^{\beta} \leq 1$$

$$Crt\_My = \left(\frac{M_{y.Ed}}{M_{Ny.Rd}}\right)$$

$$Crt\_Mz = \left(\frac{M_{z.dE}}{M_{Nz.Rd}}\right)$$

Where $M_{Ny.Rd}$ and $M_{Nz.Rd}$ are the design moment resistance of the cross-section, reduced by the presence of the axial force:

$$M_{Ny.Rd} = My_{pl.Rd}\left[1 - \left({N_{Ed}}/{N_{pl.Rd}}\right)^2\right]$$

$$M_{Nz.Rd} = Mz_{pl.Rd}\left[1 - \left({N_{Ed}}/{N_{pl.Rd}}\right)^2\right]$$

Where α and β are constants, which may take the following values:

For I and H sections:

$$\alpha = 2 \quad \text{and } \beta = 5n \quad \beta \geq 1$$

For circular tubes:

$$\alpha = 2 \quad \text{and } \beta = 2$$

For rectangular hollow sections:

$$\alpha = \beta = \frac{1.66}{1-1.13n^2}$$

$$\text{but} \quad \alpha = \beta \leq 6$$

For solid rectangles and plates (the rest of sections):

$$n = \left(\frac{N_{Ed}}{N_{pl.Rd}}\right)$$

Furthermore, the code specifies that in the case of rolled shapes for I or H sections or other sections with flanges, it is not necessary to reduce the design plastic strength for bending around the y-y axis due to the axial force if the following two conditions are fulfilled:

$$N_d \leq 0.25 \cdot N_{pl.Rd} \cdot y$$

$$N_d \leq \frac{0.5 \cdot h_w \cdot t_w \cdot f_y}{\gamma_{M0}}$$

(if it does not reach half the tension strength of the web)

The same is applicable for bending around the z-z axis due to the axial force. There is no reduction when the following condition is fulfiled:

$$N_d \leq \frac{h_w \cdot t_w \cdot f_y}{\gamma_{M0}}$$

In absence of $M_{z.d}$, the previous check can be reduced to:

$$\left(\frac{M_{y.Ed}}{M_{Ny.Rd}}\right) \leq 1$$

Condition equivalent to:

$$Crt\_TOT = Crt\_My = \left(\frac{M_{y.Ed}}{M_{Ny.Rd}}\right)$$

Class 3 sections (without holes for fasteners):

$$\left(\frac{N_{Ed}}{Af_{yd}}\right)+\left(\frac{M_{y.Ed}}{W_{el.y}f_{yd}}\right)+\left(\frac{M_{z.Ed}}{M_{el.z}f_{yd}}\right)\leq 1$$

Condition equivalent to:

Crt_TOT = Crt_N + Crt_My + Crt_Mz ≤ 1

$$Crt\_N=\left(\frac{N_{Ed}}{Af_{yd}}\right)$$

$$Crt\_My=\left(\frac{M_{y.Ed}}{W_{el.y}f_{yd}}\right)$$

$$Crt\_Mz=\left(\frac{M_{z.Ed}}{W_{el.y}f_{yd}}\right)f_{yd}={}^{f_y}/_{\gamma_{M0}}$$

Where $M_{el.y}$ is the elastic resistant modulus about the y axis and $W_{el.z}$ is the elastic resistant modulus about the z axis.

In absence of $M_{z.d}$, the above criterion becomes:

$$\left(\frac{N_{Ed}}{Af_{yd}}\right)+\left(\frac{M_{y.Ed}}{W_{el.y}f_{yd}}\right)\leq 1$$

Which is equivalent to:

Crt_TOT = Crt_N + Crt_My ≤ 1

$$Crt\_N\left(\frac{N_{Ed}}{Af_{yd}}\right)$$

$$Crt\_My=\left(\frac{M_{y.Ed}}{W_{el.y}f_{yd}}\right)$$

Class 4 sections:

$$\left(\frac{N_{Ed}}{A_{eff}f_{yd}}\right)+\left(\frac{M_{y.Ed}+N_{Ed}e_{Ny}}{W_{eff.y}f_{yd}}\right)+\left(\frac{M_{z.Ed}+N_{Ed}e_{Ny}}{W_{eff.z}f_{yd}}\right)$$

Condition equivalent to:

Crt_TOT = Crt_N + Crt_My + Crt_Mz ≤ 1

$$Crt\_N=\left(\frac{N_{Ed}}{A_{eff}f_{yd}}\right)$$

$$Crt\_My=\left(\frac{M_{Ey.d}+N_{Ed}e_{Ny}}{W_{eff.y}f_{yd}}\right)$$

$$Crt\_Mz=\left(\frac{M_{z.Ed}+N_{Ed}e_{Ny}}{W_{eff.z}f_{yd}}\right)$$

Where:

A_{eff}	effective area of the cross-section
$W_{eff.y}$	effective section modulus of the cross-section when subjected to a moment about the y axis
$W_{eff.z}$	effective section modulus of the cross-section when subjected to a moment about the z axis
e_{Ny}	shift of the center of gravity along the y axis
e_{Nz}	shift of the center of gravity along the z axis

Without $M_{z.d}$, the above criterion becomes:

$$\left(\frac{N_{Ed}}{A_{eff}f_{yd}}\right)+\left(\frac{M_{y.Ed}+N_{Ed}e_{Ny}}{W_{eff.y}f_{yd}}\right)+\left(\frac{N_{Ed}e_{Ny}}{W_{eff.z}f_{yd}}\right)\leq 1$$

which is equivalent to:

Crt_TOT = Crt_N + Crt_My + Crt_Mz ≤ 1

$$\text{Crt\_N}=\left(\frac{N_{Ed}}{A_{eff}f_{yd}}\right)$$

$$\text{Crt\_My}=\left(\frac{M_{y.Ed}+N_{d}e_{Ny}}{W_{eff}f_{yd}}\right)$$

$$\text{Crt\_Mz}=\left(\frac{N_{Ed}e_{Ny}}{W_{eff.z}f_{yd}}\right)$$

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
NED	N_{Ed}	Design value of the axial force.
MYED	$M_{y.Ed}$	Design value of the bending moment about Y axis.
MZED	$M_{z.Ed}$	Design value of the bending moment about Z axis.
NCRD	$A\cdot f_{yd,}$ $A_{eff}\cdot f_{yd}$	Design compression resistance of the cross-section

Result	Concepts	Description
MNYRD	$M_{Ny\cdot Rd,} W_{el.y} \cdot f_{yd,}$ $W_{eff\cdot y} \cdot f_{yd}$	Reduced design moment resistance of the cross-section about Y axis
MNZRD	$M_{Nz.Rd,} W_{el.z} \cdot f_{yd,}$ $W_{eff\cdot} Z \cdot f_{yd}$	Reduced design moment resistance of the cross-section about Z axis
CRT_N	N_{Ed} / N_{cRd}	Axial criterion
CRT_MY	M_{yEd} / M_{NyRd}	Bending criterion along Y
CRT_MZ	M_{zEd} / M_{NzRd}	Bending criterion along Z
ALPHA	α	Alpha constant
BETA	β	Beta constant
CRT_TOT	Crt_tot ≤ 1	Structural Code global criterion
CLASS		Section Class
AREA	A, A_{eff}	Area of the section utilized (Gross or Effective)
WY	$W_{el.y,} W_{pl.y,} W_{eff.y}$	Used section Y modulus (Elastic, Plastic or Effective)
WZ	$W_{el.z,} W_{pl.z,} W_{eff.z}$	Used section Z modulus (Elastic, Plastic or Effective)
SIGXED	$\sigma_{X.Ed}$	Maximum longitudinal stress
ENY	e_{Ny}	Shift of the Z axis in Y direction
ENZ	e_{Nz}	Shift of the Y axis in Z direction
USE_MY	$M_{y.Ed} + N_{Ed} \cdot e_{Ny}$	Modified design value of the bending moment about Y axis
USE_MZ	$M_{z.Ed} + N_{Ed} \cdot e_{Nz}$	Modified design value of the bending moment about Z axis
PARM_N	n	Parameter n

7.1.13. Checking of Members under Bending, Shear and Axial Force

Corresponds to chapter 6.2.10 in Structural Code (Annex 22)..

1. Forces and moments selection. The forces and moments considered for this checking type are:

$N_{Ed} = FX$	Design value of the axial force.
$V_{y.Ed} = FY \text{ or } FZ$	Design value of the shear force perpendicular to the secondary axis of bending.
$V_{y.Ed} = FY \text{ or } FZ$	Design value of the shear force perpendicular to the relevant axis of bending.
$M_{y.Ed} = MY \text{ or } MZ$	Design value of the bending moment about the relevant axis of bending.
$M_{z.Ed} = MZ \text{ or } MY$	Design value of the bending moment about the secondary axis of bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of the sections with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. The entire calculation is accomplished with the gross section properties.
3. Criteria calculation.
   For members subjected to bending, axial and shear force, the same conditions of the bending +axial force and bi-axial bending are checked at each section, reducing the design plastic resistance moment for the presence of shear force.
   The shear force effect is taken into account when it exceeds 50% of the design plastic resistance of the cross-section. In this case, both the axial and the shear force are taken into account.
   The axial force effects are included as stated in the previous section, and the shear force effects are taken into account considering a yield strength for the cross-section, reduced by the factor (1-ρ), as follows:

$$f_{yd} = f_y(1-\rho)/\gamma_{M\,0}$$

where:

$$\rho = \left(2V_{Ed}/V_{pl.Rd} - 1\right)^2 \quad \text{for} \quad V_{Ed}/V_{pl.Rd} > 0.5$$

$$\rho = 0 \qquad \text{for } V_{Ed}/V_{pl.Rd} < 0.5$$

This yield strength reduction is selectively applied to the resistance of the cross-section along each axis, according to the previous conditions.

Note: The yield strength reduction is applied to the entire cross-section; however, Structural code only requires the reduction to be applied to the shear area. Thus, it is a conservative simplification.

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
NED	N_{Ed}	Design value of the axial force.
VZED	V_{Ed}	Design value of the shear force.
VYED	V_{Ed}	Design value of the shear force.
MYED	$M_{y.Ed}$	Design value of the bending moment about Y axis.
MZED	$M_{y.Ed}$	Design value of the bending moment about Z axis.
NCRD	$A \cdot f_{yd,}$ $A_{eff} \cdot f_{yd}$	Design compression resistance of the cross-section.
MNYRD	$M_{Ny.Rd,}$ $W_y \cdot f_{yd} \cdot (1-\rho)$	Reduced design moment Y resistance of the cross-section.
MNZRD	$M_{Nz \cdot Rd,}$ $W_z \cdot f_{yd} \cdot (1-\rho)$	Reduced design moment Z resistance of the cross-section.
CRT_N	N_{Ed}/N_{cRd}	Axial criterion.
CRT_MY	M_{yEd}/M_{NyRd}	Bending Y criterion.
CRT_MZ	M_{zEd}/M_{NzRd}	Bending Z criterion.
ALPHA	α	Alpha constant.
BETA	β	Beta constant.

Result	Concepts	Description
RHO_Y	ρ	Reduction factor for MNYRD.
RHO_Z	ρ	Reduction factor for MNZRD.
CRT_TOT	$Crt\_tot \leq 1$	Structural code global criterion.
AREA	A, A_{eff}	Used area of the section (Gross or Effective).
WY	$W_{el.y,} W_{pl.y,} W_{eff.y}$	Used section Y modulus (Elastic, Plastic or Effective).
WZ	$W_{el.z,} W_{pl\cdot z,} W_{eff\cdot z}$	Used section Z modulus (Elastic, Plastic or Effective).
SIGXED	$\sigma_{x.Ed}$	Maximum longitudinal stress.
ENY	e_{Ny}	Shift of the Z axis in Y direction.
ENZ	e_{Nz}	Shift of the Y axis in Z direction.
USE_MY	$M_{y\cdot Ed} + N_{Ed} \cdot e_{Nz}$	Modified design value of the bending moment about Y axis.
USE_MZ	$M_{Z.Ed} + N_{Ed} \cdot e_{Ny}$	Modified design value of the bending moment about Z axis.
SHY_AR	A_v	Shear Y area.
SHZ_AR	A_v	Shear Z area.
PARM_N	n	Parameter n.

7.1.14. Checking for Buckling of Members in Compression

Corresponds to chapter 6.3.1 in Structural Code (Annex 22)..

1. Forces and moments selection.
   The forces and moments considered in this checking type are:
   $N_{Ed} = FX$ Design value of the axial force (positive if compressive, otherwise element is not processed).

2. Class definition and effective section properties calculation.
   The section class is determined by the sections general processing with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. The entire calculation is accomplished with the gross section properties.

3. Criteria calculation.
   When checking the buckling of compression members, the criterion is given by:

$$N_{Ed} \leq N_{b.Rd} \quad \rightarrow \quad Crt\_TOT = Crt\_CB = \frac{N_{Ed}}{N_{p.Rd}} \leq 1$$

where:

$N_{b.Rd}$ Design buckling resistance. $N_{b.Rd} = \chi\ \beta\ A\, f_y/\gamma_{M1}$

β = 1 for class 1, 2 or 3 sections.

$\beta = A_{eff}/A$ for class 4 sections.

χ Reduction factor for the relevant buckling mode, the program does not consider the torsional or the lateral-torsional buckling.

The χ calculation in members of constant cross-section may be determined from:

$$\chi = \frac{1}{\emptyset + \left(\emptyset^2 - \bar{\lambda}^2\right)^{1/2}} \leq 1$$

$$\emptyset = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

where α is an imperfection factor that depends on the buckling curve. This curve depends on the cross-section type, producing the following values for α:

Section type	**Limits**	**Buckling axis**	**Steel f_y**	**Buckling curve**	**α**
Rolled I	h/b>1.2 and t≤40mm	y – y	< 460 MPa	a	0.21
			≥ 460 MPa	a0	0.13
Rolled I	h/b>1.2 and t≤40mm	z – z	< 460 MPa	b	0.34
			≥ 460 MPa	a0	0.13
Rolled I	h/b>1.2 and 40mm<t≤ 100mm	y – y	< 460 MPa	b	0.34
			≥ 460 MPa	a	0.21
Rolled I	h/b>1.2 and 40mm<t≤ 100mm	z – z	< 460 MPa	c	0.49
			≥ 460 MPa	a	0.21
Welded I	h/b≤1.2 and t≤100mm	y – y	< 460 MPa	b	0.34
			≥ 460 MPa	a	0.21
Welded I	h/b≤1.2 and t≤100mm	z – z	< 460 MPa	c	0.49

			≥ 460 MPa	a	0.21
Rolled I	t>100mm	y – y	< 460 MPa	d	0.76
			≥ 460 MPa	c	0.49
Rolled I	t>100mm	z – z	< 460 MPa	d	0.76
			≥ 460 MPa	c	0.49
Welded I	t≤40mm	y – y	all	b	0.34
Welded I	t≤40mm	z – z	all	c	0.49
Welded I	t >40mm	y – y	all	c	0.49
Welded I	t >40mm	z – z	all	d	0.76
Pipes	Hot finished	all	< 460 MPa	a	0.21
			≥ 460 MPa	a0	0.13
	Cold formed	all	all	c	0.49
Reinforced box sections	Thick weld: a/t>0.5 b/t<30 h/tw<30	all	all	c	0.49
	In other case	all	all	b	0.34
U, T, plate	-	all	all	c	0.49
L	-	all	all	b	0.34

$$\bar{\lambda} = \left[\beta_A A f_y / N_{cr}\right]^{1/2}$$

Where N_{cr} is the elastic critical force for the relevant buckling mode. (See section for Critical Forces and Moments Calculation).

In the case of angular sections, the buckling length will be taken as the highest among the buckling lengths on the Y and Z axis.

4. The elastic critical axial forces are calculated in the planes XY (Ncr_{xy}) and XZ (Ncr_{xz}) and the corresponding values of χ_{xy} and χ_{xz}, and the correspondent to the principal axis Ncr_u and Ncr_v and the values for χ_u and χ_v taking the smaller one as the final value for χ.

$$\chi = \min(\chi_{xy}, \chi_{xz}, \chi_u, \chi_v)$$

5. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
NED	N_{Ed}	Design value of the compressive force.
NBRD	$N_{b.Rd}$	Design buckling resistance of a compressed member.
CRT_CB	$N_d/N_{b.Rd}$	Compression buckling criterion.
CRT_TOT	$N_d/N_{b.Rd}$	Structural code global criterion.
CHI	$Min\{\chi_y, \chi_z\}$	Reduction factor for the relevant buckling mode.
BETA_A	β_A	Ratio of the used area to gross area.
AREA	A	Area of the gross section.
CHI_Y	χ_y	Reduction factor for the relevant My buckling mode.
CHI_Z	χ_z	Reduction factor for the relevant Mz buckling mode.
CHI_V	χ_v	Reduction factor for the principal axis V.
CHI_U	χ_u	Reduction factor for the principal axis U.
CLASS		Section Class.
PHI_Y	$\emptyset_y$	Parameter Phi for bending My.
PHI_Z	$\emptyset_z$	Parameter Phi for bending Mz.
PHI_V	$\emptyset_v$	Parameter Phi for the principal axis V.
PHI_U	$\emptyset_u$	Parameter Phi for the principal axis U.
LAM_Y	λ_y	Non-dimensional reduced slenderness for bending My.
LAM_Z	λ_z	Non-dimensional reduced slenderness for bending Mz.
LAM_V	λ_v	Non-dimensional reduced slenderness for the principal axis V.
LAM_U	λ_u	Non-dimensional reduced slenderness for the

Result	Concepts	Description
		principal axis U.
NCR_Y	N_{cr}	Elastic critical force for the relevant My buckling mode.
NCR_Z	N_{cr}	Elastic critical force for the relevant Mz buckling mode.
NCR_V	N_{cr}	Elastic critical force for the principal axis V.
NCR_U	N_{cr}	Elastic critical force for the principal axis U.
ALP_Y	α_y	Imperfection factor for bending My.
ALP_Z	αz	Imperfection factor for bending Mz.

7.1.15. Checking for Lateral-Torsional Buckling of Beams Subjected to Bending

Corresponds chapter 6.3.2 in EN Structural Code (Annex 22)..

1. Forces and moments selection.
   The forces and moments considered for this checking type are:
   $M_{Ed} = \text{MY or MZ}$ Design value of the bending moment about the relevant axis of bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of sections with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. The entire calculation is accomplished with the gross section properties.
3. Criteria calculation.
   When checking for lateral-torsional buckling of beams, the criterion shall be taken as:

$$|M_{Ed}| \leq M_{b.Rd} \quad \rightarrow \quad Crt\_TOT = Crt\_LT = \left|\frac{M_{Ed}}{M_{b.Rd}}\right| \leq 1$$

where:

$M_{b.Rd}$ Design buckling resistance moment of a laterally unrestrained beam. $M_{b.Rd} = \chi_{LT}\beta_w W_{pl.y} f_y \; / \; \gamma_{M1}$

β_w = 1 for class 1and 2 sections.

$\beta_w = W_{el.y}/W_{pl.y}$ for class 3 sections.

$\beta_w = W_{eff.y}/W_{pl.y}$ for class 4 sections.

χ_{LT} Reduction factor for lateral-torsional buckling.

The value of χ_{LT} is calculated as:

$$\chi_{LT} = \frac{1}{\emptyset_{LT} + \left(\emptyset_{LT}^2 - \bar{\lambda}_{LT}^2\right)^{1/2}} \leq 1$$

$$\emptyset_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right]$$

$$\bar{\lambda}_{LT} = \left[\beta_w W_{pl.y} f_y / M_{cr}\right]^{1/2}$$

Where:

α_{LT} is the imperfection factor for lateral-torsional buckling:

Section type	Limits	Buckling curve	α
Rolled I	h/b≤2 h/b>2	a b	0.21 0.34
Welded I	h/b≤2 h/b>2	c d	0.49 0.76
Others			0.76

M_{cr} is the elastic critical moment for lateral-torsional buckling.

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
MED	M_{Ed}	Design value of the bending moment.

MBRD	$M_{b.Rd}$	Buckling resistance moment of a laterally unrestrained beam.
CRT_LT	$M_d/M_{b.Rd}$	Lateral-torsional buckling criterion.
CRT_TOT	$M_d/M_{b \cdot Rd}$	Structural code global criterion.
CLASS		Section Class.
CHI_LT	χ_{LT}	Reduction factor for lateral-torsional buckling.
BETA_W	β_W	Ratio of the used modulus to plastic modulus.
WPL	$W_{pl.y}$	Plastic modulus.
PHI_LT	$\emptyset_{LT}$	Parameter Phi for lateral-torsional buckling.
LAM_LT	λ_{LT}	Non-dimensional reduced slenderness.
MCR	Mcr	Elastic critical moment for lateral-torsional buckling.
ALP_LT	α_{LT}	Imperfection factor for lateral-torsional buckling.

7.1.16. Checking for Lateral-Torsional Buckling of Members Subjected to Bending and Axial Compression

Corresponds to chapter 6.3.3 in Structural Code (Annex 22).

1. Forces and moments selection.
   The forces and moments considered in this checking type are:

   N_{Ed}= FX — Design value of the axial compression (positive if compressive, otherwise element not processed if tensile).

   $M_{y.Ed}$= MY or MZ — Design value of the bending moment about the relevant axis of bending.

   $M_{z.Ed}$ = MZ or MY — Design value of the bending moment about the secondary axis of bending.

2. Class definition and effective section properties calculation.
   The section class is determined by the general processing of sections with the previously selected forces and moments if the selected option is partial, or with all the forces and moments if the selected option is full. The entire calculation is accomplished with the gross section properties.
3. Criteria calculation.

The following criterion will always be calculated:

$$\left(\frac{N_{Ed}}{N_{b,Rd\,1}}\right)+\left(K_y C_{my}\frac{M_{y,Ed}+e_{N,y}N_{Ed}}{M_{yb,Rd\,1}}\right)+\left(\alpha_z K_y C_{mz}\frac{M_{z,Ed}+e_{N,z}N_{Ed}}{M_{zb,Rd\,1}}\right)\leq 1$$

Crt_1 = Crt_N1 + Crt_My1 + Crt_Mz1 ≤ 1

Elements without torsional buckling:

$$\left(\frac{N_{Ed}}{N_{b,d\,2}}\right)+\left(\alpha_y K_y C_{my}\frac{M_{y,Ed}+e_{N,y}N_{Ed}}{M_{yb,Rd\,2}}\right)+\left(K_z C_{mz}\frac{M_{z,Ed}+e_{N,z}N_{Ed}}{M_{zb,Rd\,2}}\right)\leq 1$$

Elements which may have torsional buckling:

$$\left(\frac{N_{Ed}}{N_{b.Rd\,2}}\right)+\left(K_{yLT}\frac{M_{y,Ed}+e_{N,y}N_{Ed}}{M_{yb,Rd\,2}}\right)+\left(K_z C_{mz}\frac{M_{z,Ed}+e_{N,z}N_{Ed}}{M_{zb,Rd\,2}}\right)\leq 1$$

→ Crt_2 = Crt_N2 + Crt_My2 + Crt_Mz2 ≤ 1

→ Crt_TOT = Max (Crt_1, Crt_2)

Where:

$\text{Crt\_N1} = \left(\frac{N_{Ed}}{N_{b,Rd\,1}}\right)$	Axial force criterion 1.
$\text{Crt\_My1} = \left(K_y C_{my}\frac{M_{y,Ed}+e_{N,y}N_{Ed}}{M_{yb,Rd\,1}}\right)$	Bending moment criterion for principal axis 1.
$\text{Crt\_Mz1} = \left(K_z C_{mz}\frac{M_{z,Ed}+e_{N,z}N_{Ed}}{M_{zb,Rd\,2}}\right)$	Bending moment criterion for secondary axis 1
Crt_TOT1	General criterion 1.
$\text{Crt\_N2} = \left(\frac{N_{Ed}}{N_{b,Rd\,2}}\right)$	Axial force criterion 2.
$\text{Crt\_My2}\left(\alpha_y K_y C_{my}\frac{M_{y,Ed}+e_{N,y}N_{Ed}}{M_{yb,Rd\,2}}\right)$	Bending moment criterion 2 for principal axis without torsional buckling
$\text{Crt\_My2} = \left(K_{yLT}\frac{M_{y,Ed}+e_{N,y}N_{Ed}}{M_{yb,Rd\,2}}\right)$	Bending moment criterion 2 for principal axis when torsional buckling is considered.

$Crt\_Mz2 = \left(K_z C_{mz} \frac{M_{z,Ed} + e_{N,z} N_{Ed}}{M_{zb,Rd\,2}}\right)$	Bending moment criterion 2 for secondary axis.
Crt_TOT2	Criterion 2
Crt_TOT=max (Crt_TOT1, Crt_TOT2)	Global criterion.

Where:

$N_{b,Rd1} = \chi_y A f_y/\gamma_{M1} \quad M_{b,Rdy1} = \chi_{LT} W_y f_y/\gamma_{M1} \quad M_{b,Rdz1} = W_z f_y/\gamma_{M1}$

$N_{b,Rd2} = \chi_y A f_y/\gamma_{M1} \quad M_{b,Rdy2} = \chi_{LT} W_y f_y/\gamma_{M1} \quad M_{b,Rdz2} = W_z f_y/\gamma_{M1}$

($\chi_{LT} = 1.0$ when torsional buckling is not considered).

χ_y and χ_z are the reduction factors defined for the section corresponding to the check for Buckling of Compression Members.

χ_{LT} lateral buckling factor according to 6.3.2.2. Assumes the value of 1 for members not susceptible to torsional deformations.

$e_{N,y}$ and $e_{N,z}$ shifts of the centroid of the effective area relative to the centre of gravity of the gross section in class 4 members for y, z axes.

$C_{m,y}$, $C_{m,z}$ and $C_{m,LT}$ are equivalent uniform moment factors for flexural bending. These factors are entered as member properties at member level. (See C_{My}, C_{My} and C_{Mz}).

Checking Parameters:

Class	A	W_Y	W_z	α_Y	α_z	$e_{N,y}$	$e_{N,z} e_{N,z}$
1	A	$W_{pl,y}$	$W_{pl,z}$	0.6	0.6	0	0
2	A	$W_{pl,y}$	$W_{pl,z}$	0.6	0.6	0	0
3	A	$W_{el,y}$	$W_{el,z}$	0.8	1	0	0
4	A_{eff}	$W_{eff,y}$	$W_{eff,z}$	0.8	1	Depending on members and stresses	Depending on members and stresses

Interaction Factors:

Class	Section type	K_y	K_z	K_{yLT}
1 y 2	I, H	$1+(\bar{\lambda}_y-0.2)\frac{N_{Ed}}{\chi_y N_{C,Rd}}$	$1+(2\bar{\lambda}_z-0.6)\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	$1-\frac{0.1.\bar{\lambda}_z}{(C_{mLT}-0.25)}.\frac{N_{Ed}}{\chi_z N_{C,Rd}}\leq 0.6+\bar{\lambda}_z$
	RHS		$1+(\bar{\lambda}_z-0.2)\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	
3 y 4	All sections	$1+0.6\bar{\lambda}_y\frac{N_{Ed}}{\chi_y N_{C,Rd}}$	$1+0.6\bar{\lambda}_z\frac{N_{Ed}}{\chi_z N_{C,Rd}}$	$1-\frac{0.5.\bar{\lambda}_z}{(C_{mLT}-0.25)}.\frac{N_{Ed}}{\chi_z N_{C,Rd}}$

where:

λ_y y λ_z Limited slenderness values for y-y and z-z axes, less than 1.

$$N_{C,Rd}=A^*\frac{f_y}{\gamma_{M1}}$$

4. Output results are written in the CivilFEM results file (.CRCF). Checking results: criteria and variables are described in the following table.

Result	Concepts	Description
NED	N_{Ed}	Design value of the axial compression force.
MYED	$M_{y.Ed}$	Design value of the bending moment about Y axis.
MZED	$M_{z.Ed}$	Design value of the bending moment about Z axis.
NBRD1	$\chi_y\cdot A\cdot f_y/\gamma_{M1}$	Design compression resistance of the cross-section.
MYRD1	$\chi_{LT}\cdot W_y\cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Y axis.
MZRD1	$W_z\cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Z axis.
NBRD2	$\chi_z\cdot A\cdot f_y/\gamma_{M1}$	Design compression resistance of the cross-

Result	Concepts	Description
		section.
MYRD2	$\chi_{LT} \cdot W_y \cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Y axis.
MZRD2	$W_z \cdot f_y/\gamma_{M1}$	Reduced design moment resistance of the cross-section about Z axis.
K_Y	K_y	Parameter $K_{y.}$
K_Z	K_z	Parameter $K_{z.}$
K_LT	K_{LT}	Parameter $K_{LT.}$
CRT_N1	N_{Ed}/N_{cRd1}	Axial criterion.
CRT_MY1	$K_y C_{my}(M_{y,Ed} + N_{Ed} \cdot e_{Ny})/M_{yb,Rd1}$	Bending Y criterion.
CRT_MZ1	$\alpha_z \cdot K_z \cdot C_{mz}(M_{z,Ed} + N_{Ed} e_{Nz})/M_{zb,Rd1}$	Bending Z criterion.
CRT_1	CRT_N1+CRT_MY1+CRT_MZ1	Criterion 1
CRT_N2	N_{Ed}/N_{cRd2}	Axial criterion.
CRT_MY2	$KC_{my}(M_{y,Ed} + N_{Ed} \cdot e_{Ny})/M_{yRd2}$	Bending Y criterion. K=K_{LT}/C_{my} if torsion exists and if not present K=$\alpha_y K_y$
CRT_MZ2	$K_z \cdot C_{mz}(M_{z,Ed} + N_{Ed} e_{Nz})/M_{zb,Rd2}$	Bending Z criterion.
CRT_2	CRT_N2+CRT_MY2+CRT_MZ2	Criterion 2
CRT_TOT	Crt_tot $\leq$ 1	Structural code global criterion.
CLASS		Section Class.
CHIMIN	$\text{Min}\{\chi_y, \chi_z\}$	Reduction factor for the relevant buckling mode.
CHI_Y	χ_y	Reduction factor for the relevant My buckling mode.
CHI_Z	χ_z	Reduction factor for the relevant Mz buckling mode.

Result	Concepts	Description
CHI_LT	χ_{LT}	Reduction factor for lateral-torsional buckling.
AREA	$A, \ A_{eff}$	Used area of the section (Gross or Effective).
WY	$W_{el.y}, W_{pl,y}, W_{eff.y}$	Used section Y modulus (Elastic, Plastic or Effective).
WZ	$W_{el.z}, W_{pl,z}, W_{eff.z}$	Used section Z modulus (Elastic, Plastic or Effective).
ENY	e_{Ny}	Shift of the Z axis in Y direction.
ENZ	e_{Nz}	Shift of the Y axis in Z direction.
NCR_Y	N_{cr}	Elastic critical force for the relevant My buckling mode.
NCR_Z	N_{cr}	Elastic critical force for the relevant Mz buckling mode.
MCR	M_{cr}	Elastic critical moment for lateral-torsional buckling.
LAM_Y	λ_y	Non-dimensional reduced slenderness for bending My.
LAM_Z	λ_z	Non-dimensional reduced slenderness for bending Mz.
LAM_LT	λ_{LT}	Non-dimensional reduced slenderness for lateral-torsional buckling.

7.1.17. Critical Forces and Moments Calculation

The critical forces and moments $N_{cr\,xy}$, $N_{cr\,xz}$ and M_{cr}, are needed for the different types of buckling checks. They are calculated based on the following formulation:

$$N_{cr\,xy} = \frac{AE\pi^2}{\lambda_{xy}^2} = AE\left(\frac{\pi\, i_{xy}}{L_{xy}}\right)^2$$

$$N_{cr\,xz} = \frac{AE\pi^2}{\lambda_{xz}^2} = AE\left(\frac{\pi\, i_{xz}}{L_{xz}}\right)^2$$

where:

$N_{cr\,xy}$	Elastic critical axial force in plane XY.
$N_{cr\,xz}$	Elastic critical axial force in plane XZ.
A	Gross area.
E	Elasticity modulus.
λ_{xy}	Member slenderness in plane XY.
λ_{xz}	Member slenderness in plane XZ.
i_{xy}	Radius of gyration of the member in plane XY.
i_{xz}	Radius of gyration of the member in plane XZ.
L_{xy}	Buckling length of member in plane XY.
L_{xz}	Buckling length of member in plane XZ.

The buckling length in both planes is the length between the ends restrained against lateral movement and it is obtained from the member properties, according to the following expressions:

$$L_{xy} = L \cdot Cfbuckxy$$

$$L_{xz} = L \cdot Cfbuckxz$$

where:

Cfbuckxy	Buckling factor in plane XY.
Cfbuckxz	Buckling factor in plane XZ.

For the calculation of the elastic critical moment for lateral-torsional buckling, Mcr, the following equation shall be used. This equation is *only valid for uniform symmetrical cross-sections about the minor axis* .

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(kL)^2}\left\{\left[\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z} + \left[C_2 Z_g - C_3 Z_j\right]^2\right]^{1/2} - \left[C_2 Z_g - C_3 Z_j\right]\right\}$$

$$Z_j = Z_s - \frac{0.5}{I_y}\int_A (y^2 + z^2)\ z\, dA$$

where:

M_{cr}	Elastic critical moment for lateral-torsional buckling.
C_1, C_2 y C_3	Factors depending on the loading and end restraint conditions.
k y k_w	Effective length factors.
E	Elasticity modulus.
I_y	Moment of inertia about the principal axis.
I_y	Moment of inertia about the minor axis.
L	Length of the member between end restraints.
G	Shear modulus.
Z_g	$Z_a - Z_s$
Z_a	Coordinate of the point of load application. By default the load is applied at the center of gravity, therefore: $Z_a = 0$.
Z_s	Coordinate of the shear center.
A	Cross-section area.

Factors C and k are read from the properties at structural element level.

The integration of the previous equation is calculated as a summation extending to each plate. This calculation is accomplished for each plate according to its ends coordinates: y_1, z_1 and y_2, z_2 and its thicknesses.

$$\int_A (y^2 + z^2)\, z\, dA = \sum_{i=1}^{n\ plates} S_i{}^* \int_{L_i} (y^2 + z^2)\, z\, dl$$

where:

s_i = thickness of plate i

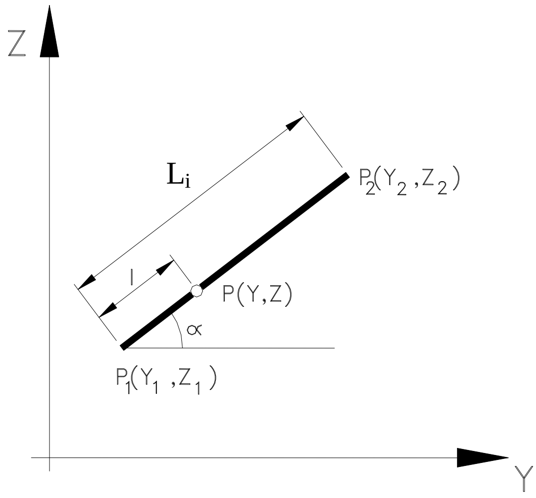
$$dA = s_i \ * dl$$

$$y = y_1 + I^* \ \cos\alpha$$

$$z = z_1 + I^* \sin\alpha$$

$$\alpha = \ \arctan\frac{z_2 - z_1}{y_2 - y_1}$$

$$L_i = \sqrt{(y_1 - y_2)^2 + \ (z_1 - z_2)^2} = \text{ plate width}$$



Chapter 8
Seismic Design

8.1. Introduction

Seismic design with CivilFEM provides the user a set of tools to analyze seismic action on structures, according to the provisions of:

- User response spectrum
- Eurocode 8
- The Spanish code NCSE-02

Aspects considered for calculations:

1. Spectrum definition.
2. Calculation of mode shapes.
3. Modal combination.

8.2. Spectrum Calculation according to Eurocode 8

8.2.1. Input data

The data required to define the response spectrum for Eurocode 8 (EN-1998-1:2004) are listed below:

AG	Design ground acceleration for the reference return period [a_g].
SPTYPE	Spectrum type defined. Elastic or Design.
C	Ground type coefficient [S].
QH	Horizontal behavior factor [q_h].
QV	Vertical behavior factor [q_v].
DMPRAT	Ratio of viscous damping ratio of the structure [ξ] (in %).

Once the data have been input, the fraction α is obtained by dividing the design ground acceleration a_g by the gravity acceleration g, displayed below:

$$\alpha = \frac{a_g}{g}$$

8.2.2. Spectrum calculation

8.2.2.2 Horizontal Spectra

The values of the parameters which describe the horizontal response spectrum are given in the table below in accordance with the type of subsoil and type of spectrum:

Subsoil types for Type 1 Elastic	**A**	**B**	**C**	**D**	**E**
S	1.00	1.20	1.15	1.35	1.40
β_0	2.50	2.50	2.50	2.50	2.50
T_B(s)	0.15	0.15	0.20	0.20	0.15
T_C(s)	0.40	0.50	0.60	0.80	0.50

Subsoil types for Type 1 Elastic	**A**	**B**	**C**	**D**	**E**
T_D(s)	2.0	2.0	2.0	2.0	2.0
T_E(s)	3.50	3.50	3.50	3.50	3.50
K_{d1}	2/3	2/3	2/3	2/3	2/3
K_{d2}	5/3	5/3	5/3	5/3	5/3
K_1	1.00	1.00	1.00	1.00	1.00
K_2	2.00	2.00	2.00	2.00	2.00

Subsoil types for Type 2 Elastic	**A**	**B**	**C**	**D**	**E**
S	1.00	1.35	1.50	1.80	1.60
β_0	2.50	2.50	2.50	2.50	2.50
T_B(s)	0.05	0.05	0.10	0.10	0.05
T_C(s)	0.25	0.25	0.25	0.30	0.25
T_D(s)	2.0	2.0	2.0	2.0	2.0
T_E(s)	3.50	3.50	3.50	3.50	3.50
K_{d1}	2/3	2/3	2/3	2/3	2/3
K_{d2}	5/3	5/3	5/3	5/3	5/3
K_1	1.00	1.00	1.00	1.00	1.00
K_2	2.00	2.00	2.00	2.00	2.00

If the spectrum is elastic, the ordinates of the horizontal spectrum are obtained as follows:

$$S_d(T) = \alpha \cdot S \cdot \left[1 + \frac{T}{T_B}\left(\frac{\beta_0}{q} - 1\right)\right] \qquad T_A < T \leq T_B$$

$$S_d(T) = \alpha \cdot S \cdot \frac{\beta_0}{q} \qquad T_B < T \leq T_C$$

$$S_d(T) = \alpha \cdot S \cdot \frac{\beta_0}{q}\left[\frac{T_C}{T}\right]^{K_{d1}} \qquad T_C < T \le T_D$$

$$S_d(T) = \alpha \cdot S \cdot \frac{\beta_0}{q}\left[\frac{T_C}{T_D}\right]^{K_{d1}}\left[\frac{T_D}{T}\right]^{K_{d2}} \qquad T_D < T \le T_E$$

Where:

q	=	behavior factor. The values for this factor differ for the horizontal seismic action and for the vertical seismic action. Therefore, this factor assumes two different values q_h and q_v depending on the material type.
K_{d1}, K_{d2}	=	exponents which influence the shape of the design spectrum for a vibration period greater than T_C, T_D respectively.

If the spectrum is the design spectrum, the ordinates of the horizontal spectrum are obtained as follows:

$$S_d(T) = a_g \cdot S \cdot \left[\frac{2}{3} + \frac{T}{T_B}\left(\frac{2.5}{q} - \frac{2}{3}\right)\right] \qquad 0 < T \le T_B$$

$$S_d(T) = a_g \cdot S \cdot \frac{2.5}{q} \qquad T_B < T \le T_C$$

$$S_d(T) = a_g \cdot S \cdot \frac{2.5}{q}\left[\frac{T_C}{T}\right] \qquad T_C < T \le T_D$$

$$\ge \beta \cdot a_g$$

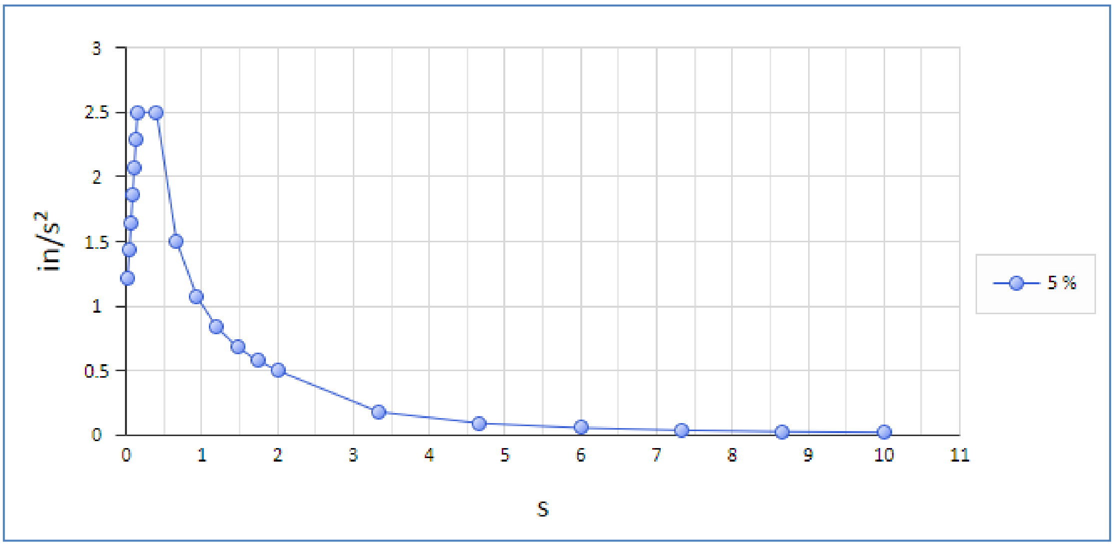
$$S_d(T) = a_g \cdot S \cdot \frac{2.5}{q}\left[\frac{T_C \cdot T_d}{T^2}\right] \qquad T_D < T$$

$$\ge \beta \cdot a_g$$

Where:

q	=	behavior factor. The values of this factor are different for the horizontal seismic action and for the vertical seismic action. Therefore, this factor assumes two different values q_h and q_v

depending on the material type.



8.2.2.2 Vertical Spectra

Subsoil types	Type 1	Type 2
a_{vg}/a_g	0.90	0.45
$T_B(s)$	0.05	0.05
$T_C(s)$	0.15	0.15
$T_D(s)$	1.0	1.0

For the elastic spectrum, the ordinates of the vertical spectrum are obtained as follows:

$$S_{ve}(T) = a_{vg} \cdot \left[1 + \frac{T}{T_B}(\eta \cdot 3.0 - 1)\right] \qquad 0 < T \leq T_B$$

$$S_{ve}(T) = a_{vg} \cdot \eta \cdot 3.0 \qquad T_B < T \leq T_C$$

$$S_e(T) = a_{ve} \cdot \eta \cdot 3.0 \cdot \left[\frac{T_C}{T}\right] \qquad T_C < T \leq T_D$$

$$S_{ve}(T) = a_{vg} \cdot \eta \cdot 3.0 \cdot \left[\frac{T_C \cdot T_D}{T^2}\right] \qquad T_D < T \leq 4s$$

Where:

η	=	damping correction factor with reference value of η = 1 for a viscous damping of 5%.

To obtain the vertical design spectrum, the same expressions of the horizontal design spectrum are utilized with S=1 and the recommended values of a_g ,Tc and Td in the vertical elastic response spectra table.

8.3. Spectrum Calculation according to NCSE-2002

8.3.1. Input data

The data required to define the response spectrum are listed below:

AB	Ratio of the basic seismic acceleration to the gravity acceleration $\left[\frac{a_b}{g}\right]$.
SPTYPE	Spectrum type to be calculated (Linear or Simplified).
RO	Dimensionless risk coefficient [ρ].
C	Coefficient of the ground type.
K	Coefficient of contribution.
OMEGA	Structure type [Ω].
MU	Ductility coefficient [μ].

Once the data have been input, T_A and T_B are calculated by:

$$T_A = k \cdot \frac{c}{10}$$

$$T_B = k \cdot \frac{c}{2.5}$$

In addition, the amplification coefficient of soil S is calculated by:

$$S = \frac{C}{1.25} \qquad \text{for } \rho \cdot a_b \leq 0.1 \cdot g$$

$$S = \frac{C}{1.25} + 3.33\left(\rho \cdot \frac{a_b}{g} - 0.1\right) \cdot \left(1 - \frac{C}{1.25}\right) \qquad \text{for } 0.1 \cdot g \leq \rho \cdot a_b \leq 0.4 \cdot g$$

$$S = 1.0 \qquad \text{for } 0.4 \cdot g \leq \rho \cdot a_b$$

Finally, the modification factor of the spectrum υ is calculated as a function of the damping by:

$$\nu = (5/\Omega)^{0.4}$$

8.3.2. Spectrum calculation

The value of the ordinate of the spectrum α(T) is defined as the quotient of the absolute acceleration of an elastic linear oscillator (S_a) and the maximum acceleration of the movement applied on its basis (a):

$$\alpha(T) = \frac{S_a}{a}$$

The design spectrum S_d is given by (Art. 3.6.2.2):

$$S_d = (T_i) = \alpha_i \cdot S \cdot \rho \cdot g \cdot \left(\frac{a_b}{g}\right)$$

where:

S is the soil amplification factor

$\alpha_i = \alpha(T_i) \cdot \beta$ if $T_i \geq T_A$

$\alpha_i = 1 + (2.5\beta - 1) \cdot \frac{T_i}{T_A}$ if $T_i \leq T_A$

$$\beta = \frac{\nu}{\mu}$$

$\alpha(T_i)$ is the normalized spectrum of elastic response (Art. 2.3):

$\alpha(T) = 1 + 1.5 \cdot T/T_A$ if $T < T_A$

$\alpha(T) = 2.5$ if $T_A < T < T_B$

$\alpha(T) = K \cdot C/T$ if $T > T_B$

A total of 20 values of the period T are calculated as specified below:

1. The first 10 values for periods T_i between $1/10 \cdot T_A$ and T_A are calculated by:

$$T_i = \frac{i \cdot T_A}{10}$$

where: i = 1 to 10

a. If the spectrum type entered is linear, then the ordinates of the spectrum $\alpha(T_i)$ are obtained with the following equation:

$$\alpha(T) = 1 + 1.5 \cdot T / T_A$$

where: i = 1 to 10

b. If the spectrum type is simplified, then the ordinates of the spectrum $\alpha(T_i)$ are obtained by:

$$\alpha(T_i) = \alpha(T_A)$$

where: i = 1 to 10

2. The remaining values of the period and of the ordinates of both spectrum types are calculated as follows:

a. Values of the period:

$$T_i = \frac{10 \cdot T_B}{(21 - i)}$$

where: i = 10 to 20

b. Values of the ordinates of the spectrum, using the following equation:

$$\alpha(T_i) = \alpha(T_A)\ {}^{T_B}/_{T_i}$$

where: i = 10 to 20

Once the values of the period and the ordinates of the spectrum are calculated, the spectral accelerations are obtained for two orthogonal directions consisting of the X and Y global axes by applying:

$$S_d(T_i)_x = S_d(T_i)$$

$$S_d(T_i)_y = S_d(T_i)$$

For vertical movements, the ordinates of the spectrum will be reduced by a factor of 0.7.

$$S_d(T_i)_z = 0.70 \cdot S_d(T_i)$$

8.4. Combination of modes and directions

The modes *r* and ϕ_i and the natural vibration frequencies ω_i of the structure are calculated by performing the modal analysis using the Block Lanczos method.

Once the vibration modes are obtained they are combined to obtain the response of the structure.

CivilFEM provides two options for the combination of modes:

8.4.1. Complete Quadratic Combination Method (CQC)

The total modal response is calculated by:

$$Rm = \sqrt{\sum_{1}^{N}\sum_{1}^{N}\varepsilon_{ij}\left|R_i \cdot R_j\right|}$$

Where:

N= total number of modes

ε_{ij}= Coupling coefficient.

R_i= modal response in the ith mode.

R_j=modal response in the jth mode.

Coupling coefficient is evaluated by means of:

$$\varepsilon_{ij} = \frac{8 \cdot \sqrt{\xi_i \cdot \xi_j} \cdot (\xi_i + r_{ij} \cdot \xi_j) \cdot (r_{ij})^{3/2}}{\left(1-(r_{ij})^2\right) + 4 \cdot \xi_i \cdot \xi_j \cdot r_{ij} \cdot \left(1+(r_{ij})^2\right) + 4 \cdot (\xi_i^2 + \xi_j^2) \cdot (r_{ij})^2}$$

Where:

$$r_{ij} = \frac{w_j}{w_i}$$

ξ_i = Damping ratio of the ith mode.

ξ_j = Damping ratio of the jth mode.

8.4.2. Square Root of the Sum of the Squares

The SRSS method is from the NRC Regulatory Guide, for this case, the total mode response is performed by:

$$Rm = \sqrt{\sum_{1}^{N} R_i^2}$$

8.4.3. Combination of maximum modal values

Once the mode combination is performed, then the maximum modal responses from the three directions must be combined as well. Two methods may be used:

8.4.3.1 Square Root of the Sum of the Squares

$$(R^{max})_i = \sqrt{\sum_{\alpha} ((R_\alpha^{max})_i)^2}$$

Where:

i=X, Y and Z direction (the three components are calculated separately).

8.3.3.2. Newmark method

The maximum seismic response attributable to seismic loading in three orthogonal directions is given by the following equations:

$$\mathbf{R_1 = P \cdot R_X + S \cdot R_Y + S \cdot R_Z}$$
$$\mathbf{R_2 = S \cdot R_X + P \cdot R_Y + S \cdot R_Z}$$
$$\mathbf{R_3 = S \cdot R_X + S \cdot R_Y + P \cdot R_Z}$$
$$\mathbf{R_4 = -P \cdot R_X - S \cdot R_Y - S \cdot R_Z}$$
$$\mathbf{R_5 = -S \cdot R_X - P \cdot R_Y - S \cdot R_Z}$$
$$\mathbf{R_6 = -S \cdot R_X - S \cdot R_Y - P \cdot R_Z}$$

Where P and S are the primary and secondary combination factor defined by the user.

Chapter 9
Miscellaneous Utilities

9.1. Parameters and Expressions

Parameters are variables and their type must be declared in the ***Parameter list*** window. The available types are the following:

- Real number.
- Integer number.
- 2D point (x, y).
- 2D vector (x, y).
- 3D point (x, y, z).
- 3D vector (x, y, z).

Parameters can be used (instead of a literal number) as a property to any CivilFEM property; the parameter is evaluated and its current value is used for that property (i.e. Material's Young Modulus).

Parameters can be defined as expressions made up of constants, operators, functions and other previously evaluated parameters.

9.1.1. Naming rules

Parameter names must start with a letter and can only contain letters, numbers, and underscores. All letters included in the Unicode Standard scripts are permitted. Example:

	Parameter
area	**A**
circle area	**Ac**
box number 3	**box_3**
steel thermal expansion	**α_steel**

Distinction is made between upper and lower case letters. Example: Different parameter names because of upper and lower cases.

	Parameter
area	**a**

angle	**A**

Reserved words cannot be used as parameter names. Reserved words are the function and constant names defined in the next sections. Example: Invalid parameter name.

	Parameter
maximum area	~~**max**~~
tangent line	~~**tan**~~

Full list of **reserved** words:

AND, E, NOT, OR, PI, pi, abs, acos, acosd, acosu, arccos, arccosd, arccosu, arcsin, arcsind, arcsinu, arctan, arctand, arctanu, asin, asind, asinu, atan, atan2, atand, atanu, ceil, cos, cosd, cosh, coshd, coshu, cosu, cross, distance, division, dot, e, exp, fact, factorial, floor, fmod, g_SI, g_ft, ln, log, max, middlePoint, min, mod, norm, oneX2d, oneX3d, oneY2d, oneY3d, oneZ3d, ones2d, ones3d, percent, percentage, pow, projectionXY, projectionXZ, projectionYZ, rotate, rotateCW, rotateX, rotateXCW, rotateY, rotateYCW, rotateZ, rotateZCW, round, roundUp, sin, sind, sinh, sinhd, sinhu, sinu, sqrt, sum, tan, tand, tanh, tanhd, tanhu, tanu, trunc, truncate, unitary, zero2d, zero3d, Σ, π, √

There is also a list of parameter names already used for **predefined local parameters** (parameters for individual material, section or other properties…) that neither can be used for global parameters:

b, bfbot, bftop, h, mc, od, tf, tfbot, tftop, tk, tw

9.1.2. Constants

Numbers assigned to parameters.

	Parameter	Input
area	A	5.1
number of bars	n_bars	24

Reserved words associated with a specific numerical value.

Predefined constants		
Constant Name	Value	Description
π / **PI, pi**	3.1415926	The ratio of a circle's circumference to its diameter.
e	2.71828	Euler's constant.
g_SI	9.80665	Earth's gravity in the International System of Units.
g_ft	32.174	Earth's gravity in Imperial Units (feet per square second).

A predefined constant assigned to a parameter.

	Parameter	Input
Earth's gravity	g	g_SI

9.1.3. Operators

Arithmetic operators:

- Addition (**+**)
- Subtraction (**-**)
- Multiplication (*****)
- Division (**/**)
- Module (***mod***): the remainder of the first number when divided by the second.
- Power (^)

Example:

Parameter	Input	Output
A	41	41
B	7	7

C	A*B	287
D	A ***mod*** B	6

Relational and Equality operators:

- Less than (**<**): “A<B” returns true if parameter “A” is less than parameter “B”.
- Greater than (**>**): “A>B” returns true if parameter “A” is greater than parameter “B”.
- Less than or Equal to (**<=**): “A<=B” returns true if parameter “A” is equal or less than parameter “B”.
- Greater than or Equal to (**>=**): “A>=B” returns true if parameter “A” is equal or greater than parameter “B”.
- Equal (**=**): “A=B” returns true if parameter “A” is equal to parameter “B”.
- Different from (**<>**): “A<>B” returns true if parameter “A” is different from parameter “B”.

Logical operators:

- NOT (**!**): logical negation on a Boolean expression.
- AND (**&&**): logical conjunction on two Boolean expressions.
- OR (**||**): logical disjunction on two Boolean expressions.

Example:

<table>
<tr><th>A</th><th>B</th><th>A!</th><th>A && B</th><th>A || B</th></tr>
<tr><td>T</td><td>T</td><td>F</td><td>T</td><td>T</td></tr>
<tr><td>T</td><td>F</td><td>F</td><td>F</td><td>T</td></tr>
<tr><td>F</td><td>T</td><td>T</td><td>F</td><td>T</td></tr>
<tr><td>F</td><td>T</td><td>T</td><td>F</td><td>F</td></tr>
</table>

Where “T” means true and “F” means false.

Operator precedence:

When several operations occur in an expression, each part is evaluated and resolved in a predetermined order called operator precedence. Parentheses can be used to override the order of precedence and force operations within parentheses to be evaluated before those outside. Within parentheses, however, normal operator precedence is maintained.

Operators are sorted in precedence levels from highest to lowest precedence as shown in the following list. When two or more operators in an expression have the same precedence level, operations are evaluated from left to right.

I. Power (^).
II. Multiplication (*****), division (**/**), module (***mod***).
III. Addition (**+**), subtraction (**-**).
IV. Less than (**<**), less than or equal to (**<=**), greater than (**>**), greater than or equal to (**>=**), equal to (**=**), different from (**<>**).
V. NOT (**!**).
VI. AND (**&&**).
VII. OR (**||**).

Example:

Parameters	Input	Output
A	2+2*3	8
B	(2+2)*3	12
C	A/2^3+5	6

9.1.4. Functions

Absolute value (***abs***).

Parameters	Input	Output
A	-3	-3
B	**abs**(A)	**3**

Square root (***sqrt***)

Parameters	Input	Output
A	16	16
B	**sqrt**(A)	**4**

Common Logarithm (***log***)

Parameters	Input	Output
A	1000	1000
B	**log**(A)	**3**

Natural Logarithm (***ln***)

Parameters	Input	Output
A	e*e	7.3890561
B	**ln**(A)	**2**

Trigonometric functions: Sine (***sin***), Cosine (***cos***), Tangent (***tan***):

Parameters	Input	Output	Unit
B	35	35	deg
C	**sin** (B)	**0.5735**	**-**
D	**cos** (B)	**0.8191**	**-**
E	**tan** (B)	**0.7002**	**-**

IMPORTANT: trigonometric functions evaluate in the **units of the model** (degrees by default), and all angular parameters will be converted properly to the units of the model (no matter if they are in radians or degrees, proper conversions will be applied). Undefined parameters or dimensionless values will be evaluated in the units of the model, but the alternative functions: **sinr**, **cosr**, **tanr** are available to evaluate them in radians, or: **sind**, **cosd**, **tand** to evaluate them in degrees, or: **sinu**, **cosu**, **tanu** to evaluate them in the user units defined in the model (work the same as: sin, cos, tan).

Parameters	Input	Output	Unit
A	pi/2	1.5708	rad
B	90	90	deg
C	pi/2	1.5708	-
D	90	90	-
E	**sin** (A)	**1.0**	**-**
F	**sin** (B)	**1.0**	**-**
G	**sinr** (C)	**1.0**	**-**
H	**sind** (D)	**1.0**	**-**

Inverse trigonometric functions: Arcsine (**asin**), Arccosine (**acos**), Arctangent (**atan**).

The returned values will be in the units of the model (degrees by default). If the results are assigned to an angular parameter, they will be shown in the units of the parameter (proper conversion will be applied). If the functions are going to be operated with undefined parameters or dimensionless values, then the results will be operated in the units of the model, but the alternative functions: **asinr**, **acosr**, **atanr** are available to return results in radians, or: **asind**, **acosd**, **atand** to return results in degrees, or: **asinu**, **acosu**, **atanu** to return results in the user units defined in the model (work the same as: asin, acos, atan).

Parameters	Input	Output	Unit
B	0.5	0.5	-
C	**asin** (B)	**30**	deg
D	**acos** (B)	**60**	deg
E	**atan** (B)	**26.565**	deg
F	**asinr** (B)	**0.5236**	rad

Hyperbolic functions: Hyperbolic sine (***sinh***), Hyperbolic cosine (***cosh***), Hyperbolic tangent (***tanh***).

Minimum (***min***)

Parameters	Input	Output
A	1	1
B	0	0
C	min(A,B)	0

Maximum (***max***)

Parameters	Input	Output
A	1	1
B	0	0
C	***max***(A,B)	1

Round to the nearest integer (***round***)

Parameters	Input	Output
A	1.3	1.3
B	1.5	1.5
C	-1.5	-1.5
D	-2.6	-2.6

As	***round*(A)**	**1**
Bs	***round*(B)**	**2**
Cs	***round*(C)**	**-2**
Ds	***round*(D)**	**-3**

<u>Truncate to zero decimal digits</u> (***truncate***)

Parameters	Input	Output
A	-1.45	-1.45
B	0.7	0.9
CT	**truncate**(A)	**-1**
DT	**truncate**(B)	**0**

<u>Factorial</u> (***fact***)

Parameters	Input	Output
B	3	3
C	***fact***(B)	6

<u>Distance between two points</u> (***distance***): distance(p1,p2)

<u>Dot product of vectors</u> (***dot***): dot(v1,v2) = v1 * v2

<u>Cross product of vectors</u> (***cross***): cross(v1,v2) = v1 ^ v2 = v1 x v2

<u>Map a real number to the smallest following integer</u> (***ceil***): The ceil of 2.8 is 3.0. The ceil of -2.8 is -2.0.

<u>Map a real number to the largest previous integer</u> (***floor***): The floor of 2.8 is 2.0. The floor of -2.8 is -3.0.

<u>Remainder of the integer division of two real numbers</u> (***fmod***): The remainder of -10.00 / 3.00 is -1.0. fmod(-10.0,3.0)=-1.0

<u>Middle point of two points</u> (***middlePoint***).

<u>Percentage</u> (***percent*** or ***percentage***): x% = percent(x) = percentage(x) = x/100.0

<u>Exponentiation or power</u> (***pow***): pow(x,y) = x^y = x^y

<u>Projection of point into XY plane</u> (***projectionXY***): projectionXY((1,1,1)) = (1,1,0)

Projection of point into XZ plane (***projectionXZ***): projectionXZ((1,1,1)) = (1,0,1)

Projection of point into YZ plane (***projectionYZ***): projectionYZ((1,1,1)) = (0,1,1)

Round to the closest integer (***round***).

Round to the next highest integer (***roundUp***).

Summatory of components of a vector (***Σ***): Σv = v.x + v.y + v.z

9.1.5. Units

The user has great flexibility in specifying parameter units. However, it is **strongly recommended to take into account the following notes** in order to avoid unit conversion problems:

- All dimensional parameters, that is, all variables where a unit type was assigned (length, mass, etc.) are correctly **converted to consistent units** before operation. Problems may arise when operating with variables with unknown unit types.
  - By default, parameters are of "**undefined**" unit type, which means that those parameters may have units but are not specified yet. Those parameters are not converted and will be operated as dimensionless. As all the other parameters are converted to consistent units, the effect of operating the undefined parameters with them would be equivalent to considering the undefined parameters to be defined in the same consistent units. Users may be aware of this to avoid unintended results.
  - **Dimensionless** unit type parameters should be reserved to truly dimensionless variables.
- The key is to know that, before operating, all dimensional parameters are converted to a **consistent units system based on the Principal Units of the Model** (Environment -> Model Configuration -> Units), not to the visualization units.
  - The Derived Units of the model can be customized by the user for visualization or other purposes, and **may be unconsistent** with the principal units… So, only the Principal Units of the model are considered for the consistent units system used on calculations.
  - For the Derived Units of the model, the associated **consistent units are shown between brackets**. For example, if the units of the model are:

meters ("m"), seconds ("s"), and kilometers per hour ("km/h"), then the derived consistent units will be shown as "**[**m/s**]**".

- The usage of undefined and/or dimensionless parameters or values in a formula, that is also using other parameters with assigned unit types, may lead to misinterpretation in formula evaluation **if the unit system is changed** or the model is imported in a different unit system (for example, the change of a model in Imperial Units to International Units).
  - To avoid this kind of situations, **CivilFEM "remembers" the units that were used to define each formula**. If those original units are no longer the same as the units of the model, then, to avoid misinterpretations, **the original units will be shown after a semicolon** and used to evaluate the formula as originaly intended.
  - **As a rule**, formulas are always **evaluated in the consistent unit system** based on the principal units of the model, **unless** other interpretation units are specified after **a semicolon in the formula**. In that case, units after the semicolon will be used for the consistent unit system of that particular formula.
  - The effect of this is **formulas will never be misinterpreted due a change in the units system**, even when using undefined or dimensionless parameters or values, even when mixing formulas from different unit systems, as the values will still be calculated as originaly intended and properly converted to the new unit system.
  - The idea behind this innovation is to **preserve the real magnitudes** of the model, no matter what changes in the unit visualization. If this is not what intended, the user may remove the units after the semicolon in the formula.
  - For **example**, in a model defined with Imperial Units with a parameter "x" in inches, you may use the formula "1+x" so the dimensionless constant "1" actually operates as 1 inch. When you import that model in another one with International Units, the parameter "x" would be converted to meters by default, but the formula will be shown as "1+x**;in**" (which means "x" must be converted to inches before operating, as originaly intended) and the result in inches will be properly converted to meters.

Example: Unit conversion in a mixed strategy (it is recommended to avoid this case, specifying the units whenever is possible).

- The following parameters are defined by the user:

	Parameter	Magnitude	Unit entered by the User
Rectangle side 1	R1	Length	cm
Rectangle side 2	R2	Undefined	?
Rectangle perimeter	Rp	Undefined	?
Triangle side A	tA	Length	in
Triangle side B	tB	Length	ft
Triangle side C	tC	Length	in
Triangle perimeter	tPer	Length	in

For those parameters with no specific unit defined by the user, the corresponding unit of the model (global system unit) will be assigned. In this example, the International Units System is considered to be the unit system of the model (global unit system). Therefore, *R2* and *Rp* should be in *m*.

– The user enters the parameter inputs as follow:

	Parameters	Input	Units
Rectangle side 1	R1	5	cm
Rectangle side 2	R2	6	? *(m)*
Rectangle perimeter	Rp	2*(R1 + R2)	? *(m)*
Triangle side A	tA	3	in
Triangle side B	tB	7	ft
Triangle side C	tC	4	in
Triangle perimeter	tPer	tA + tB + tC	in

– The parameter inputs are **evaluated in a consistent unit system** based on the principal units of the model (*meters*, in this case; **no unit conversion is performed on parameters *R2* and *Rp***):

	Param	Input	Consistent values	Output
Rectangle side 1	R1	5 cm	0.05 m	5 cm
Rectangle side 2	R2	6 *(m)*	6	6 *(m)*
Rect. perimeter	Rp	2*(R1 + R2)	12.1	12.1 *(m)*
Triangle side A	tA	3 in	0.0762 m	3 in
Triangle side B	tB	7 ft	2.1336 m	7 ft
Triangle side C	tC	4 in	0.1016 m	4 in
Tri. perimeter	tPer	tA + tB + tC	2.3114 m	91 in

As you can see, the rectangle and triangle perimeters are **properly computed**:

Parameter list

	Parameter...	Parameter type	Unit type	Coordin...	Formula	Value	Unit
	R1	Real number	Length			5	cm
	R2	Real number	Undefined			6	?
	Rp	Real number	Undefined		2*(R1 + R2)	12.1	?
	tA	Real number	Length			3	in
	tB	Real number	Length			7	ft
	tC	Real number	Length			4	in
	tPer	Real number	Length		tA + tB + tC	91	in

– Now, if the **unit system of the model** (global system of units) is **set to Imperial Units (inches)**, the parameters expressed in the units of the model will be computed in a consistent unit system based on the Imperial Units, unless other units are specified after a semicolon. That **could be a problem for the undefined parameters** of this example, as proper conversions could only be performed on parameter with properly defined units, but all formulas involving parameters with units will "remember" those original units to evaluate as originally intended (so those original units will be shown after a **semicolon**). Again, the rectangle and triangle perimeters are **properly computed**:

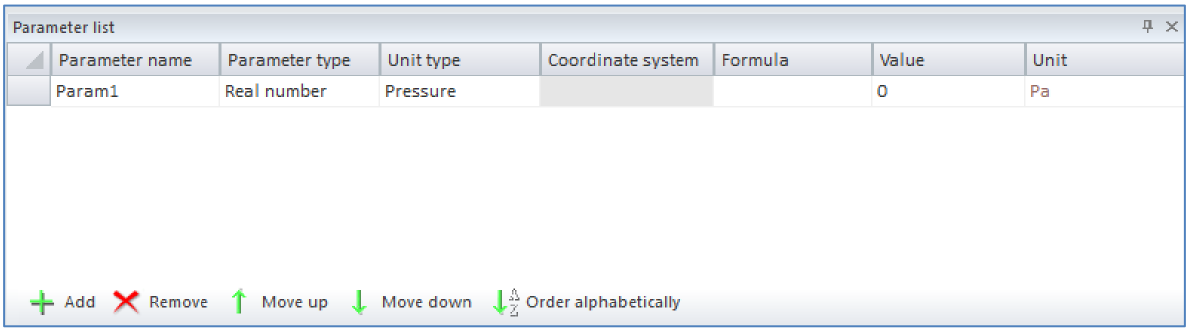
Parameter list

	Parame...	Parameter type	Unit type	Coordin...	Formula	Value	Unit
	R1	Real number	Length			5	cm
	R2	Real number	Undefined			6	?
	Rp	Real number	Undefined		2*(R1 + R2);m	12.1	?
	tA	Real number	Length			3	in
	tB	Real number	Length			7	ft
	tC	Real number	Length			4	in
	tPer	Real number	Length		tA + tB + tC;m	91	in

Anyway, it is **strongly recommended to define units for the parameters that store formula results** (like "*Rp*" in the example), so that CivilFEM can avoid problems with undefined parameters.

9.1.6. Parameter List Window

The user can parameterize any part of the modeling process using the Parameter List window. The parameter list window has the following distribution:



The Parameter List window columns are:

- **Parameter name**: In this column the parameter name will be set. Any occurrence of the parameter name inside any CivilFEM form will be detected and CivilFEM will substitute the parameter for its value or formula.
- **Parameter type**: The user can choose between the following parameter types: 2D Point, 2D Vector, 3D Point, 3D Vector, Integer number, Real number. This is useful when the parameterized value is of the point or vector class. The user just needs to define the point instead of its two or three individual components.
  - There is a key **difference between a point and a vector**: when changing from one coordinate system to another, points are transformed taking into account the change of origin and vectors are unaffected by the origin.
- **Unit type**: A list of unit types is available so any unit can be chosen so the user can define its unit later.
- **Coordinate system**: As different coordinate sytems can be defined in CivilFEM, this column lets the user choose the system used to define a point or vector parameter. This *makes the point or vector **referenced** to the coordinate system*, so modifying the coordinate system will modify the global position of the point or vector (but not its coordinates in the local coordinate system).
- **Formula**: The user is not restricted to entering values for defining a parameter. An arbitray formula (the formula must begin with an equal sign) can be entered to define the final value. This is a very powerful feature as another parameter can be used inside a formula, linking several parameters in the definition. A whole model can be changed in this way just by changing a single value.
- **Value**: A fixed numerical value can be entered directly. If the user has used the formula option, the calculated value will be displayed here.

- **Unit**: The unit can be chosen using this option, so the user has total flexibility when defining the parameter using a mix unit approach.

The following figures show an example of the Parameter List window usage in conjunction with geometry creation.

Parameter list

Parameter name	Parameter type	Unit type	Coordinate sys...	Formula	Value	Unit
Vertical	Real number	Length			5	m
Horiz	Real number	Length		Vertical*1.5	7.5	m
P1	3D Point	Length	Global Cartesian	(Vertical, Horiz,0)	(5, 7.5, 0)	m

In the previous figure, three parameters are created. Vertical is a real number of the type length that is defined using a direct value of 5. Then a Horiz parameter is created using a formula: =Vertical*1.5 . The resulting value is 7.5. Finally a 3D point parameter called P1 is created using the formula =(Vertical, Horiz, 0) . The resulting point can be directly used when creating geometry, entering the point with the formula =P1 .

Point	
Name	Point
Coordinates	(5, 7.5, 0) m P1

9.2. CivilFEM Python Programming

Python is an interpreted, interactive object-oriented programming language sometimes compared to Perl, Java, and Tcl. It has interfaces to IP networking, windowing systems, audio, and other technologies. Integrated with CivilFEM, it provides a more powerful scripting language than procedure files since it contains conditional logic and looping statements such as if, while, and for.

To start using Python in CivilFEM just activate the ***Script editor*** window.

Always refer to ***CivilFEM Python Manual*** and ***CivilFEM Script Manual*** to know all available commands.

One of the biggest differences between the Python language and other programming languages is that Python does not denote blocks of code with reserved words or symbols such as if..then..endif (FORTRAN) or if { ... } (curly braces in C). Instead, indenting is used for this purpose. For example, the take following block of FORTRAN code:

```
if(jtype.eq.49) then
  ladt=idt+lofr
endif
```

The block of FORTRAN code would need to be coded as follows in Python:

```
if jtype == 49:
  ladt=idt+lofr
```

Python matches the amount of indenting to the block of code. The colon at the end of the if statement denotes that it is a compound statement. All the lines that are to be in that block of code need to be at the same indent level. The block of code is ended when the indenting level returns to the level of to the compound statement. The examples in the following chapters will show you more about the Python syntax.

9.2.1 Python Data Types

When programming in Python, you don't explicitly declare a variable's data type. Python determines the data type by how the variable is used. Python supports the following implied data types:

- Basic Data Types:

1. **String**: A character string similar to the char data in C and character in FORTRAN. A string may be specified using either single or double quotes.
2. **Float:** A floating point number similar to the double data type in C and the real*8 data type in FORTRAN.
3. **Integer**: An integer or fixed point number similar to the long int data type in C and the integer*8 data type in FORTRAN.

- Extended Data Types:

1. **List:** A Python list is essentially a linked list that can be accessed like an array using the square bracket operators [ ]. The list can be composed of strings, floats, or integers to name a few.

The material covered in this tutorial is very basic and should be easy to access and understand for the first time Python user.
A multi-dimension list is created by first creating a single dimensional list, and then creating the other dimensions, as follows (a 3x2 array):

```
A = [None] * 3
for i in range(3)
  A[i] = [None] * 2
```

Always refer to ***CivilFEM Python Manual*** to know all available commands.

9.2.2 Python Example

Python files have .py extension.

- Polyline:

```
1.   # Points
2.   p1 = pnt("Point1", [1,2,3])
3.   p2 = pnt("Point2", [2,3,4])
4.   p3 = pnt("Point3", [3,4,5])
5.   p4 = pnt("Point4", [4,5,6])
6.   # Polyline
7.   polyline([p1, p2, p3, p4])
```

Lines 1-5: To add a commentary, the # symbol must be inserted first. To create a point command, createPoint (or alias pnt) is used. If ***CivilFEM Python Manual*** is opened then createPoint needs two arguments:

1) GeomName (str): Name.

2) Pnt (Point): Coordinates of the point.

Each point is saved into a variable to be used later as a list (p1, p2, …)

Lines 6-7: To create a polyline, command createPolyline (or alias polyline) is used. If ***CivilFEM Python Manual*** is opened then createPolyline needs two arguments:

1) GeomName (str): Name.

2) POINT ([Entity]): List of points (between square brackets [ ]) to define the polyline.

The material covered in this tutorial is very basic and should be easy to access and understand for the first time Python user.

A multi-dimension list is created by first creating a single dimensional list, and then creating the other dimensions, as follows (a 3x2 array):

```
A = [None] * 3
for i in range(3)
  A[i] = [None] * 2
```

Always refer to ***CivilFEM Python Manual*** and ***CivilFEM Script Manual*** to know all available commands.